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Rock armour slope stability under wave attack in shallow water

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Abstract

The Van der Meer stability formula for rock slopes under wave attack has recently been revised, replacing the mean period T_m with the spectral period $T_{m-1,0}$. This rewritten Van der Meer formula closely resembles the Modified Van der Meer formula for shallow water as in the Rock Manual, with differences primarily in coefficients and the use of $H_{2\%}$ in the Rock Manual and $H_{1/3}$ or H_{m0} in the rewritten Van der Meer formula.

The wave characteristics changes significantly in shallow water due to nonlinearities and wave breaking. The result is a significant change in the wave height and period, especially when severe breaking occurs and infragravity waves become significant or even dominate the spectrum. This may lead to very large breaker parameters $\xi_{m-1,0}$. At a certain point, existing stability formulae may thus become inaccurate, both the original, rewritten and the Modified formula for shallow water. The primary objective of this paper is to identify when and where shallow water stability results deviate from established formulae and how these deviations can be described.

The analysis involves an in-depth examination of the dataset that led to the Modified Van der Meer formula, and relevant data from other sources to increase the understanding of waves in shallow water and how they affect rock slope stability.

The use of $H_{2\%}$ in the Modified Van der Meer formula gives some difficulties as no reliable prediction method is available for that parameter when the relative depth is small, $h/H_{m0\,deep} < 1.5$. The rewritten Van der Meer formula applies the significant wave height, and it may be chosen as either H_{m0} or $H_{1/3}$. These two parameters are almost identical in deep water for which the formula was derived, but significant differences may occur in shallow water. The application of the rewritten Van der Meer formula in shallow water indicates a preference for use of H_{m0} as it describes nonlinear waves better. The main conclusion is that this rewritten formula seems valid much further into the shallow water region than what the Rock Manual recommends and at least to relative water depths of $h/H_{m0\,deep} > 1.5$. For shallow water with $h/H_{m0\,deep} < 1.5$, no systematic trend with the energy period is observed anymore and constant combined stability numbers are given for guidance in preliminary design.

Keywords

Coastal structures, stability, rock armour slopes, damage, spectral period, relative water depth, shallow foreshores

1 Introduction

1.1 Background

The Van der Meer formula on rock slope stability against wave attack has been given in major design manuals such as the Rock Manual (2007) and the Coastal Engineering Manual (CEM 2002). The original formula was developed without a sloping foreshore and without depth-induced wave breaking (Van der Meer, 1987, 1988a and 1988b). Although a limited number of tests were performed on a foreshore of 1:30. Research by Smith et al. (2002) and Van Gent et al. (2003) resulted in the Modified Van der Meer formula for shallow water conditions. The main modifications to the Van der Meer formula were that: the spectral wave period $T_{m-1,0}$ was used instead of the mean period T_m; the wave height $H_{2\%}$ was used instead of the significant wave height H_s; and new coefficients were fitted. The essence of both formulae, however, is very much the same, such as the influence of permeability of the structure, the number of waves, the slope angle, the wave steepness and damage development with wave height.

The Rock Manual (2007) gives the transition from the original formula for deep water to the Modified formula for shallow water conditions at $h_{toe}/H_{s\ toe} = 3$. Here h_{toe} is the water depth at the toe of the structure and $H_{s\ toe}$ the significant wave height at the same location. The limit $h_{toe}/H_{s\ toe} = 3$, or a little larger ($h_{toe}/H_{s\ toe} = 4$, Hofland et al. 2017), is well known for describing the shallowness of the foreshore and dividing deep water from shallow water *wave* conditions. But no validation has been given in Van Gent et al. (2003), nor in the Rock Manual (2007) that it should also yield for *stability of rock slopes*. It might well be that the original Van der Meer formula is applicable for much smaller values than $h_{toe}/H_{s\ toe} = 3$. This paper will look into that aspect.

