



A review of the research developments on inventory management of growing items

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Abstract – Growth outlines the behavior of a class of inventories whose weight and size increase during their storage. Fast-growing young animals such as broiler chickens, ducks, and calves are examples of this class, prevalent in many food industries. Despite their presence in various food supply chains, studying these items in the context of inventory control, supply chain, and operations management is still in its early stage. This study provides the essential grounds of the knowledge in this area and further reviews the related research developments to provide the researchers with clear insights into the structure of the current literature, its main gaps, and promising directions for future research. In order to guarantee the comprehensiveness of our review, we have incorporated a systematic search procedure to identify the relevant literature. Through this procedure, 23 papers were identified that were used in our further analysis throughout the paper. Our investigations show that the literature on growing inventories is very confined, and this body of the literature gets even more limited when it comes to the problems in the context of the supply chain. Our findings introduce the topic as a promising direction in inventory and supply chain management with high potential for insightful future research works.

Keywords: Growing Inventory; Inventory Management; Supply Chain Management; Amelioration; Deterioration

1. Introduction

Inventory management is a fundamental tool in operations management due to its key impact on cost, wastage and sustainability, supply and demand balance, and production as well as retail decisions. Concerning its crucial role, inventory control policies can achieve their ultimate purpose in handling costs only if they are capable of capturing the pragmatic and real-world conditions of the existing problems. In classic inventory models, it is usually assumed that the nature of the items remains unchanged during their storage, and the inventory level is depleted to zero just due to demand. However, the introduction of deteriorating items has opened up a new path in inventory management problems, where demand is not the only driving force of inventory level changes.

Deterioration is a well-known concept in the area and is understood as spoilage, decay, damage, evaporation, obsolescence, pilferage, and loss of utility or marginal value. The items undergoing deterioration become useless, which is usually modeled in the form of a gradual inventory level decrease. The inventory level reduction is not the only possible alteration, and one can expect level increases for certain types of inventories. Growing items are an important category of such inventory types. Growth is a common phenomenon in the poultry and livestock industry and is introduced as natural development leading to physical changes such as size and weight increase (Pourmohammad-Zia and Karimi 2020).

Growth describes a process under which the newborn animals enter the system of a rearing farm at the beginning of the period. They are fed and nourished, resulting in their physical weight increase and thereby inventory level increase. The items are raised until they reach a slaughtering point where the growth phase terminates. The slaughtered items such as beef and chicken meat are used as raw materials or final products in

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various Food Supply Chains (FSCs) and mostly fall in the category of deteriorating items. Figure 1. illustrates the behavior of a growing inventory system in a company-level structure.

As the figure depicts, an inventory cycle of the growing items is comprised of two phases: the breeding (growth) and consumption periods. The breeding period starts as the newborn inventory enters the system. During this period, the inventory level keeps increasing since the items grow larger. The breeding period terminates as the items get slaughtered. Afterward, the consumption period begins, during which the items are prone to demand and deterioration. Once the inventory level is depleted to zero, the consumption period comes to an end.

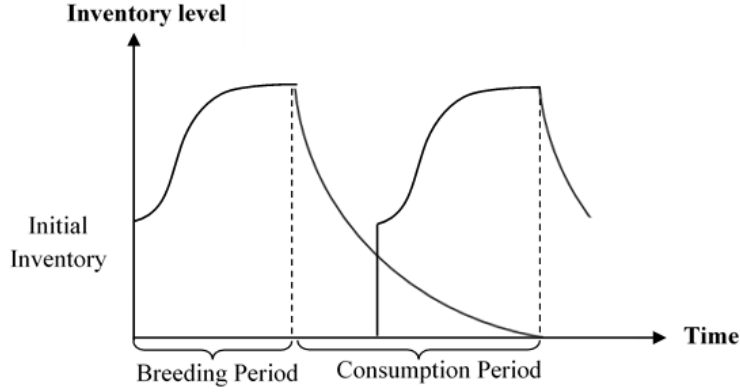


Figure 1. Growing Inventory System

A biological weight function usually governs the inventory level increase during the breeding period, illustrating the weight of the items at any instant point of the time. Richards (1959) has introduced different weight functions in this respect, among which the Autocatalytic function has shown good performance in reflecting the growth behavior and was applied by Rezaei (2014) for the first time to reflect the inventory level increase of growing items. In this function, the weight of a unit item at time t is formulated as $w_t = A(1 + be^{-gt})^{-1}$, where A is the ultimate limiting value ($A > 0$) representing the maximum possible weight of the item; b is the integration constant reflecting the choice of zero time ($b > 0$); g is a constant rate determining the spread of the growth curve during the time axis ($0 < g < 1$) and governs the growth speed. In this formulation, time is expressed in days. The basic model provided by Rezaei (2014) is represented here to provide the readers with further insights into the feature of inventory control problems.

Suppose y unit items are purchased at time zero. Each item weights $w_0 = A(1 + b)^{-1}$. Then the initial inventory level (order quantity) is:

$$Q = I(0) = yw_0 = yA(1 + b)^{-1} \quad (1)$$

Considering the breeding period as T_p , the inventory level at the end of the breeding period when the items get slaughtered yields:

$$I(T_p) = yw_{T_p} = yA(1 + be^{-kT_p})^{-1} \quad (2)$$

During the consumption period (T_D), the inventory level is depleted to zero due to demand (D). Therefore, the inventory level at the beginning of the consumption period is $D \times T_D$. Obviously, at the time T_p , $D \times T_D$ should be equal to Eq. (2); that is:

$$DT_D = yA(1 + be^{-kT_p})^{-1} \quad (3)$$

Using this equation, y can be represented as a function of T_p and T_D .

$$y = \frac{DT_D}{A}(1 + be^{-kT_p}) \quad (4)$$

By Substituting Eq. (4) into the previous ones, the formulations are expressed as a function of the two decision variables T_p and T_D . The components of the total cost of the system are outlined as follow:

1. Purchasing cost:

$$PC = C_p Q = C_p \frac{DT_D}{(1+b)} (1 + be^{-kT_p}) \tag{5}$$

Where C_p represents the unit purchasing cost.

