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Economic order quantity for growing items in the presence of mortality

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Abstract – Growth depicts the behavior of a class of inventories whose weight and size increase during their storage. Although the phenomenon is fundamental in the food industry, it still faces a lack of academic heed. Our research sheds new light on this area by illustrating the inventory system of a rearing farm that breeds fast-growing newborn animals (like broiler chickens) and finally slaughters them. The items are prone to different diseases during their growth which can lead to their death. This is modeled by applying a mortality rate that can be controlled by investing in preventive practices. The slaughtered items (such as chicken meat) that fall into the category of deteriorating inventory are quality controlled and exploited to satisfy demand. An analytic solution approach is developed to derive the optimal order quantity of newborn animals, their growth period, and action level for preventive practices. Experimental results show that choosing a hatchery with the best purchasing cost should be the first priority of the system to manage its costs. It is also depicted that the breeding period is not influenced by all input parameters of the model, and the nature of the items plays a key role in this regard.

Keywords: Economic order quantity; Growing items; Inventory control; Growth; Mortality

1. Introduction

Inventory control is a central problem in the area of supply chain management, which attracts notable research endeavors due to the substantial role of the inventories in managing costs in the manufacturing and retailing industries. The introduction of new concepts in the related context opens up the path to dealing with the raised problems more efficiently. Growth is a recently heeded concept in the broad area of inventory management (Rezaei 2014), by investigation of which industrial animal farming is expected to experience a transformation in handling its inventories.

Growth is generally referred to as natural development leading to positive physical changes such as size and weight increase. Growing items depict the behavior of a group of inventories whose level increases due to the weight gain of the items during the stocking. This is prevalent in poultry and livestock industries, and the items are the main components of fresh food supply chains. The inventory cycle of a growing item embraces two sub-cycles: breeding and consumption. At the starting point of the breeding period, newborn animals enter the system of the rearing farm and subsequently are fed and raised. Growth is outlined by inventory level increase which keeps enhancing until the items are slaughtered, and the sub-cycle is brought to an end. Afterward, the consumption period starts, during which the slaughtered items (such as chicken meat) satisfy demand and the inventory level is depleted to zero. The slaughtered items usually tend to get spoiled during the time and are classified as deteriorating inventory.

Growing items are prone to different diseases during their breeding, resulting in their death or lower quality of the slaughtered inventory. The disease can be controlled by incorporating preventive practices. As an instance,

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poultry disease at the rearing farms can be effectively controlled by early vaccination, parasitic inspection, biosecurity procedures, and multi-age flock separation (Humphrey 2006).

The inventory control of growing items is of great importance. The items have a propensity to undergo quality losses, disease, and even death. So, the costs incurred by any inappropriate replenishment policy are not comparable to the case of classic inventories. Besides, this is directly linked to food safety which might threaten the health of the consumers. Motivated by the mentioned significance, this research addresses the novel concept of growth in the area of inventory management. The paper investigates replenishment policies for growing items in a rearing farm. The growth of the inventory is outlined by weight increase which is measured by a weight function. To depict a pragmatic condition, the developed model accounts for the disease and death of the items during their breeding, and deterioration during the consumption period. It is considered that preventive practices can control the mortality rate. The negative impact of over-breeding is also taken into consideration by assuming quality degradations.

Addressing growth in the context of inventory management is an emerging and young direction. There are limited related papers, most of which fail to outline the interactive impacts of the number of initial newborn animals and the length of their breeding period on the final inventory level. This is heeded in our model by firstly incorporating a weight function that can effectively project the growth behavior of the newborn animals and secondly taking the number of newborn animals and the breeding period as decision variables. As another contribution of our model, the mortality of the items and its link to preventive actions are taken into account. Moreover, an age-dependent cost term comprising feeding and holding expenditures is applied. Finally, the negative impact of overbreeding, disease, and quality losses is considered to preserve the quality standards of the items.

The remainder of the paper is structured as follows. In Section 2, relevant literature is reviewed. Section 3 provides the problem statement and the mathematical model. The solution approach is outlined in Section 4. Experimental results are derived in Section 5, and finally, Section 6 concludes the paper and proposes directions for future research.

2. Literature Review

In contrast to growth, deterioration is a rich area of academic research that has attracted numerous studies since its introduction. In this regard, the paper refrains from covering the literature body of the deterioration, and interested readers are referred to Rabbani et al. (2014), Rabbani et al. (2015), Janssen et al. (2016), Rabbani et al. (2016), Rabbani et al. (2017), Shi et al. (2019), Acevedo-Ojeda et al. (2019), Taleizadeh et al. (2019), Agi and Soni (2020), and Al-Amin et al. (2020) for studies in this area.