Traditionally, in wave mechanics deep and shallow water refers to the behaviour of the waves and if the wave is influenced by the bottom or not. However, shallow water may also refer to the breaking of the waves due to depth limitation. The first phenomenon is described by the water depth to wavelength ratio while the latter is described by the wave height to water depth ratio. In the present paper, shallow water refers to waves being influenced by depth-induced breaking. Hofland et al. (2017) characterise the waves in the depth-induced breaking zone by the shallowness of the foreshore, using the ratio $h/H_{m0\ deep}$, where h is the local water depth and $H_{mo\ deep}$ is the spectral significant wave height at deep(er) water. Note that unless the wave steepness in deep water is small the difference is small between the wave heights $H_{s\ toe}$ and $H_{m0\ deep}$ for a value of $h/H_{m0\ deep} > 3$. This is because depth-induced breaking does not occur and when $h/H_{m0\ deep} > 3$, then nonlinear shoaling is only large for low steepness waves. The nonlinear shoaling refers to the growth of the bound sub- and superharmonics that especially effects the highest waves in the sea state. The classification of Hofland et al. (2017) gives deep water (no breaking) for $h/H_{m0\ deep} = 4$; shallow water for $h/H_{m0\ deep} = 1\text{-}4$; very shallow water for $h/H_{m0\ deep} = 0.3\text{-}1$; and extremely shallow water for $h/H_{m0\ deep} < 0.3$. For very shallow and extremely shallow water infragravity waves come into play or even dominate the process or the spectrum. Infragravity waves are caused by nonlinear interactions that generate subharmonic wave components with wave frequencies below the wind generated sea and the swell waves. The infragravity waves are long waves with periods typically with an order of 100 s in prototype (Hofland et al. 2017). Thus, $h/H_{m0\ deep}$ is an important parameter to describe the behaviour of waves over a foreshore. Therefore, it may also be an important parameter to analyse stability of rock slopes in shallow water.

Due to wave breaking at shallow foreshores, the wave height may reduce significantly and due to infragravity waves the wave period may increase drastically. This all may lead to large and extremely large breaker parameters, $\xi_{m-1,0}$, much larger than in the application area of the stability formula. It is expected that with such large breaker parameters, the stability formula will not any longer be correct, not for the original Van der Meer formula, nor for the Modified Van der Meer formula for shallow water. This paper will study this behaviour.

Van der Meer (2021) has revisited the Van der Meer formula and has rewritten it to include the spectral wave period $T_{m-1,0}$ instead of the mean period T_m. A guideline was also presented on how to integrate new research for the formula's application areas. In this way, a direct comparison becomes possible between the original rewritten and the Modified Van der Meer formula, as they now only differ in coefficients and the use of H_s or $H_{2\%}$. This comparison will be described in this paper.

A comparison with the rewritten Van der Meer formula should first start with a validation of two essential relationships in the formula (Van der Meer, 2021). These are a gradually developing damage curve according to a power function and secondly the fixed relationship between damage development and storm duration. Van der Meer (2021) gives two examples of such a comparison. The work of Bradbury et al. (1988) and Latham et al. (1988) on the influence of rock shape, could indeed be described by the rewritten Van der Meer formula including modification factors. But the work of Stewart et al. (2002, 2003) on individual and dense rock armour placing showed that there was no gradually developing damage curve according to a power function, but a kind of brittle failure: no damage up to a high wave height and then suddenly damage and failure. Such behaviour cannot be described by the Van der Meer formula and an alternative method of analysis was given in Van der Meer (2021).

Van der Meer (2021) gave the application area of the rewritten Van der Meer formula in a so-called stability versus breaker parameter graph. That graph has been repeated here as Figure 1, as it is the basis for comparison with data on stability of rock slopes in shallow water. For explanation of parameters, please look at the Notation list at the end of the paper. The notional permeability factor $P = 0.1$ gives a structure with an impermeable core, $P = 0.5$ is a structure with a permeable core and $P = 0.6$ is a homogeneous structure (only large armour stones). The curves in Figure 1 cover the tested ranges and are not applied outside these ranges. The stability for plunging waves (the curves on the left side) decreases with decreasing permeability of the structure. The curves in this area are parallel (shifted vertically by the P-influence). The stability of permeable structures increases significantly in the surging waves region (right side of the graph), if the breaker parameter increases. The curves are almost horizontal for an impermeable structure. A larger value of P gives a steeper curve and it is obvious that large breaker parameters outside the application range (as may happen at shallow foreshores) may give way too optimistic stability results in this case.