2. Breeding cost: As the items flourish in the system, their breeding costs increase. That is because the items' feeding costs increase due to weight increase. Moreover, as they grow older, they are more prone to diseases. Then related health and holding costs increase as well. The impact of this cost increase could be regarded by a time-dependent function ($B(t)$) as follows:

$$BC = C_b \frac{DT_D}{A} (1 + be^{-kT_p}) \int_0^{T_p} B(t) dt \tag{6}$$

Where C_b represents the unit item breeding cost during the breeding period.

3. Holding cost: This cost is charged during the consumption period.

$$HC = C_h \int_{T_p}^{T_p+T_D} I(t) dt = C_h \int_{T_p}^{T_p+T_D} D(T_D - t) dt \tag{8}$$

Where C_h represents the unit holding cost per unit time during the consumption period.

4. Ordering/Set up cost

$$OC = C_o \tag{9}$$

As shown in Figure 1, the inventory cycle is repeated every T_D units of time. Then the total unit cost is given by:

$$TUC = \frac{PC + BC + HC + OC}{T_D} \tag{10}$$

This cost function is the basic cost function represented by Rezaei (2014) that was later extended by different researchers incorporating various other assumptions such as deterioration, price-dependent demand, mortality, the negative impact of overbreeding, functional constraints, shortages and each will be discussed in Section 4.

Growth has some similarities with the well-grounded concept of amelioration, which was first introduced by Hwang (1997). The idea is frequent in the vinegar and wine industry. Both of the concepts illustrate the inventory level increase of the items during the storage. Amelioration is mainly taken as the opposite procedure of deterioration, where the utility enhancement is expressed in the form of an inventory level increase. However, growth depicts a biological process where the items undergo weight and size increase, leading to inventory level change. This depends on the number of initial young animals entering the system and the period during which the items are grown, which is not the case for the amelioration. As discussed, the growing inventory level change is usually governed by a biological weight function, while we mainly use the Weibull rate to model amelioration. Furthermore, growing inventories are prone to diseases and even mortality during their breeding period, which is not authentic for ameliorating inventory. Finally, the growing inventory system necessarily starts with a non-zero initial inventory, while there is no such limitation for the ameliorating items.

Considering the Economic Production Quantity (EPQ) with non-zero initial inventory, a resemblance between its performance and the growing inventory system can also be imagined. As we know, each period of replenishment of the inventory production system consists of two phases of production and consumption. But there are two major differences between the two systems. First, the amount of inventory added to the system during the production phase in EPQ is independent of the initial inventory and depends only on the production rate and the length of the production phase. However, as stated earlier, the increase in growing inventory depends not only on the growth function and the length of the breeding period but also on the initial inventory. The second distinction

arises from the change in the nature of inventory in the growth system. In other words, the type of inventory varies during the growth and consumption phase. For instance, during the growth phase, it is the stock of the livestock system, and by slaughtering it, the inventory changes into the meat products in the consumption phase. Therefore, if we assume that the product offered to the customer is the slaughtered inventory, demand does not enter the system during the growth phase. However, in the standard state of the EPQ, due to the same nature of inventory during production and consumption periods, demand affects the level of inventory both in the production and consumption phases.

Growing items are the initial inputs of various FSCs, and thereby any managerial decisions on these items have a direct impact on their entire supply chain. An estimated 60 percent of food losses occur during different activities through FSCs and before the items are received by the end consumers (Göbel et al. 2015). Besides, growing inventories are prone to different diseases and quality degradations. Then, any improper decision on these items not only leads to huge financial losses but also is directly linked to food safety issues. These highlight the significant position of appropriate inventory and revenue management policies for these items. Motivated by this significance, our study provides the essential grounds of the knowledge in growing inventory management and reviews the related research developments in this area to provide the researchers with clear insights into the structure of the current literature, its main gaps, and promising directions for future research.

The remainder of the paper is organized as follows. In Section 2, the review methodology, including the scope and classification, is provided. Section 3 presents a descriptive analysis of the existing literature, and this literature is reviewed in Section 4. Finally, Section 5 brings the paper to an end with discussions and conclusions.

2. Methodology

This paper reviews all relevant publications in the area of inventory, revenue, operations, and supply chain management of growing inventories. In order to guarantee the comprehensiveness of our review, we have incorporated a systematic search procedure to identify the relevant literature. Since the related literature is confined, we did not put any limits on the time range of publications, and all publication years until 2022 were included. We first searched Scopus and Google Scholar databases for the relevant literature based on the following keywords: {'Growth' OR 'Growing'} AND {'Inventory' OR 'Replenish' OR 'Order' OR 'Supply Chain' OR 'Revenue'} appearing in the title, abstract, or keywords, where OR and AND represent logical links between the terms. This search provided us with 31 papers, among which 12 were irrelevant and thereby were removed from our review set. We further replaced {'Growth' OR 'Growing'} with {'Amelioration' OR 'Ameliorating'} and, among the results, identified the publications relevant to fast-growing animals. Through this procedure, we identified three more relevant papers. Finally, the reference lists of identified publications were investigated to find remaining related research works that provided us with one further relevant paper.

Afterward, we analyzed the abstract of identified publications in a structured screening process. The first screening criterion was limiting the publications to those written in English and published in peer-reviewed scientific journals. Then, the abstracts were studied to ensure that the contents fall in the category of the mentioned keywords and the methodology is scoped with different streams of operations research, involving the terms 'Optimizations' OR 'Mathematical Programming' OR 'Nonlinear Programming' OR 'Integer Programming' OR 'Mixed-Integer Programming' OR 'Dynamic Programming' OR 'Convex Programming' OR 'Game Theory'.