Rezaei (2014) addressed the problem for the first time by developing an EOQ model for the growing items. A mathematical weight function measures the growth, and the feeding procedure is highlighted by linking the costs to the age of the items. The results suggest that the breeding period is highly affected by feeding costs. Sebatjane and Adetunji (2018) treated the breeding period of the items as a fixed value by defining a targeted final weight for each unit item. A portion of the slaughtered inventory is regarded as an imperfect quality that needs to be salvaged. This portion is considered to be probabilistic. They applied three growth functions, including linear, split linear, and logistics, among which the logistics function can illustrate the growth pattern more accurately. Sebatjane and Adetunji (2019) extended their previous research by taking an incremental discount scheme into account. Nobil et al. (2019) investigated the replenishment policies in a poultry farm while allowing for shortages. They treated the breeding period as a known parameter by specifying the initial and final weight of the chickens. Khalilpourazari and Pasandideh (2019) developed a multi-item EOQ model for the growing items under on-hand budget, warehouse capacity, and total allowable holding limitations. They applied sequential quadratic programming and two novel meta-heuristics to obtain near-optimal solutions for small and large sizes, respectively. Malekitabar et al. (2019) proposed an inventory model for Rainbow trout. They considered the initial inventory level of the growing items to be known and applied a deterioration rate to depict mortality and an increase rate to project growth. The nature of the items is identical during the cycle, which means the demand is for the growing items, not the slaughtered inventory. They optimized the periodic profit of the system, which implicitly suggests that the problem is analyzed for a one-period case. Gharaei and Almehdawe (2020) developed an economic quantity model for growing inventory where a fraction of the items die during their growth. They considered the weight of the items to be a linear function of the time. As shown by Sebatjane and Adetunji (2018), it is not an appropriate estimation to reflect the weight increase pattern of the growing items. They showed that the system is highly dependent on the initial weight of the items as well as feeding and purchasing costs.

As the items grow, the ratio of useless weight (such as fat) to their whole weight increases. Accordingly, a fraction of the slaughtered inventory is discarded, which is linked to the length of the breeding period. Pourmohammadzia and Karimi (2020) took this point into account by conducting an instantaneous quality control process of the slaughtered items at the end of the breeding period. They defined an exponentially breeding-time-dependent function for a fraction of the items being discarded after their slaughter. Pourmohammad-Zia et al.'s work (2021a) is among the few studies in the context of the supply chain, where the simultaneous impact of the initial number of newborn animals and the length of the breeding period on the final inventory level are considered. The growing items usually fall in the category of deteriorating inventory after slaughter. However, this is mostly overlooked in the related literature. Pourmohammad-Zia et al. (2021a) took this into account by modelling a continuous inventory level decline at the retailer. Sebatjane and Adetunji (2020) investigated optimal pricing, ordering, and shipment decisions in a three-level FSC, where the customer demand is price and freshness-dependent. Pourmohammadzia et al. (2021b) proposed the other research in this area, where pricing, breeding, ordering, and production decisions are studied in a three-level FSC consisting of a rearing farm, a processed food manufacturer, and multiple retailers. The manufacturer applies Vendor Managed Inventory (VMI) to handle the inventory systems' of its multiple retailers. Pourmohammadzia (2021c) has reviewed research developments in the area of growing inventory and showed that modeling growth of the inventory in the context of operations management is still in its infancy, and simplifications, as well as drawbacks, exist in the area.

2. Model Development

Notations

C_P	Unit purchasing cost
C_B	Breeding (feeding and holding) cost per unit item during the breeding period
C_H	Unit holding cost per unit time during the consumption period
C_O	Fixed ordering cost per cycle
i	Action level of preventive practices
C_{PA}^i	Preventive actions cost for level i per unit item
D	Annual demand rate
w_t	Weight of a unit item at time t
$I(t)$	Inventory level at time t
$\lambda(.)$	Fraction of the slaughtered items which do not pass quality standards
η_i	Mortality rate under preventive actions of level i during the breeding period
θ	Deterioration rate during the consumption period
y	Number of purchased newborn animals purchased (unit items)
$Y(t)$	Number of animals at time t (unit items)
Q	Order quantity of the newborn animals (units)
T_1	Breeding period
T_2	Consumption period
T	Replenishment cycle ($T = T_1 + T_2$)
TUC	Total unit cost

Assumptions

The following basic assumptions are applied to form the structure of the model:

1. The planning horizon is infinite.
2. Shortages are not allowed.
3. Replenishment is instantaneous with an infinite rate and negligible lead-time.
4. The growth and death of the items start to occur from the point they are effectively in stock.
5. Deterioration and mortality rates are constant.

Mathematical formulation

Consider the system of a rearing farm which buys y newborn animals at the beginning of each inventory cycle. The items are raised during the breeding period, and growth is outlined by measuring the weight of the animals through time. In this regard, a mathematical function is applied which projects the weight of a unit item at time t as: $w_t = A(1 + be^{-jt})^{-1}$ (Richards 1959). A is the ultimate limiting value ($A > 0$) reflecting the maximum threshold for the weight of a unit item. b is the integration constant which specifies the weight of a unit item at time zero ($b > 0$), and j is a constant rate illustrating the spread of growth curve during the time, which outlines the speed of growth ($0 < j < 1$). Note that in this formulation, time is expressed in days. In order to change this to year, which is the time basis in our inventory model, $k = 365j$ is substituted (i.e. $w_t = A(1 + be^{-kt})^{-1}$, t in years).

As the growing items are raised, the system bears higher feeding costs for each unit item which is due to their weight increase. This is heeded by applying an age-dependent function to drive the breeding costs of the growing items during the breeding period. The polynomial and exponential functions are the most vastly applied ones in the literature of animal farming (Goliomytis et al. 2003). In this paper, the exponential function ($B_t = e^{\beta t},\ \beta > 0$) is applied.

The growing items might experience disease, which results in their death during the breeding period. The mortality rate of the items can be controlled by carrying out preventive practices. Pragmatically, the preventive practices embrace several distinct action levels rather than a continuous scheme. In this paper, the preventive practices might hold various action levels ranging from very lenient control to very strict one, each of which imposes different costs on the system. We have considered five austerity levels for the preventive practices involving very lenient (1), lenient (2), normal (3), strict (4), and very strict (5). Apparently, as the austerity level increases, preventive practices lead to higher costs and a lower mortality rate.