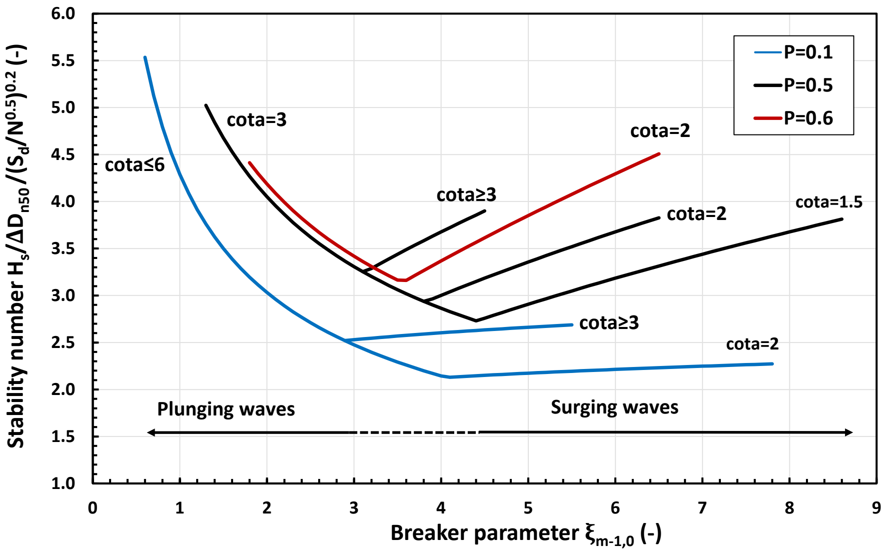

Figure 1: Application area of the Van der Meer formula in a $H_s/\Delta D_{n50} / (S_d/N^{0.5})^{0.2}$ versus $\xi_{m-1,0}$ graph.

There is not much research on rock stability at shallow foreshores. The relevant work is summarised below, describing shortly the available data and formulae and how the work has been used in this paper.

- The main work in shallow water is by Smith et al. (2002), later extended by Van Gent et al. (2003), that led to the Modified Van der Meer formula. That work will be referenced throughout this paper as Van Gent et al. (2003). The actual data were kept confidential for about fifteen years. On request, the data became eventually available as the directors of Deltares decided that scientific work of Delft Hydraulics (before 2008) should be made publicly available. The data was provided as a link to a spreadsheet without further report, referenced here as Van Gent (2017). The work includes two foreshore slopes of 1:100 and 1:30, four structures with slope angles of 1:2 and 1:4 and each of them with a permeable and an impermeable core. Tests were performed for shallow water and in a few cases for very shallow water. Besides the Modified Van der Meer formula, also a simpler formula was developed by Van Gent et al. (2003), assuming that the wave period or steepness had no influence anymore in shallow water. All data will be used for comparison.

- Eldrup (2019) and Eldrup et al. (2019) investigated the estimation of the notional permeability factor *P* and Eldrup and Lykke Andersen (2019a) looked at the stability of rock slopes in shallow water. But the wave conditions of this work were never into the (severe) breaking zone as $h/H_{m0\ deep}$ > 1.5. They used foreshore slopes of 1:100 and 1:30, similar to the ones in Van Gent et al. (2003). Part of the work focused on highly nonlinear waves by shoaling low steepness waves. Only the relevant part of their tests will be used. Eldrup and Lykke Andersen (2019a) proposed a modified formula for the surging area (the right part in Figure 1), where their curve became horizontal.

  The tests were not performed as individual tests, as the armour layer was only rebuilt after a number of tests with cumulative damage. This is in contrast to the original tests of Van der Meer (1987, 1988a and 1988b) and the tests by Van Gent et al. (2003). By using the cumulative damage development method, described in Box 5.18 of the Rock Manual (2007), it was possible to relate most of the tests to individual tests (incorporating the damage of the previous test condition in the actual test condition). For a small number of tests this was not possible as the increment in wave height between tests was too small.

- Herrera et al. (2017) performed stability tests at a foreshore slope of 1:50 and a breakwater cross-section with a rock armour slope of 1:1.5 on an underlayer and core. Their test conditions did also not include a lot of wave breaking as the relative depth $h/H_{m0\ deep}$ > 1.5. Unfortunately, they performed their tests with about 10% increase of wave height for the next test, without rebuilding the armour layer. This cumulative damage depends significantly on the previous test, and it is not possible to uncouple the tests to individual tests, like for the tests of Eldrup and Lykke Andersen (2019a). The stability formula proposed by Herrera et al. (2017) is therefore only applicable to a test set-up with small increments of the wave height and cumulative damage. Their stability data cannot be compared with the rewritten Van der Meer formula, but the wave data will be used in this paper to study wave transformation over a foreshore slope of 1:50.

- Recently Marino et al. (2022) performed stability tests at a foreshore slope of 1:30 and a breakwater cross-section with a rock armour slope of 1:2 on an underlayer and core. The data is (not yet) publicly available and as such the data of that paper cannot be used for comparison.

- Sallaudin et al. (2017) performed stability tests at a foreshore slope of 1:30 and a structure with an armour layer of concrete units. As such the stability results are not interesting for this paper, but the wave data at a foreshore slope of 1:30 will be combined with the other data on such a foreshore slope.