This procedure led us to 23 publications that were used in our further analysis throughout the paper. The full texts of these research works were analyzed to extract the relevant data for classifying the literature. Then, the literature was thematically classified based on the context the growing inventory was studied, which was the most distinct feature differentiating the existing publications. Accordingly, the literature body of the problem was classified into three research topics, including company-level, two-level supply chain, and three (or more)-level supply chain structures.

3. Descriptive Analysis

In order to provide insights into the structure of our review database, the descriptive analysis of the articles is provided based on their publication year, contributing journals, and different thematic and subject-oriented factors. Figure 2. represents the distribution of the existing papers based on their publication year.

As Figure 2. illustrates, there is no considerable number of publications in the early years. Only 17% of the papers have been published before 2015, whereas 78% belong to the recent three years (2019-2021). This implies that growing inventory management is still in its very early stage, despite its essential role in the food industry.

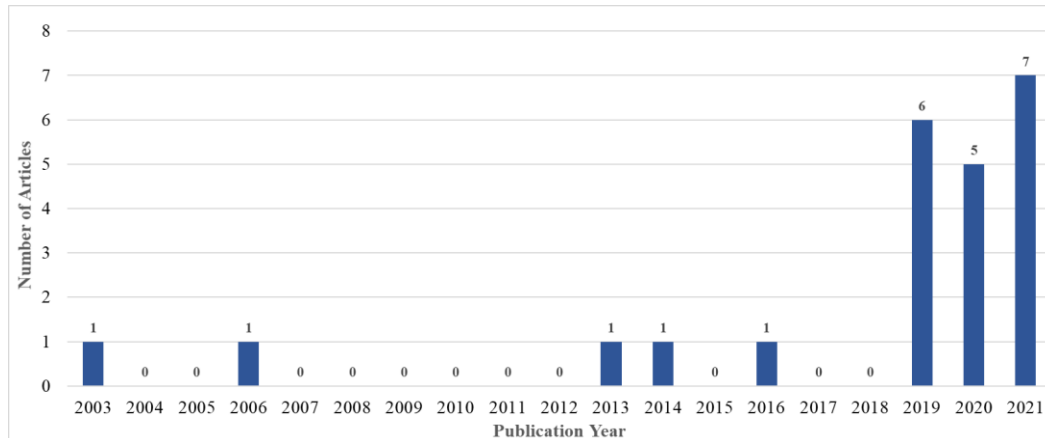


Figure 2. Distribution of the papers by their publication year

On the other hand, this literature deficiency introduces growing inventories as a promising direction in the inventory management area with various unexplored potentials to be investigated. The three early publications (2003, 2006, and 2013) investigated ameliorating inventories with a case study from the fast-growing animal industry. In 2014, Rezaei (2014) opened up the path in growing inventory management by extending the Economic Order Quantity model to the area of growth and applying a biologic weight function to model this growth for the first time. From 2019, this new path has gained fairly notable attention compared to the past, which may stem from the increasingly highlighted necessity of reducing food losses, enhanced food safety, and sustainable food production.

In order to analyze the existing literature further, Table 1 provides the distribution of the papers of our review database by their publishing journals.

Table 1. Distribution of the papers by their publishing journals

Journal	Number of publications
1 International Journal of Production Economics	3
2 Arabian Journal for Science and Engineering	3
3 Applied Mathematical Modelling	2
4 Opeartions Research Perspectives	2
5 Mathematical and Computer Modelling	2
6 Annals of Operations Research	1
7 Computers and Industrial Engineering	1
8 Journal of Cleaner Production	1
9 International Journal of Procurement Management	1
10 Knowledge-Based Systems	1
11 Journal of Industrial Engineering International	1
12 Journal of Modelling in Management	1
13 Production and Manufacturing Research	1
14 International Journal of Applied and Computational Mathematics	1
15 Mathematical Problems in Engineering	1
16 Journal Teknik Industri	1
Total number of publications	23

There exist 16 contributing journals in our review database, which mainly publish in the areas of management science and operations research, applied mathematics, and industrial engineering. International Journal of Production Economics and Arabian Journal for Science and Engineering, each with three articles, are the top two contributing journals.

The structure of the problem, including company-level, two-level supply chain, and three (or more)-level supply chain, is the other factor in our analysis. Figure 3 outlines the distribution of the papers by their problem structure.

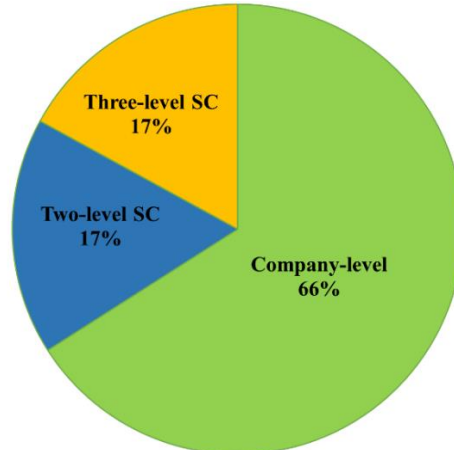


Figure 3. Distribution of the papers by their problem structure

As Figure 3 illustrates, the majority of the papers deal with the growing inventory management at a company level. Studying growth in the context of supply chain management is limited to only eight articles, with mostly simple chain structures. This is while the food supply chain of growing items requires a high deal of academic attention due to its direct link to food safety, food waste, and sustainability issues.

As previously mentioned, we have focused on the growing inventory management problems, where different streams of operations research are applied to model and solve the problem. Figure 4 shows the contribution of different methods in modeling and solving growing inventory management. Please note that some papers may fall into more than one category.