As the items grow in the system, the useless portion of their weight (such as fat) rises (Jensen et al. 1974). Furthermore, overbreeding can lead to quality losses due to diseases. In order to take these negative impacts of overbreeding into account, the slaughtered items are quality controlled, and a portion of them are regarded as useless and discarded. Overbreeding raises the risk of disease and fat deposition. Accordingly, the fraction of discarded items after quality control should be an increasing function of the breeding period. It also needs to hold two other features: First, in time zero, this fraction is negligible (i.e., $\lambda(0) = 0$). Second, as the breeding period takes very large values, this fraction approaches one (i.e. $\lim_{T_{1\to\infty}} \lambda(T_1) = 1$).

Accordingly, $\lambda(T_1) = 1 - e^{-\alpha T_1},\ \alpha > 0$ holds the mentioned features. The process of quality control is assumed to be instantaneous.

The status of the number of animals at time $t \in [0, T_1)$ is governed by the following differential equation:

$$\frac{dY(t)}{dt} = -\eta_i Y(t) \qquad 0 \le t < T_1 \tag{1}$$

Solving this equation with the boundary condition $Y(0) = y$ yields:

$$(t) = ye^{-\eta_i t} \tag{2}$$

The weight of each unit item at time t is $w_t = A(1 + be^{-kt})^{-1}$. Accordingly, the inventory level during $t \in [0, T_1)$ is illustrated by:

$$I(t) = Y(t)w_t = yAe^{-\eta_i t}(1 + be^{-kt})^{-1},\ 0 \le t < T_1 \tag{3}$$

So, the initial inventory level is obtained as:

$$Q = I(0) = yA(1 + b)^{-1} \tag{4}$$

Eq. (4) gives $y = \frac{Q(1+b)}{A}$. Then Eq. (3) can be reformulated as:

$$I(t) = Q(1 + b)e^{-\eta_i t}(1 + be^{-kt})^{-1},\ 0 \le t < T_1 \tag{5}$$

The inventory level just before the quality control is $Q(1 + b)e^{-\eta_i T_1}(1 + be^{-kT_1})^{-1}$

Therefore, the quantity of the discarded items yields:

$$\lambda(T_1)Q(1 + b)e^{-\eta_i T_1}(1 + be^{-kT_1})^{-1} = Q(1 + b)\frac{1-e^{-\alpha T_1}}{e^{\eta_i T_1}(1+be^{-kT_1})} \tag{6}$$

Consequently, the inventory level at time T_1 is outlined as:

$$I(T_1) = \frac{Q(1+b)}{e^{\alpha T_1}e^{\eta_i T_1}(1+be^{-kT_1})} \tag{7}$$

This is the ending point of the breeding period when the system enters the consumption period, and the inventory is exposed to demand and deterioration. The status of the inventory level at any instant time $t \in [T_1, T]$ ($T = T_1 + T_2$) is ruled by the following differential equation:

$$\frac{dI(t)}{dt} = -D - \theta I(t) \qquad T_1 \le t \le T \tag{8}$$

Solving Eq. (8) with the boundary condition $I(T) = 0$ gives:

$$I(t) = \frac{D}{\theta}\left(e^{\theta(\mathrm{T}-\mathrm{t})} - 1\right) \qquad \mathrm{T}_1 \le t \le \mathrm{T} \tag{9}$$

Eq. (9) at $t = T_1$ should provide the same value as in Eq. (7):

$$\frac{D}{\theta}\left(e^{\theta T_2} - 1\right) = \frac{Q(1+b)}{e^{\alpha T_1}e^{\eta_i T_1}(1+be^{-kT_1})} \tag{10}$$

Then, Q can be expressed as a function of T_1 and T_2:

$$Q = \frac{D}{\theta(1+b)}\ e^{\alpha T_1}e^{\eta_i T_1}(1 + be^{-kT_1})(e^{\theta T_2} - 1) \tag{11}$$

The total cost of the system involves the following components:

1. **Purchasing cost:**

$$PC = C_P\mathrm{Q} = C_P\frac{D}{\theta(1+b)}\ e^{\alpha T_1}e^{\eta_i T_1}(1 + be^{-kT_1})(e^{\theta T_2} - 1) \tag{12}$$

2. Breeding cost: This cost is charged during the breeding period

$$\begin{aligned} BC &= C_B\int_0^{T_1} Y(t)B_t dt = C_B y\int_0^{T_1} e^{-\eta_i \mathrm{t}}e^{\beta \mathrm{t}}dt = C_B\frac{Q(1+b)}{A}\int_0^{T_1} e^{(\beta-\eta_i)\mathrm{t}}dt \\ &= C_B\frac{D}{A\theta(\beta-\eta_i)}e^{\alpha T_1}e^{\eta_i T_1}(1 + be^{-kT_1})(e^{(\beta-\eta_i)T_1} - 1)(e^{\theta T_2} - 1) \end{aligned} \tag{13}$$

3. Preventive cost: This cost is charged per each unit item of the newborn animals

$$PAC = C_{PA}^i y = C_{PA}^i\frac{Q(1+b)}{A} = C_{PA}^i\frac{D}{A\theta}\ e^{\alpha T_1}e^{\eta_i T_1}(1 + be^{-kT_1})(e^{\theta T_2} - 1) \tag{14}$$

4. Holding cost: This cost is charged during the consumption period

$$HC = C_H \int_{T_1}^{T} I(t)dt = C_H \frac{D}{\theta} \int_{T_1}^{T} (e^{\theta(\mathrm{T-t})} - 1)dt = C_H \frac{D}{\theta^2} (e^{\theta T_2} - \theta T_2 - 1) \tag{15}$$

5. Ordering/Set up cost

$$\mathrm{OC} = \mathrm{C_O} \tag{16}$$

The inventory cycle is repeated every T_2 units of time. So, the total unit cost is given by:

$$TUC = \frac{PC + BC + PAC + HC + OC}{T_2} \tag{17}$$

Solution Approach

The first-order optimality conditions for TUC are $\frac{\partial TUC}{\partial T_1} = 0$ and $\frac{\partial TUC}{\partial T_2} = 0$.