Some researchers have looked at the influence of water depth on stability and used part of the data sets described above. None of them describe the possibility that a different situation exists for (very) shallow water and that it may not be possible to come to one overall stability formula, and that a new formula may be needed for the specific situation of (very) shallow water.

Etemad-Shahidi et al. (2020) gathered the data of Thompson and Shuttler (1975), Van der Meer (1988b), Van Gent et al. (2003) and Vidal et al. (2006). The main purpose of their paper is to come with one unified formula that describes deep and shallow water stability all together. They come to a Van der Meer type formula (in their paper Equations 16a and 16b), where the notional permeability factor has been deleted, a different wave height – damage relationship was given and different influences of wave steepness and slope angle. The developed formulae have a discontinuity in the transition point from plunging to surging waves. They had in total 1199 tests in the database, where 865 (72% of the database) were related to deep water and these tests were the basis of the Van der Meer formula. Only 17% of the tests is related to the tests of Van Gent et al. (2003). They do not show any stability curve and do not explain why using the same data, their formula should be better than the Van der Meer formula, nor where the analysis of Van der Meer (1988b) was different from theirs. They also do not prove that one formula should fit for the whole range from deep water to very shallow water. The paper gives the usual graphs of measured versus predicted, which does not show any physical explanation.

Verhagen and Mertens (2009) combined the work of Van der Meer (1988b) and Van Gent (2003). They also aim to develop one overall formula, but at that time the dataset of Van Gent (2017) was not available.

Losada (2021) took data from four structure slopes with impermeable cores from Van der Meer (1988b) and looked at the influence of the relative water depth (*h/L*). But the majority of the tests of Van der Meer (1988b) were performed without a foreshore and with fairly deep water. Thus, the relative water depth mainly shows the influence of the wavelength. The analysis never went to a relative depth of $h/H_{m0\ deep} \leq 2$ and although the paper describes a possible influence of water depth, it does not describe stability in shallow water. Losada (2021) acknowledges the scarcity of data in this area.

1.2 Objectives

Stability of rock slopes at shallow or very shallow foreshores with heavy wave breaking has not been considered in Van der Meer (2021) as it was seen as a field of research that had not yet fully been explored. This paper will review and re-analyse all available and relevant data on shallow foreshores.

Waves develop over a sloping foreshore and it is important to have an understanding of this behaviour before stability results of rock armoured slopes are analysed. Such behaviour of waves in shallow water has been described by many researchers, but validation of such theories is not the subject of this paper. Main references are Goda (2010a and 2010b), Kamphuis (1991) and Allsop et al. (1998). Recently, Lashley et al. (2020) and Lashley (2021) came to an empirical prediction method for wave heights at shallow foreshores, including infragravity waves. The first part of this paper will describe the behaviour of waves over tested foreshores and the second part will analyse stability of rock armoured slopes. The main objectives are then:

- Describe how waves develop over shallow foreshores, for the data that is also available for stability of rock slopes;
- Analyse stability of rock armoured slopes at shallow foreshores, using Van Gent et al. (2003) with data from Van Gent (2017) and partly tests from Eldrup and Lykke Andersen (2019a) and compare the data with relevant formulae.
- In this analysis on stability the main objective is then: when and where do shallow water stability results deviate from the rewritten Van der Meer formula and how can deviating results be described. This all happens to come to an improved understanding of rock slope stability in shallow water.

The analysis will mainly be performed with the relative depth, $h/H_{m0\ deep}$ and wave steepness as leading parameters, as this gives a clear indication of (deep or shallow) non-breaking, shoaling or (severe) breaking of the waves.

2 Development of waves over tested shallow foreshores

2.1 Introduction

The wave height can be described by many characteristic values, where for most stability formulae the significant wave height from the time domain $H_{1/3}$; the spectral significant wave height H_{m0}; and the 2% exceedance wave height from the time domain ($H_{2\%}$) are used. Sometimes, the significant wave height is defined as H_s and here it is unclear whether $H_{1/3}$ or H_{m0} should be used. In deep water $H_{1/3}$ and H_{m0} are very similar, but they may deviate significantly in shallow water. In order to avoid misunderstanding $H_{1/3}$ and H_{m0} will explicitly be used in this paper and not just H_s.