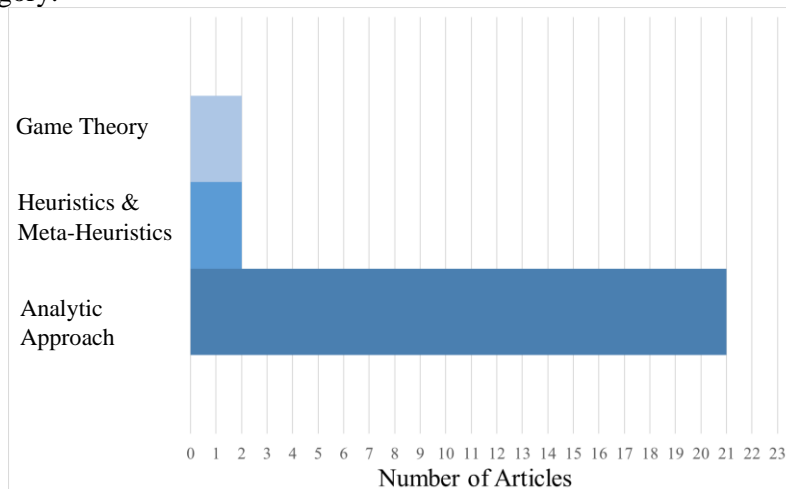


Figure 4. Distribution of the papers by their methodology

Figure 4 shows that most of the papers have incorporated an analytic approach, the most widely used method in nonlinear programming and thereby inventory control problems. Heuristics and Meta-Heuristics appear in two papers, where different constraints are taken into account. Game theory is a rigorous implement in modeling the interactions between members of complex supply chains structures. However, as the figure depicts, this is applied in two papers.

In addition to the mentioned criteria, the existing literature highly differs based on their modeling assumptions. Table 2 provides an overview of these papers concerning several classification factors.

Table 2. Distribution of the papers by their publishing journals

Reference	Structure			Growth	Mortality	Shortage	Pricing	Deterioration	Simultaneous impact of BP & IIL	Age-dependent breeding cost	Impact of overbreeding	Uncertainty	Solution Approach
	CL	2L SC	3L SC										
Mondal et al. (2003)	✓			WR			✓	✓					Analytic
Law and Wee (2006)		✓		WR				✓					Analytic
Goyal et al. (2013)	✓			WR		✓							Meta-Heuristics
Rezaei (2014)	✓			BWF					✓	✓			Analytic
Zhang et al. (2016)	✓			BWF					✓	✓			Analytic
Nobil et al. (2019)	✓			LR		✓							Analytic
Malekitabar et al. (2019)		✓		BWF	✓		✓			✓			Analytic Game Theory
Sebatjane and Adetunji (2019a)	✓			BWF LR									Analytic
Sebatjane and Adetunji (2019b)	✓			BWF									Analytic
Khalilpourazari and Pasandideh (2019)	✓			BWF					✓	✓			Meta-Heuristics
Sebatjane and Adetunji (2019c)		✓		BWF									Analytic
Sebatjane and Adetunji (2020a)			✓	BWF								✓	Analytic
Gharaei and Almehdawe (2020)	✓			LR	✓				✓				Analytic
Hidayat et al. (2020)	✓			LR					✓				Analytic

Table 2. Continued

Reference	Structure			Growth	Mortality	Shortage	Pricing	Deterioration	Simultaneous impact of BP & IIL	Age-dependent breeding cost	Impact of overbreeding	Uncertainty	Solution Approach
	CL	2L SC	3L SC										
Pourmohammad-Zia and Karimi (2020)	✓			BWF				✓	✓	✓	✓		Analytic
Sebatjane and Adetunji (2020b)			✓	BWF			✓	✓					Analytic
Alfares and Afzal (2021)	✓			LR		✓						✓	Analytic
De-la-Cruz-Márquez et al. (2021)	✓			BWF		✓	✓					✓	Analytic
Pourmohammad-Zia et al. (2021a)		✓		BWF			✓	✓	✓	✓	✓		Analytic
Mittal and Sharma (2021)	✓			BWF									Analytic
Pourmohammad-Zia et al. (2021b)			✓	BWF			✓	✓	✓	✓	✓		Analytic Game Theory
Sebatjane and Adetunji (2021)			✓	BWF				✓					Analytic
Gharaei and Almehdawe (2021)	✓			LR	✓	✓			✓				Analytic

WR: Weibull Rate, LR: Linear Rate, BWF: Biological Weight Function

As the table projects, several areas in the related literature face serious deficiency, while some directions are more heeded. A complete analysis of this literature will be provided later.

4. Review of The Literature

The studies on growing inventory are very confined compared to that of the deteriorating inventory. There exists a massive body of literature on revenue and inventory management of deteriorating items (interested readers are referred to Rabbani et al. (2014), Janssen et al. (2016), Rabbani et al. (2016), Rabbani et al. (2017), Taleizadeh et al. (2019), and Al-Amin et al. (2020)). This suggests that, due to similarities shared, one can expect a high deal of new research developments on growing inventory in upcoming years. In this section, the existing literature on growing inventory management is reviewed based on the problems' basis, including company-level, two-level supply chain, and three level supply chains.

4.1. Company-Level Models

The majority of the literature on growing inventory models is dedicated to company-level structure, where the inventory system of a rearing farm is studied.

Mondal et al. (2003) proposed one of the early studies in this area. Although they studied an inventory system of the ameliorating items, their developed model shares similarities with that of the growing inventories. They simultaneously considered amelioration and deterioration to model the growth and death of the items. The Weibull distribution rate is applied in their model to reflect the increase and decrease of the inventory level. They used a prescribed cycle length threshold to decrease the complexity of their problem, which is the main limitation of this study. They showed that varying amelioration and deterioration rates has a notable impact on the breeder's profit. However, no clue is introduced on the reasons for changes in these two rates. Goyal et al. (2013) developed a similar model in which amelioration reflects the behavior of fast-growing animals such as ducks, pigs, and broiler chickens. Despite the previous research, in this model, shortages are allowed and partially backlogged. Due to its complexity, their developed model falls in the class of non-convex programming, where the optimality of the solutions cannot be guaranteed. A Genetic Algorithm (GA) is developed to solve this model, but the quality of the obtained solutions cannot be evaluated. This is one of the main challenges in nonlinear models, for which a slight change in assumptions may lead to a non-convex programming outcome.