The conditions under which these equations provide unique optimal solutions should also be established. Due to the complexity of the formulations, this cannot be done by incorporating the Hessian matrix. In this regard, a solution approach similar to (Pentico and Drake 2009) is applied.

The necessary optimality condition, $\frac{\partial TUC}{\partial T_1} = 0$ gives:

$$\frac{\partial TUC}{\partial T_1} = \left(\frac{C_P D}{\theta(1+b)T_2} + \frac{C_{PA}^i D}{A\theta T_2}\right)\left(e^{\theta T_2} - 1\right)\begin{bmatrix}(\alpha+\eta_i)e^{(\alpha+\eta_i)T_1}\left(1+be^{-kT_1}\right)\\ -bke^{(\alpha+\eta_i)T_1}e^{-kT_1}\end{bmatrix}$$

$$+\frac{C_B D}{A\theta(\beta-\eta_i)T_2}\left(e^{\theta T_2} - 1\right)\begin{bmatrix}(\alpha+\eta_i)e^{(\alpha+\eta_i)T_1}\left(1+be^{-kT_1}\right)(e^{(\beta-\eta_i)T_1} - 1)\\ -bke^{(\alpha+\eta_i)T_1}e^{-kT_1}(e^{(\beta-\eta_i)T_1} - 1)\\ +(\beta-\eta_i)e^{(\alpha+\eta_i)T_1}\left(1+be^{-kT_1}\right)e^{(\beta-\eta_i)T_1}\end{bmatrix} = 0 \tag{18}$$

Then:

$$\left(\frac{C_P}{(1+b)} + \frac{C_{PA}^i}{A}\right)\left[(\alpha+\eta_i) + b(\alpha+\eta_i - k)e^{-kT_1})\right]$$

$$+\frac{C_B}{A(\beta-\eta_i)}\begin{bmatrix}(\alpha+\eta_i)\left(1+be^{-kT_1}\right)(e^{(\beta-\eta_i)T_1} - 1)\\ -bke^{-kT_1}(e^{(\beta-\eta_i)T_1} - 1)\\ +(\beta-\eta_i)\left(1+be^{-kT_1}\right)e^{(\beta-\eta_i)T_1}\end{bmatrix} = 0 \tag{19}$$

As shown, the value provided by Eq. (19) for T_1 is independent of T_2. That is to say, under the conditions that $\frac{\partial TUC}{\partial T_1}$ provides a unique optimal solution, different values of T_2 establish an identical $T_1{}^*$. So, we need to focus on the convexity of TUC when T_2 is treated as a fixed value.

$$\frac{\partial^2 TUC}{\partial {T_1}^2} = \left(\frac{C_P D}{\theta(1+b)T_2} + \frac{C_{PA}^i D}{A\theta T_2}\right)\left(e^{\theta T_2} - 1\right)\begin{bmatrix}(\alpha+\eta_i)^2 e^{(\alpha+\eta_i)T_1}\left(1+be^{-kT_1}\right) \\ -2bk(\alpha+\eta_i)e^{(\alpha+\eta_i)T_1}e^{-kT_1} \\ +bk^2 e^{(\alpha+\eta_i)T_1}e^{-kT_1}\end{bmatrix}$$

$$+\frac{C_B D}{A\theta(\beta-\eta_i)T_2}\left(e^{\theta T_2} - 1\right)\begin{bmatrix}(\alpha+\eta_i)^2 e^{(\alpha+\eta_i)T_1}\left(1+be^{-kT_1}\right)\left(e^{(\beta-\eta_i)T_1} - 1\right) \\ -2bk(\alpha+\eta_i)e^{(\alpha+\eta_i)T_1}e^{-kT_1}\left(e^{(\beta-\eta_i)T_1} - 1\right) \\ +2(\alpha+\eta_i)(\beta-\eta_i)e^{(\alpha+\eta_i)T_1}\left(1+be^{-kT_1}\right)e^{(\beta-\eta_i)T_1} \\ +bk^2 e^{(\alpha+\eta_i)T_1}e^{-kT_1}\left(e^{(\beta-\eta_i)T_1} - 1\right) \\ -2bk(\beta-\eta_i)e^{(\alpha+\eta_i)T_1}e^{-kT_1}e^{(\beta-\eta_i)T_1} \\ +(\beta-\eta_i)^2 e^{(\alpha+\eta_i)T_1}\left(1+be^{-kT_1}\right)e^{(\beta-\eta_i)T_1}\end{bmatrix} \tag{20}$$

To guarantee the convexity of TUC, Eq. (20) should be non-negative. It can be shown that if $\frac{C_P}{(1+b)} + \frac{C_{PA}^i}{A} \geq \frac{C_B}{A(\beta-\eta_i)}$ or $(2\alpha+\eta_i+\beta)(1+b) \geq 2bk$, this is met. See Appendix A for details. Having ${T_1}^*$ on hand, the necessary optimality condition for T_2 gives:

$$\frac{\partial TUC}{\partial T_2} = \left(\frac{C_P D}{(1+b)} + \frac{C_{PA}^i D}{A} + \frac{C_B D e^{(\beta-\eta_i)T_1}}{A(\beta-\eta_i)}\right)e^{(\alpha+\eta_i)T_1}\left(1+be^{-kT_1}\right)\left(\frac{\theta T_2 e^{\theta T_2} - e^{\theta T_2} + 1}{\theta\, {T_2}^2}\right)$$
$$+\frac{C_H D}{\theta^2}\left(\frac{\theta T_2 e^{\theta T_2} - e^{\theta T_2} + 1}{{T_2}^2}\right) - \frac{C_O}{{T_2}^2} = 0 \tag{21}$$