The tests by Van Gent et al. (2003) were performed with single peaked spectra as well as with double peaked or bi-modal spectra. Van Gent et al. (2003) have proven that by using the spectral wave period $T_{m-1,0}$ the spectra show the same behaviour in wave development over the foreshore as well as in stability results. In this paper tests with single and double peaked spectra have been taken together to describe stability and wave transformation.

The behaviour of waves over the tested foreshores will be described in this section. The analysed straight foreshores are 1:100, 1:50 (only waves, no stability) and 1:30. The consequence is that description outside this range, gentler than 1:100 or steeper than 1:30 is not part of the analysis. The description of waves over the straight foreshore is made by the relative depth $h/H_{m0\ deep}$. In case a structure is present, this could also be the depth at the toe, h_{toe}. Sometimes, the wave steepness $s_{m-1,0}$ also plays a role. Furthermore, Goda (2010a) has been used to describe the change in the wave height when they propagate from deep water over the foreshore to very shallow water, see Figure 2. Based on these graphs the following zones are identified:

- non-breaking and no significant shoaling (roughly $h/H_{m0\ deep}$ > 4-6);
- shoaling and slightly breaking (roughly $h/H_{m0\ deep}$ = 2-5), mainly determined by the wave steepness;
- depth-limited wave conditions with wave breaking ($h/H_{m0\ deep}$ < 1.5-2), start of breaking mainly determined by the wave steepness;
- severely breaking, infragravity (IG) waves come into play ($h/H_{m0\ deep}$ < 1);
- broken waves dominated by infragravity (IG) waves ($h/H_{m0\ deep}$ < 0.5).

Above limits are valid for a wave steepness above 1%, see Figure 2.

In the next sections, the development of H_{m0} and $H_{1/3}$ over the foreshores will be described by the available data sets. Then the ratio of significant wave heights $H_{m0}/H_{1/3}$ and the ratio $H_{2\%}/H_{1/3}$ and finally the change in spectral wave period $T_{m-1,0}$ is studied.

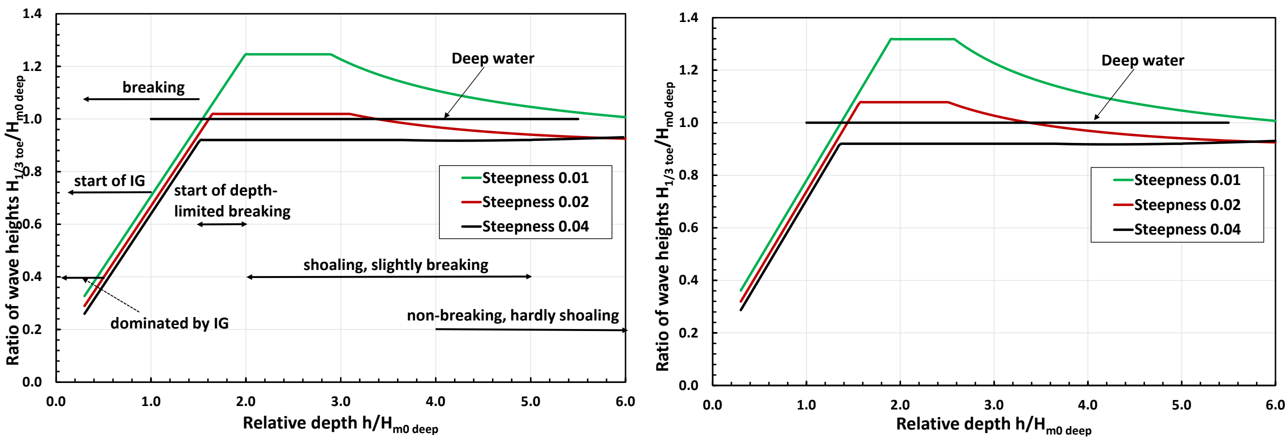

Figure 2: Wave transformation from deep to shallow water using Equations 3.31 and 3.47 in Goda (2010a). Left graph with a 1:100 foreshore and right graph with a 1:30 foreshore.

2.2 Wave breaking over a foreshore 1:100

Figure 3 shows the development of H_{m0} (left graph) and $H_{1/3}$ (right graph) along the 1:100 foreshore. Data from Van Gent (2017) have only conditions with relative depth $h/H_{m0\ deep}$ < 2.5, where Eldrup and Lykke Andersen (2019a) give conditions with $h/H_{m0\ deep}$ > 2.5. There is a nice continuation of the two data sets to a consistent trend. Data points deviate in wave steepness and in most of the area of relative depths ($h/H_{m0\ deep}$ > 3 and $h/H_{m0\ deep}$ < 1.5) the wave steepness does not show significant influence. In the zone 1.5 < $h/H_{m0\ deep}$ < 3 the data with a wave steepness $s_{m-1,0}$ = 0.04 has a wave height ratio close to 0.9 and the data with a wave steepness $s_{m-1,0}$ = 0.02 have a ratio close to 1. Those data points fit well with the curves according to Goda (2010a) in Figure 2.