Despite for deterioration, Weibull distribution is not able to capture the behavior of growth properly. There exist biological time-dependent weight functions for fast-growing animals that accurately measure the items' weight at any instant point of the time. Rezaei (2014) opened the path for growing inventory management, establishing a link between a number of biological features of fast-growing animals and operations research. Accurately, instead of using Weibull distribution, he modeled growth as a common weight increase function in poultry and livestock literature, which was introduced in Section 1. Moreover, the breeding procedure is highlighted by considering age-dependent feeding costs and production function. While the paper has incorporated the concept in a simple EOQ structure, it can be regarded as the basis for later extensions in this area. The results indicated that breeding costs are the most influential parameter in this model. Later, Zhang et al. (2016) extended this research by taking into account carbon emission and environmental factors.

An important point in modeling growing inventory control problems is heeding the simultaneous impact of the initial number of newborn animals and the length of the breeding period on the final inventory level. These two factors are the two key decision variables in the system of a rearing farm affecting its profit. Following the direction initiated by Rezaei (2014), Nobil et al. (2019) extended the model by allowing for shortages in the form of full backorders. In order to manage the complexity raised by allowing for shortages, they treated the breeding period as a known parameter by specifying the initial and final weight of the items. In this way, the model is transformed into a variant of the EOQ model, where the optimal initial order size should be determined.

Sebatjane and Adetunji's (2019a) research is the other extension in this area that has considered the breeding period as a fixed parameter by taking a customer-preferred final weight. A fraction of the items is distinguished as imperfect quality after being slaughtered. Additionally, they applied linear and split linear functions instead of biological logistic weight function and showed that the errors raised by applying these two simplified functions are considerable. This suggests that the value of accuracy outweighs the cost of increased complexity by applying the logistic function. Later, Sebatjane and Adetunji (2019b) extended their previous work by incorporating quantity discounts. Alfares and Afzal (2021) proposed similar research taking the imperfect quality of the items into account and allowing for shortages. In a similar manner, they neglected the simultaneous impact of the

newborn young animals' number and growth period on the final inventory level by taking the slaughtering weight of the items as a known value.

Embedding operational constraints that many systems face into the structure of complex inventory control problems is not trivial. This is because we may transform the problem into non-convex programming in this way, for which optimality is not guaranteed. In such cases, the development of an appropriate solution algorithm is a key challenge that gives rise to the scarcity of constrained growing inventory control models. Khalilpourazari and Pasandideh (2019) developed a multi-item and multi-constraint EOQ model for the growing inventories. They considered three operational limitations in their model, including an on-hand budget, warehouse capacity, and total allowable holding cost constraints. Sequential Quadratic Programming is utilized to solve the problem in small size instances, and two hybrid metaheuristics are developed for medium and large size instances. The authors have claimed that their proposed solution approaches perform notably well. However, since the exact solution cannot be necessarily achieved in even small-size problems, their claim cannot be validated. Similarly, Hidayat et al. (2020) took notice of capacitated storage facility and limited budget in their growing inventory model. In order to manage the complexities raised by incorporating these two constraints, they applied a linear growth function.

During growth, the items are prone to different diseases that can lead to their death. Therefore, mortality of the growing inventories is an assumption, heeding which captures a more realistic behavior of the growth. Despite its prevalence in reality, mortality has been mostly neglected in the existing literature. Gharaei and Almehdawe (2020) proposed one of these studies in the area. To model the mortality of the items, they considered two uniform density functions for survival and mortality. The growth function is taken as a linearly time-dependent function to decrease model complexities. They showed that the initial weight of each unit item plays a key role in their model. Gharaei and Almehdawe (2021) extended their previous work by considering the environmental effects of greenhouse gases emitted from produced manure, fermentation process, and transportation. Their results indicated that taking sustainability into account increases the order quantity and shortens the growth cycle, bringing a lower weight for each slaughtered unit item.

The diseases during the growth do not necessarily lead to the mortality of the items but can impact the quality of the inventory being slaughtered. That is to say, the items may lose the quality standards due to illnesses and overbreeding. On the other hand, as the items grow, the ratio of useless weight (such as fat) to their whole weight increases (Jensen et al. 1974). Accordingly, a fraction of the slaughtered inventory is discarded, which is directly linked to the length of the breeding period. Pourmohammadzia and Karimi (2020) took this point into account by conducting an instantaneous quality control process of the slaughtered items at the end of the breeding period. They defined an exponentially breeding-time-dependent function for a fraction of the items being discarded after their slaughter. They investigated the developed structure for the case of the broiler chickens and showed that this assumption shortens the length of the breeding period by roughly 25% compared to that of the classical value in poultry farms (which is 42 days).

Revenue management policies such as pricing, advertisement, discounting, and trade credit policies usually have a direct impact on customer demand. By the fluctuations in customer demand, ordering and inventory holding decisions will require update and adaptation. Consequently, joint inventory and revenue management policies can considerably improve the performance of the firms and have been vastly investigated for different inventory problems such as deteriorating items. However, the related papers for the growing inventory are few. De-la-Cruz-Márquez et al. (2021) investigated optimal replenishment and pricing decisions for growing items, where the demand is a price-sensitive polynomial function, and shortages are fully backordered. Similar to the majority of the existing papers, they treated the breeding period as a fixed value to handle model complexities. They showed that allowing for shortages and the firm's ability to set the appropriate selling prices has a positive influence on their developed inventory model. Mittal and Sharma (2021) studied the inventory system of a rearing farm that can benefit from the delay in payments for its ordered items. They showed that trade credit financing could considerably enhance the profit of the firm and suggested incorporating this policy in the structure of more complex growing inventory models.