Consider $\chi = e^{(\beta-\eta_i)T_1}$ and $\varphi = e^{(\alpha+\eta_i)T_1}\left(1+be^{-kT_1}\right)$. Eq. (21) can be rewritten as:

$$\frac{\partial TUC}{\partial T_2} = \frac{D}{\theta\, {T_2}^2}\begin{bmatrix}\left(\frac{C_P}{(1+b)} + \frac{C_{PA}^i}{A} + \frac{C_B \chi}{A(\beta-\eta_i)}\right)\varphi\left(\theta T_2 e^{\theta T_2} - e^{\theta T_2} + 1\right) \\ +\frac{C_H}{\theta}\left(\theta T_2 e^{\theta T_2} - e^{\theta T_2} + 1\right) - \frac{\theta C_O}{D}\end{bmatrix} = 0 \tag{22}$$

Motivated by Eq. (22), the auxiliary function $\phi(T_2)$ can be defined as the phrases in []. Since $\frac{\partial TUC}{\partial T_2} = 0$ and $\phi(T_2) = 0$ are equivalent, it is enough to show that $\phi(T_2) = 0$ gives a unique optimal solution.

$$\frac{d\phi(T_2)}{dT_2} = \left[\left(\frac{C_P}{(1+b)} + \frac{C_{PA}^i}{A} + \frac{C_B \chi}{A(\beta-\eta_i)}\right)\varphi + \frac{C_H}{\theta}\right]\theta^2 T_2 e^{\theta T_2} \tag{23}$$

Suggested by Eq. (23), $\forall T_2 \in (0,\infty)$ $\frac{d\phi(T_2)}{dT_2} > 0$. That is $\phi(T_2)$ is a strictly increasing function of T_2. On the other hand, $\lim_{T_2\to 0}\phi(T_2) = -\frac{\theta C_O}{D} < 0$ and $\lim_{T_2\to\infty}\phi(T_2) = \infty > 0$. Therefore, there exists a unique value of T_2 where $\phi(T_2) = 0$. Since, $\frac{\partial TUC}{\partial T_2} = \phi(T_2)\frac{D}{\theta T_2^2}$, at point $T_2 = {T_2}^*$:

$$\frac{\partial^2 TUC}{\partial T_2^2}\Big|_{T_2={T_2}^*} = \frac{D}{\theta}\frac{\phi' - 2\phi}{{T_2}^3}\Big|_{T_2={T_2}^*} = \frac{D}{\theta}\frac{\phi'}{{T_2^*}^2} > 0 \tag{24}$$

So, there exists a unique optimal value for T_2 which minimizes $TUC({T_1}^*, T_2)$.

The following simple algorithm is applied to drive the optimal solutions of the problem:

Algorithm

Step 0- *i=1*

Step 1- *If* $i \leq 5$, *go to step 2; otherwise go to step 6*

Step 2- *Apply a numerical root-finding approach to solve Eq. [19] and obtain* $T_1^{i^*}$.

Step 3- *Apply a numerical root-finding approach to solve Eq. [22] and obtain* $T_2^{i^*}$ *for,* $T_1^i = T_1^{i^*}$.

Step 4- *Calculate* $TUC^i\left(T_1^{i^*}, T_2^{i^*}\right)$ *by Eq. [17].*

Step 5- $i = i + 1$ *and go to step 1.*

Step 6- $i^* = \arg\min_i(TUC^i), TUC^* = TUC^{\,i^*}, (T_1^*, T_2^*) = \left(T_1^{i^{**}}, T_2^{i^{**}}\right)$.

Step 7- *End*

3. Experimental Results

The proposed structure is illustrated through numerical experiments for a specific type of newborn animals named “broiler chickens”. The parameters of the weight function are estimated by function approximation techniques based on a real data set of an industrial rearing farm, according to which the parameters are outlined as:
$A = 3200$, $b = 69.4$ and $g = 0.12$, $k = 0.12 * 365 = 43.8$. Then, the weight function is outlined as $w_t = 3200(1 + 69.4e^{-43.8t})^{-1}$. Moreover, the exponential breeding function $B(t)$ and the disposal rate $\lambda(T_1)$ are ruled by: $B(t) = e^{76t}$ and $\lambda(T_1) = 1 - e^{-T_1}$ respectively. Identical parameters of the problem are taken from Rezaei (2014) and adapted to our model. The values of the applied parameters are as follows:

$\theta = 0.2$, $\eta = (0.27, 0.25, 0.2, 0.16, 0.12)$, $C_P = 0.005$ €/gr, $C_B = 0.02$ €/unit item,
$C_H = 0.001$ €/gr/year, $C_O = 500$ €/cycle, $C_{PA} = (0.04, 0.05, 0.06, 0.07, 0.08)$ €/unit item and $D = 100 \times 10^6$gr/year.