In the area of continuous wave breaking ($h/H_{m0\ deep}$ < 1.5) H_{m0} is slightly larger than $H_{1/3}$, which could be caused by the development of infragravity waves. A rule of thumb for gentle foreshores is that the depth-limited significant wave height is about half of the water depth ($H_{1/3}$ = 0.5h). The line of this rule is given in Figure 3 (right graph) and fits very well for this gentle foreshore and for $H_{1/3}$. As H_{m0} is a little larger, the line gives a small underprediction in the left graph of Figure 3. The prediction of Goda for $H_{1/3}$ in the right graph of Figure 3 is significantly higher than the data. Also, the empirical prediction of $H_{m0\ toe}$ by Lashley (2021 – Equations 4.16-4.18) for conditions with $h/H_{m0\ deep}$ < 1 is given in the left graph of Figure 3 for H_{m0}. The predictions are a little on the high side. The graphs clearly show that wave breaking and depth-limited wave conditions are present for $h/H_{m0\ deep}$ < 1.5.

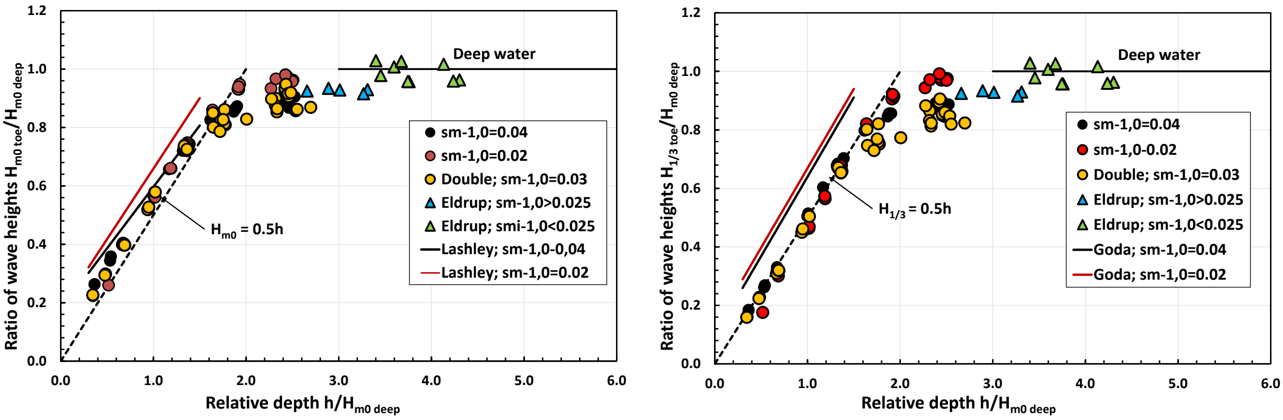

Figure 3: Wave breaking over the 1:100 foreshore given as the ratio of wave heights in shallow and deep water versus the relative depth. Left graph with the spectral wave height H_{m0} and the prediction by Lashley et al. (2021). Right graph with the significant wave height $H_{1/3}$ and the prediction by Goda (2010a). In both graphs the "rule of thumb" of "depth limited wave height is half of the water depth", is given.

2.3 Wave breaking over steeper foreshores 1:30 and 1:50

Figure 4 shows similar graphs as in Figure 3, but now for the waves measured at a foreshore slope of 1:30. During analysis an inconsistency of data was found, which seems to be an administrative mistake in creating the data spreadsheet Van Gent (2017). The inconsistency and the possible cause have been described in Appendix A and the spreadsheet was corrected as far as possible.