4.2. Two- Echelon Supply Chain Models

Growth in the context of supply chain problems is even a more limited literature stream compared to company-level problems. In the two-level supply chains, usually, a supplier-retailer scenario is studied where the supplier is the rearing farm, and the retailer is the point selling the slaughtered items to the customers. The literature in this

area is initiated by Law and Wee's (2006) work for ameliorating and deteriorating items that share some similar features with growing inventories. They investigated a two-echelon supply chain, where the supplier buys growing items and raises them to use as raw material in the production system. Then, the items are sent to the retailer to satisfy customer demand. This study confronts serious shortcomings. The most notable weakness is the inconsistency of the ordering policy throughout the chain. Precisely, fixed batches of finished items are sent to the retailer, but the stock level of the manufacturer declines continuously due to demand. Furthermore, the uniqueness of the obtained results is not justified in their solution approach.

Malekitabar et al. (2019) proposed a growing inventory model for rainbow trout. They considered the initial inventory level of the items to be known and the nature of the items to be unchanged through time. Expressly, the items are not slaughtered at a point in time, and the demand is for the growing items. They took the mortality of the inventory into account by considering a deterioration rate and optimized the periodic profit of the system, which implicitly suggests that the problem is analyzed for a one-period case. The problem is studied under centralized and decentralized supply chain structures, and revenue and cost-sharing contracts are applied as coordination mechanisms in their developed chain. Sebatjane and Adetunji (2019c) studied the inventory system of a farmer and a retailer, for which the final weight of the growing items and thereby the breeding period are again predetermined. Pourmohammad-Zia et al.'s work (2021a) is among the few studies in the context of the supply chain, where the simultaneous impact of the initial number of newborn animals and the length of the breeding period on the final inventory level are considered. As previously discussed, the growing items usually fall in the category of deteriorating inventory after slaughter. However, this is mostly overlooked in the related literature. Pourmohammad-Zia et al. (2021a) took this into account by modeling a continuous inventory level decline at the retailer. Furthermore, they defined the customer demand at the retailer as a price-sensitive function and, as the only paper in the area, applied dynamic pricing to atone for the negative impact of deterioration.

4.3. Three-Echelon Supply Chain Models

In three-echelon supply chains, usually, a rearing farm as the supplier, a manufacturer (processor) or a distributor, and one or more retailers form the chain's structure. Due to their complexities, three-echelon chains are rarely studied, even in classic or deteriorating inventory problems. That is because, in complex chains, modeling the interactions among the members and the flow of the inventory with its varying nature is not trivial, and it gets even more complicated when growing inventories come to play. In these supply chains that are usually referred to as Food Supply Chains (FSCs), coordination mechanisms are required to ensure a smooth flow of the product from the initial suppliers to the final consumers, enhance the performance of the chain, and minimize the food losses. Game Theory is known as a rigorous implement in this regard, where the competition, coalition, collaboration, and cooperation of the members are outlined.

Sebatjane and Adetunji (2020a) extended the idea of imperfect quality items proposed in Sebatjane and Adetunji's (2019a) to the context of a three-level supply chain, including a farmer, a food processor, and a retailer. Despite their previous research, here the imperfect quality items are not discarded but sold at a discounted price. They showed that this assumption could considerably enhance the performance of the proposed supply chain. Later, Sebatjane and Adetunji (2021) studied a similar supply chain, where the slaughtered items are considered to deteriorate over time, and the demand rate is dependent on the inventory level and expiration date of the items. They relaxed the traditional zero-ending inventory policy at the retail end of the supply chain and assumed that the retailer always keeps on-hand inventory and starts a new replenishment cycle once the inventory level drops to a certain minimum value. Furthermore, the retailer has a clearance sale at the end of the cycle to clear out the ending on-hand inventory. Their results indicate that in terms of profitability enhancement, this policy outperforms the traditional zero-ending inventory policy.

When revenue management policies are coupled with inventory management, the performance of FSCs is greatly improved due to the lower food losses and mismatches, increased profit, higher clearance rate, and enhanced customer service level. Despite the benefits, the related studies are very few since the resulting models get intricate. Sebatjane and Adetunji (2020b) investigated optimal pricing, ordering, and shipment decisions in a three-level FSC, where the customer demand is price and freshness-dependent. Pourmohammadzia et al. (2021b) proposed the other research in this area, where pricing, breeding, ordering, and production decisions are studied in a three-level FSC consisting of a rearing farm, a processed food manufacturer, and multiple retailers. The manufacturer applies Vendor Managed Inventory (VMI) to handle the inventory systems' of its multiple retailers.

In order to create enough incentives for the retailers to join this coordination scheme, it offers inventory holding cost-sharing contract to the retailers. In addition to this interaction between the manufacturer and the retailers, the retailers compete on their selling price with each other. These interplays among the chain members are modeled by Nash and Stackelberg competitions. Their results show that the outlined VMI-based coordination mechanism enhances the profit of all chain members and is also more desirable for the customers as the selling prices undergo a drop compared to the classic retailer-managed system.

5. Discussion and Conclusion

Growth outlines the behavior of a class of inventories whose weight and size increase during their storage. Fast-growing young animals such as broiler chickens, ducks, and calves are examples of this class. Growing inventory management is a young stream of inventory control and supply chain management problems initiated by Rezaei's research in 2014. Although the concept is a fundamental ingredient in the majority of the food supply chains, the academic research in the area is very confined and in its infancy. This is while deterioration, as the other prevalent phenomenon in FSCs, has been extensively investigated by the scholars in the area, which suggests that growing inventory management has the potential to become an active area of academic research in the field of food supply chain management.