Solving this problem provides the following results:

Preventive action level: Very lenient

$T_1 = 0.08345$ year $T_2 = 0.09203$ year $Q = 409901.96$ gr $TUC =$ € 50601.37

The results suggest that 409.9 kg (roughly 9109 unit items) of newborn chicks are bought at the beginning of each cycle. These are bred for 31 days, during which the final weight of each chicken reaches 1.15 kg, and around 2.5% of the growing items die due to disease. The items get slaughtered, and then quality is controlled when an additional 8% of the inventory is discarded due to quality losses. The slaughtered items, which are prone to deterioration, satisfy the customer demand for 33 days. This indicates that the inventory cycle recurs every 33 days. The rearing farm takes “Very lenient” preventive practices as its optimal policy. This may stem from a very healthy breeding environment or considerably high preventive costs. Later, this will be discussed in further detail. Figure 1 projects the total unit cost for this problem, which, as shown, is a convex function.

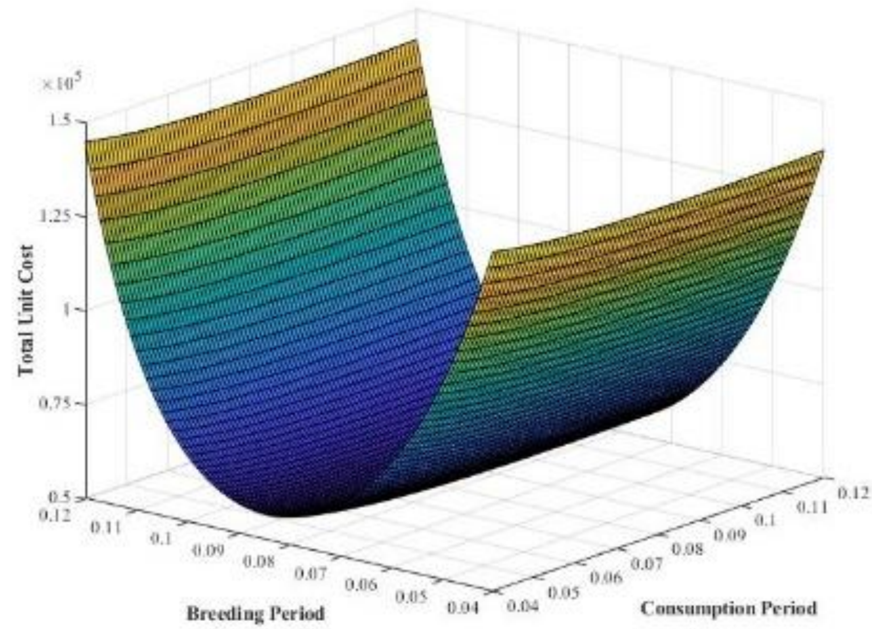

Figure 1. The Total Unit Cost.

The average age of broiler chickens in the EU (and some other regions) is 40-42 days (Mebratie et al. 2018), while our results suggest reducing it to 31 days. Solving the problem under known $T_1 = 42$ days, yields the following results:

Preventive action level: Very lenient
$T_1 = 0.1151$ year $T_2 = 0.09534$ year $Q = 229251.93$ gr $TUC = €\ 109426.64$

This indicates that the supplier buys 5094 newborn chicks at each cycle and raises them for 42 days when the final weight of each unit item reaches 2.21 kg. Then, 3.06% of growing items die during their breeding period, and 10.87% of the slaughtered inventory is discarded as low-quality items. As projected, food waste and the costs of the system are higher in this case.

This 31-day breeding period is not a one-size-fits-all policy. The optimal slaughtering age of broiler chickens highly depends on the growth pattern of the items (outlined as the weight function) and can vary from one farm to another based on growth features and system costs. In particular, this is optimal for our data-set and the estimated weight function. This emphasizes the importance of incorporating the optimization method instead of empirical practice. The first step to use this optimization model is to accurately estimate the parameters of the weight function.

Sensitivity analysis plays an efficient role in getting a better understanding of the behavior of the model. In this regard, sensitivity analysis on the key parameters of the model, including C_P, C_B, C_H, C_O and C_{PA} is carried out by changing each parameter by -50%, -25%, +25%, and +50%, taking one at a time and keeping the others fixed. The results are provided in Table 1. Figure 2a to Figure 2d illustrate these results graphically.

Table 1. Sensitivity analysis on cost parameters.