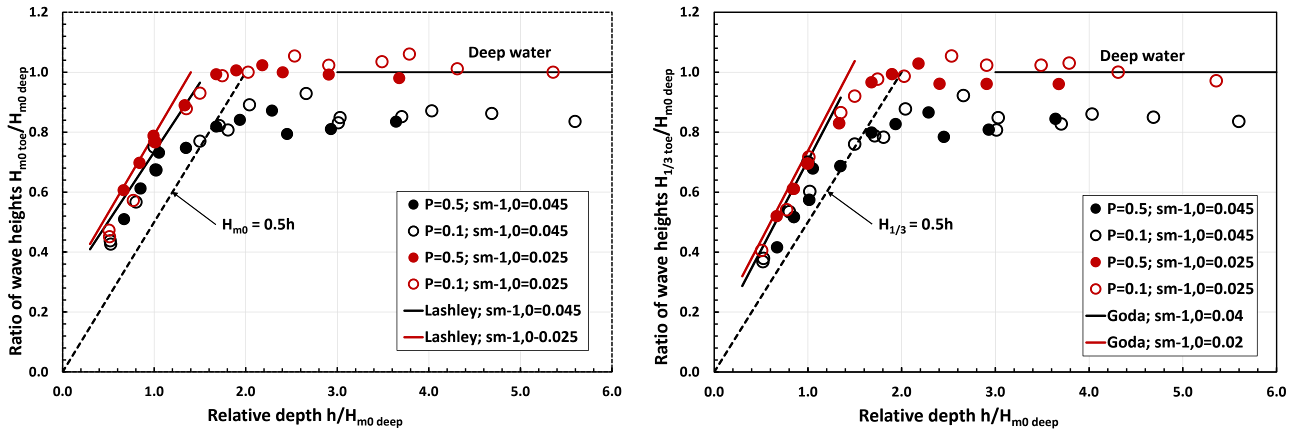

Figure 4: Wave breaking over the foreshore 1:30 given as the ratio of wave heights in shallow and deep water versus the relative depth. Left graph with the spectral wave height H_{m0} and right graph with the significant wave height $H_{1/3}$.

The foreshore of 1:30 is quite steeper than the foreshore 1:100 and now the depth limited wave heights are a little larger, also larger than the rule of thumb of significant wave height being about half of the local water depth. The predictions by Goda for $H_{1/3}$ (right graph of Figure 4) overestimate the steeper waves considerably, but are quite close to the smaller steepness of $s_{m-1,0}$ = 0.02. The data fit quite well with the Lashley curve for a foreshore 1:30 and H_{m0}.

Figure 4 shows already that for relatively deep water the largest wave steepness start to reduce in height, where the wave heights for a wave steepness of $s_{m-1,0}$ = 0.025 remains more or less similar up to a relative depth of $h/H_{m0\ deep} > 2$. Also, in the continuously breaking zone ($h/H_{m0\ deep} < 1.5$), the smaller wave steepness shows a little larger local wave height. The prediction by Lashley (2021) for $H_{m0\ toe}$ is slightly higher than the mean of the measurements.

Figure 4 shows only the (partly corrected) data of Van Gent (2017). Figure 5 shows more data for a foreshore slope of 1:30 and a few data for a foreshore of 1:50. Data of Eldrup and Lykke Andersen (2019a) were added, including tests with much smaller wave steepness and leading to highly nonlinear waves. It was also concluded that in the work of Van Gent et al. (2003) low steepness tests were actually missing. Herrera et al. (2017) performed tests with two wave steepnesses on a foreshore slope of 1:50. Salauddin et al. (2017) performed similar tests on a foreshore slope of 1:30. The data in Figure 6 have been divided in three wave steepness ranges: black points with relatively steep waves ($s_{m-1,0}$ ~ 0.04); red points with smaller wave steepness ($s_{m-1,0}$ ~ 0.02); and blue triangles with small wave steepness ($s_{m-1,0}$ < 0.01). Overall, the largest wave steepness tests give lower local wave heights than for smaller wave steepness over the whole range of local water depths, which is in line with Figure 2. This is valid for H_{m0} as well as $H_{1/3}$.

But the most important difference between the two graphs in Figure 5 is the significant shoaling of small and very small wave steepness that can be observed. This may be explained by the waves in "deep water" on a foreshore 1:30 are measured in much greater depth than for the foreshore 1:100 for the Eldrup and Lykke Andersen (2019a) data. Thus, the data for the foreshore 1:100 only show a part of the shoaling from deep water. The nonlinearity of the waves causes a significant difference between the spectral and the time domain wave heights in shallow water. Based on the results of stability tests by Eldrup and Lykke Andersen (2019a) they concluded that the spectral wave height $H_{m0\ toe}$ should be used in stability formulae, in order to overcome a significant discontinuity if using $H_{1/3\ toe}$. Note that this may be a good conclusion for nonlinear waves, it may be different for waves in the severe breaking zone with depth-limited wave conditions and relative depths $h/H_{m0\ deep}$ < 1.5. Therefore, a choice in $H_{m0\ toe}$ and $H_{1/3\ toe}$ in stability formulae for rock slopes will not be made here, but first after considering all available stability results.