By the systematic search in existing databases, only 23 publications have been identified, among which 65% are dedicated to the problems at the company level. This shows that the confined body of the literature in growing inventory management gets even more limited when it comes to the problems in the context of the supply chain. In the context of supply chains, especially when the number of involved members increases, modeling the interactions among the members and the flow of the inventory with its varying nature is very challenging and requires a high deal of attention. Coordination mechanisms are required in this regard to ensure a smooth flow of the product from the initial suppliers to the final consumers, enhance the performance of the chain, and minimize food losses.

There are several other gaps in existing papers at the company and supply chain level that can be served as promising directions for future research:

- Although it is shown that accurate estimation of growth rate is highly crucial and any over or underestimation can lead to substantial losses in the system, there are still several papers applying Weibull distribution or a linear rate to model the growth. These rates are not able to outline the behavior of the growing inventories properly, and biological weight functions should be applied in this respect. Accordingly, it is highly recommended that researchers in the area pay specific attention to this initial step in modeling growing inventory problems. In this regard, data-based methodologies such as machine learning techniques can be successfully applied for fitting the growth functions.
- A key feature in modeling growth is the simultaneous impact of the number of newborn animals entering the system and the length of the breeding period on the final inventory level. Since considering these two factors increases the complexity of the problems, it is only heeded by 36% of the research works. This is while neglecting this simultaneous impact transforms the model into a variant of the EOQ model that does not possess the features of growing inventory models.
- Growing items are prone to different diseases, leading to their death. So, mortality is an assumption, noting which captures a more realistic behavior of the growth. Despite its prevalence in reality, this assumption has been mostly neglected in the existing literature. There are only three related papers with very simplified mortality rates. Besides, preventive practices such as early vaccination and parasitic inspection can effectively control this mortality rate. Hence, modeling this interdependency is a prerequisite to illustrate a more pragmatic setting of the problem in rearing farms.
- The negative impact of overbreeding is notable on the quality of the final slaughtered inventory, which can be addressed by defining a breeding period-dependent function representing the rate of the discarded item during the quality control. This is almost overlooked and heeded only by three papers, introducing it as an important direction for future research. The quality of the slaughtered items is directly linked to food safety issues. In addition to the breeding period, a large variety of factors such as feeding components, breeding conditions, preventive practices play a fundamental role in food safety. Accordingly, taking health-related issues into account is another promising future direction.

- Animal welfare is another issue to take care of in practice, which is specifically important in farms where multiple animals are growing next to each other. High welfare farming can be less damaging to the environment, and farmers can earn more too.
- Age-dependent feeding and breeding costs and deterioration are the other two almost overlooked assumptions in the literature. The costs associated with feeding and breeding increase as the items get older. Capturing this requires the application of an age-dependent feeding cost term that is noted only by 30% of the existing papers. The items after being slaughtered fall in the category of deteriorating inventories, which are prone to spoilage during the time. However, this assumption is also heeded only by 30% of the publications.
- Considering uncertainty is one of the important directions that can resolve the challenge of different growth rates within a single flock. More precisely, the use of uncertainty in growth parameters can give a distributive value for the breeding period instead of a fixed value. This direction has not been investigated in the existing research works.
- Allowing for shortages in the form of lost sales or backorders and joint pricing and inventory control decisions (due to its capabilities in enhancing the performance of the system) are the other less studied directions requiring a higher deal of attention in future research works.
- These days concerns raised around sustainability and circular economy are tremendously increased. So, one very interesting extension is to study the water-food-energy nexus in growing items' production. In such a setting, the interactions between these three pillars are studied while taking into account the synergies and trade-offs from the management of the three resources and potential areas of conflict.
- Seasonal patterns can have a considerable impact on the operations management of growing items. This can involve seasonal changes in the feeding prices for the breeder (as in some seasons feeding could be cheaper than other seasons), together with fluctuations in demand or selling prices of slaughtered items (as an instance, the case of Turkey on Thanksgiving). Embedding these patterns into the structure of developed models opens up a promising direction for future research.
- The inventory system of growing items can face various constraints such as limited storage or breeding space, available budget, retail price threshold. Applying these assumptions may turn the model into non-convex programming, which requires the development of heuristic and meta-heuristic solution approaches to obtain a high-quality (and not necessarily optimal) solution, which has always been a major challenge in non-convex optimization problems. Transforming the model from an infinite programming horizon into a multi-period setting, which creates some simplifications and turns nonlinear relationships into linear ones, can be considered in this regard.
- In recent years, supply chain management based on integrated and hybrid models has received much attention. Among these, we can mention the adoption of integrated pricing and inventory control decisions accounting for transportation decisions. This can be considered as a suitable direction for future research. However, it is necessary to pay attention to what has been said in the previous case regarding non-convex optimization.

References

- Al-Amin, K. M., Shaikh, A. A., Panda, G. C., Bhunia, A. K., and Ioannis, K. (2020) 'Non-instantaneous deterioration effect in ordering decisions for a two-warehouse inventory system under advance payment and backlogging', *Annals of Operations Research*, 289, 243-275.
- Alfares, H. K., and Afzal, A. R. (2021) 'An economic order quantity model for growing items with imperfect quality and shortages', *Arabian Journal for Science and Engineering*, 46, 1863-1875.
- De-la-Cruz-Márquez, C. G., Cárdenas-Barrón, L. E., and Mandal, B. (2021) 'An Inventory Model for Growing Items with Imperfect Quality When the Demand Is Price Sensitive under Carbon Emissions and Shortages', *Mathematical Problems in Engineering*, In press.
- Gharaei, A., and Almehdawe, E. (2020) 'Economic growing quantity', *International Journal of Production Economics*, 223, 1-13.
- Gharaei, A., and Almehdawe, E. (2021) 'Optimal sustainable order quantities for growing items', *Journal of Cleaner Production*, 307, 1-25.
- Göbel, C., Langen, N., Blumenthal, A., Teitscheid, P., and Ritter, G. (2015) 'Cutting food waste through cooperation along the food supply chain', *Sustainability*, 7, 1429-1445.