Parameter	Changes (%)	Preventive Practice level	T_1	T_2	Q	TUC
C_p	-50%	**1**	0.07789 **(-6.66%)**	0.09422 **(+2.38%)**	490524.39 **(+19.67%)**	38646.42 **(-23.63%)**
	-25%	**1**	0.08107 **(-2.85%)**	0.09307 **(+1.13%)**	442482.97 **(+7.95%)**	44853.33 **(-11.36%)**
	+25%	**1**	0.08535 **(+2.28%)**	0.09107 **(-1.04%)**	385716.69 **(-5.9%)**	56030.28 **(+10.73%)**
	+50%	**1**	0.08127 **(+4.17%)**	0.09016 **(-2.03%)**	366710.91 **(-10.54%)**	61220.22 **(+20.98%)**
C_B	-50%	**1**	0.09019 **(+8.08%)**	0.09312 **(+1.18%)**	349660.96 **(-14.69%)**	42220.46 **(-16.56%)**
	-25%	**1**	0.08628 **(+3.39%)**	0.09252 **(+0.53%)**	382669.67 **(-6.64%)**	46761.56 **(-7.59%)**
	+25%	**1**	0.08123 **(-2.66%)**	0.09161 **(-0.45%)**	433529.39 **(+5.76%)**	53997.04 **(+6.71%)**
	+50%	**1**	0.07941 **(-4.85%)**	0.09123 **(-0.87%)**	454639.52 **(+10.91%)**	57078.49 **(+12.8%)**
C_H	-50%	**1**	0.08345 **(0%)**	0.1215 **(+32.22%)**	542565.64 **(+32.44%)**	47835.94 **(-5.46%)**
	-25%	**1**	0.08345 **(0%)**	0.1037 **(+12.68%)**	462638.16 **(+12.86%)**	49323.15 **(-2.53%)**
	+25%	**1**	0.08345 **(0%)**	0.08356 **(-9.2%)**	371861.59 **(-9.28%)**	51739.51 **(+2.25%)**
	+50%	**1**	0.08345 **(0%)**	0.07707 **(-16.25%)**	342756.65 **(-16.38%)**	52775.39 **(+4.29%)**
C_O	-50%	**1**	0.08345 **(0%)**	0.06519 **(-29.16%)**	289583.91 **(-29.35%)**	47538.2 **(-6.05%)**
	-25%	**1**	0.08345 **(0%)**	0.07977 **(-13.32%)**	354839.58 **(-13.43%)**	49199.51 **(-2.77%)**
	+25%	**1**	0.08345 **(0%)**	0.1028 **(+11.7%)**	458450.24 **(+11.84%)**	51837.41 **(+2.44%)**
	+50%	**1**	0.08345 **(0%)**	0.1126 **(+22.35%)**	502371.33 **(+22.56%)**	52955.67 **(+4.65%)**
C_{PA}	-50%	**1**	0.08268 **(-0.93%)**	0.09239 **(+0.39%)**	420171.02 **(+2.5%)**	48619.89 **(-3.91%)**
	-25%	**1**	0.08307 **(-0.45%)**	0.09221 **(+0.19%)**	414898.16 **(+1.22%)**	49615.78 **(-1.95%)**
	+25%	**1**	0.08382 **(+0.44%)**	0.09186 **(-0.18%)**	405157.65 **(-1.16%)**	51577.19 **(+1.93%)**
	+50%	**1**	0.08416 **(+0.85%)**	0.09168 **(-0.38%)**	400643.48 **(-2.26%)**	52543.78 **(+3.84%)**

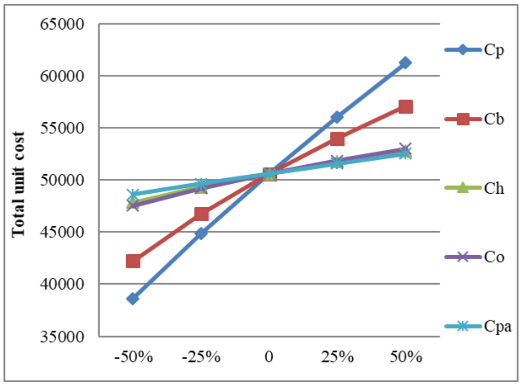

Figure 2a. Changes in the optimal TUC with variations in input parameters.

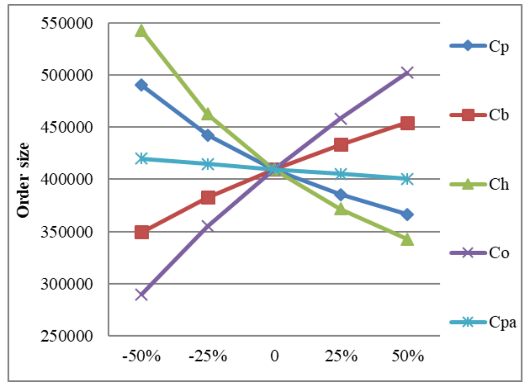

Figure 2b. Changes in the optimal Q with variations in input parameters.

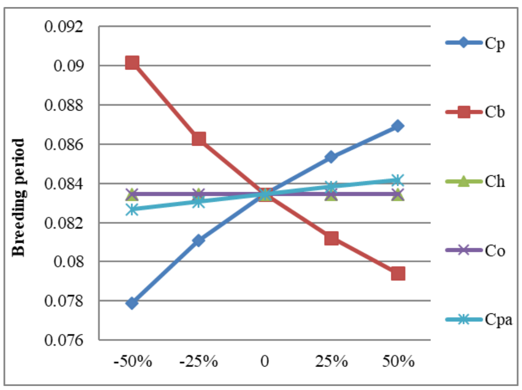

Figure 2c. Changes in the optimal T_1 with variations in input parameters.

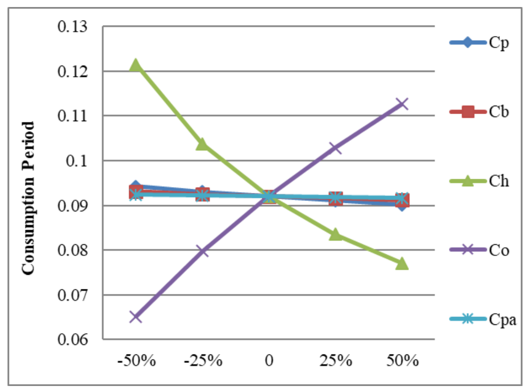

Figure 2d. Changes in the optimal T_2 with variations in input parameters.

The numerical results provide fruitful insights as follows:

1. By decreasing unit purchasing cost (C_P) an increase in order size (Q) and a decrease in total unit cost (TUC) are observed. Since a larger quantity of newborn chickens enter the system, the breeding period (T_1) gets shorter. A slight increase in T_2 is also observed, which shows that the increase in Q outweighs the decrease in T_1. The results imply that if the rearing farm has the option to choose among different hatcheries, it can effectively shorten its growth period by selecting the one with a smaller C_P. This is specifically advantageous under the conditions of a newly emerging disease among the broiler chickens or the desire of the customers to buy younger items.
2. Decreasing unit breeding cost (C_B) leads to an increase in T_1 and a decrease in Q. This is because by lowering C_B, the breeding costs of the system decrease. So it is optimal for the system to decrease the ordering quantity and increase the breeding period. The increase in T_2 implies that the impact of the rise in T_1 is higher than the decrease in Q. Decreasing TUC as the result of lowering C_B is trivial. The results also suggest that if the rearing farm confronts a limitation in the periodic purchasing budget, it can decrease the ordering quantity by decreasing the breeding costs. This is achievable by amending the holding and flourishment facilities as well as lowering the feeding costs. It also should be noted that, in comparison to other parameters, the breeding cost has the most influential effect on the breeding period.

3. The holding cost is charged during the consumption period. So, it comes as no surprise that decreasing C_H leads to an increase in T_2. The breeding period is insensitive to changes in C_H which builds a direct link between the changes in T_2 and Q. Precisely, longer T_2 implies that higher inventory is required to satisfy the demand, which results in larger Q. Since T_1 remains unchanged, the percentage of changes in T_2 and Q are almost the same.
4. Since the inventory cycle recurs every T_2 units of time, decreasing C_O leads to a significant decrease in T_2. A shorter consumption period suggests a rise in the frequency of ordering. This leads to a decrease in ordering quantity (Q). T_1 is insensitive to changes in C_O and as a result, the percentage of changes in T_2 and Q are almost identical.
5. C_{PA} is charged per each unit item of the initial inventory. Therefore, decreasing the cost of preventive practices provides the rearing farm with the opportunity to increase the initial order size (Q). Larger Q leads to a slight decrease in the breeding period. The increase in Q outweighs the decrease in T_1 and consequently a slight increase in T_2 is observed.

Regarding the costs of the system, the unit purchasing cost is the most influential cost factor (see Figure 2a). It suggests that selecting a hatchery with the most reasonable C_P should be the first priority of the rearing farm when handling the costs of the system. The breeding period is mostly dependent on in C_P and C_B (see Figure 2c). It should be noted that the nature of the items (shown in terms of growth pattern) is the most significant factor in the changes of the breeding period. The consumption period, in turn, is highly influenced by C_H and C_O (see Figure 2d). Since the ordering quantity is characterized by both the breeding and consumption period, as expected, Q is highly sensitive to changes in C_P , C_B as well as C_H and C_O (see Figure 2b).

As Table 1 suggests, the preventive practice gets optimal at level 1 (very lenient actions) for different cases. This may stem from the following reasons: (1)The breeding environment is in a great condition such that very lenient preventive practice is efficient enough to manage the mortality rates of the items (η is low). (2)The cost of preventive practice is so high that maintaining it at the lowest possible level is more beneficial than lowering the mortality rate. (3)The cost increase of applying stricter preventive actions outweighs the benefits of decreasing the mortality rate. Accordingly, it is fruitful to investigate the effect of different pairs of η and C_{PA} on the system, the numerical results of which are provided in Table 2.

Table 2. Analysis on C_{PA} and η.

C_{PA} and η	Preventive Practice level	T_1	T_2	Q	TUC
η =(0.8,0.7,0.6,0.5,0.4) C_{PA} =(0.04,0.05,0.06,0.07,0.08)	**1**	0.08362	0.08784	406897.63	50601.37
η =(0.27,0.25,0.2,0.16,0.12) C_{PA} =(0.01,0.0125,0.015,0.0175,0.02)	**1**	0.08227	0.09257	425748.92	47613.06
η =(0.8,0.7,0.6,0.5,0.4) C_{PA} =(0.01,0.0125,0.015,0.0175,0.02)	**3**	0.08241	0.08863	423700.66	48801.75
η =(0.8,0.65,0.5,0.35,0.2) C_{PA} =(0.01,0.0125,0.015,0.0175,0.02)	**5**	0.08266	0.09281	419926.29	48462.88

As the results project, either by raising the mortality rate (and keeping C_{PA} constant) or by lowering preventive action costs (and keeping η constant), the level of preventive practice does not undergo any shifts. This suggests that, among the aforementioned three reasons for the constancy of the action levels, the first and second ones are not authentic. In order to check the validity of the third one, we have simultaneously increased η and decreased

C_{PA}; so that a smaller cost increase would result in a larger reduction of mortality rate. The results depict that the action level shifts from very lenient to normal (level 3), implying that the reduction of mortality outweighs the increase in costs. By enlarging the difference of η between variant action levels, we observe another shift from the normal action level to the very strict one.

4. Conclusion

In this paper, an inventory model for growing/mortal items is introduced. It illustrates the system of a rearing farm that buys newborn animals at the beginning of each cycle and breeds them. A fraction of the items dies during their growth due to disease. This mortality rate can be controlled by investing in preventive practices. The items are quality controlled after getting slaughtered which helps to preserve the quality of the inventory and prevent overbreeding. An analytic solution approach is developed to optimize the number of newborn items and the breeding period. Numerical results show that the unit purchasing cost has the most influential role on the total cost of the system. Furthermore, it is projected that the level of preventive actions totally depends on the benefits of the practices. Precisely, the system shifts to stricter levels only when the reduction in mortality rate is considerable enough in comparison to the rise in investments.

There exist some promising directions to extend this work. Admissible shortages can be added to the assumptions to model the problem. Embedding uncertainty into the problem is another future direction. Specifically, taking the uncertainty of growth pattern and mortality rate into account helps to outline a more practical situation. Finally, the impact of the quality of the slaughtered inventory on the customers' willingness can be analyzed by linking demand to the length of the breeding period.

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