- Goyal, S., Singh, S., and Dem, H. (2013) 'Production policy for ameliorating/deteriorating items with ramp type demand', *International Journal of Procurement Management*, 6, 444-465.
- Hidayat, Y. A., Riaventin, V. N., and Jayadi, O. (2020) 'Economic order quantity model for growing items with incremental quantity discounts, capacitated storage facility, and limited budget', *Jurnal Teknik Industri*, 22, 1-10.
- Hwang, H.-S. (1997) 'A study on an inventory model for items with Weibull ameliorating', *Computers and Industrial Engineering*, 33, 701-704.
- Janssen, L., Claus, T., and Sauer, J. (2016) 'Literature review of deteriorating inventory models by key topics from 2012 to 2015', *International Journal of Production Economics*, 182, 86-112.
- Jensen, L., Falen, L., and Chang, C. (1974) 'Effect of distillers dried grain with solubles on reproduction and liver fat accumulation in laying hens', *Poultry Science*, 53, 586-592.
- Khalilpourazari, S., and Pasandideh, S. H. R. (2019) 'Modeling and optimization of multi-item multi-constrained EOQ model for growing items', *Knowledge-Based Systems*, 164, 150-162.
- Law, S. T., and Wee, H. M. (2006) 'An integrated production-inventory model for ameliorating and deteriorating items taking account of time discounting', *Mathematical and Computer Modelling*, 43, 673-685.
- Malekitabar, M., Yaghoubi, S., and Gholamian, M. R. (2019) 'A novel mathematical inventory model for growing-mortal items (case study: Rainbow trout)', *Applied Mathematical Modelling*, 71, 96-117.
- Mittal, M., and Sharma, M. (2021) 'Economic Ordering Policies for Growing Items (Poultry) with Trade-Credit Financing', *International Journal of Applied and Computational Mathematics*, 7, 1-11.
- Mondal, B., Bhunia, A. K., and Maiti, M. (2003) 'An inventory system of ameliorating items for price dependent demand rate', *Computers and Industrial Engineering*, 45, 443-456.
- Nobil, A. H., Sedigh, A. H. A., and Cárdenas-Barrón, L. E. (2019) 'A generalized economic order quantity inventory model with shortage: case study of a poultry farmer', *Arabian Journal for Science and Engineering*, 44, 2653-2663.
- Pourmohammad-Zia, N., and Karimi, B. (2020) 'Optimal replenishment and breeding policies for growing items', *Arabian Journal for Science and Engineering*, 45, 7005-7015.
- Pourmohammad-Zia, N., Karimi, B., and Rezaei, J. (2021a) 'Dynamic pricing and inventory control policies in a food supply chain of growing and deteriorating items', *Annals of Operations Research*, In press.
- Pourmohammad-Zia, N., Karimi, B., and Rezaei, J. (2021b) 'Food Supply Chain Coordination for Growing Items: A trade-off between Market Coverage and Cost-Efficiency', *International Journal of Production Economics*, In press.
- Rabbani, M., Zia, N. P., and Rafiei, H. (2014) 'Optimal dynamic pricing and replenishment policies for deteriorating items', *International Journal of Industrial Engineering Computations*, 5, 621-630.
- Rabbani, M., Zia, N. P., and Rafiei, H. (2016) 'Joint optimal dynamic pricing and replenishment policies for items with simultaneous quality and physical quantity deterioration', *Applied Mathematics and Computation*, 287, 149-160.
- Rabbani, M., Zia, N. P., and Rafiei, H. (2017) 'Joint optimal inventory, dynamic pricing and advertisement policies for non-instantaneous deteriorating items', *RAIRO-Operations Research*, 51, 1251-1267.
- Rezaei, J. (2014). 'Economic order quantity for growing items', *International Journal of Production Economics*, 155, 109-113.
- Richards, F. (1959). 'A flexible growth function for empirical use', *Journal of Experimental Botany*, 10, 290-301.
- Sebatjane, M., and Adetunji, O. (2019a) 'Economic order quantity model for growing items with imperfect quality', *Operations Research Perspectives*, 6, 1-10.
- Sebatjane, M., and Adetunji, O. (2019b) 'Economic order quantity model for growing items with incremental quantity discounts', *Journal of Industrial Engineering International*, 15, 545-556.
- Sebatjane, M., and Adetunji, O. (2019c) 'Three-echelon supply chain inventory model for growing items', *Journal of Modelling in Management*, 15, 567-587
- Sebatjane, M., and Adetunji, O. (2020a) 'Optimal inventory replenishment and shipment policies in a four-echelon supply chain for growing items with imperfect quality', *Production and Manufacturing Research*, 8, 130-157.
- Sebatjane, M., and Adetunji, O. (2020b) 'A three-echelon supply chain for economic growing quantity model with price-and freshness-dependent demand: pricing, ordering and shipment decisions', *Operations Research Perspectives*, 7, 1-15.
- Sebatjane, M., and Adetunji, O. (2021) 'Optimal lot-sizing and shipment decisions in a three-echelon supply chain for growing items with inventory level-and expiration date-dependent demand', *Applied Mathematical Modelling*, 90, 1204-1225.
- Taleizadeh, A. A., Pourmohammad-Zia, N., and Konstantaras, I. (2019) 'Partial linked-to-order delayed payment and life time effects on decaying items ordering', *Operational Research*, 21, 2077-2099.
- Zhang, Y., Li, L. Y., Tian, X. Q., and Feng, C. (2016) 'Inventory management research for growing items with carbon-constrained', *35th Chinese Control Conference (CCC)*, Chengdu, China, 9588-9593.

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