Another important conclusion of Figure 5 is that it is only Van Gent et al. (2003) who performed tests for small relative depth $h/H_{m0\ deep}$ < 1.5. The stability tests in this region may show the behaviour of rock slopes in really shallow water and a choice on preference of $H_{m0\ toe}$ or $H_{1/3\ toe}$. Thus the choice of wave height in the stability formulae for rock slopes may also depend on these results.

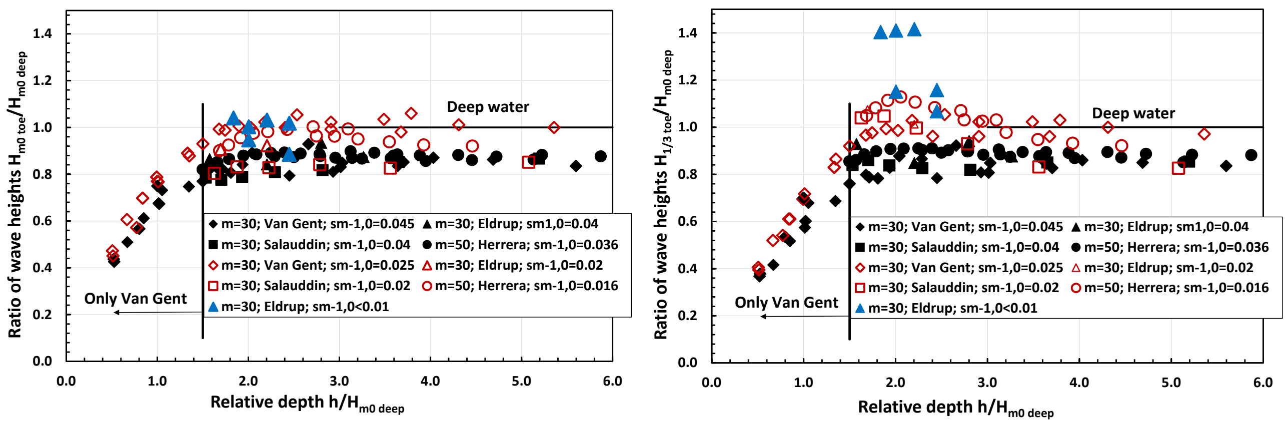

Figure 5: Wave breaking over a foreshore 1:30 and 1:50, given as the ratio of wave heights in shallow and deep water versus the relative depth and for various investigations. Left graph with the spectral wave height H_{m0} and right graph with the significant wave height $H_{1/3}$. The right graph shows significant shoaling for small wave steepness, which is not present in the left graph.

2.4 Ratios of significant and 2% wave heights

The previous section showed that differences between $H_{m0\ toe}$ and $H_{1/3\ toe}$ are usually small, but in some cases very significant differences exist especially for shoaled low steepness waves. In the Modified Van der Meer formula, the $H_{2\%}$ is used. It is therefore interesting to compare the ratios of these wave heights over the relative depth. It will give an indication beforehand on what will happen to stability results if one wave height will be replaced by the other.

Figure 6 gives the ratio $H_{m0\ toe}/H_{1/3\ toe}$, in the left graph with the foreshore slope of 1:100 and in the right graph 1:30. For relative depths $h/H_{m0\ deep}$ > 1.5-2 and not too small wave steepness ($s_{m-1,0}$ > 0.01) the ratio remains close to 1 (the horizontal line for deep water). The nonlinear waves by Eldrup and Lykke Andersen (2019a) with $s_{m-1,0}$ < 0.01 (blue triangles in the right graph) show clearly that nonlinear shoaling causes a larger increase in $H_{1/3\ toe}$ than $H_{m0\ toe}$ and therefore, a dip in the ratio. The data for shallow water from Van Gent (2017) with $h/H_{m0\ deep}$ < 1.5 show a consistent trend where the ratio $H_{m0\ toe}/H_{1/3\ toe}$ increases gradually from 1 to almost 1.5 with decreasing relative depth. Note that ratios larger than 1.2 (the dashed rectangular area in the left graph) were measured during calibration tests, but there are no stability results for the rock slopes as the wave heights became too small to cause damage.

This analysis on Figure 6 means that if stability results will be compared with $H_{m0\ toe}$ or $H_{1/3\ toe}$, there will not be a significant difference for $h/H_{m0\ deep}$ > 1.5-2 (except for the slope 1:30 with very low wave steepness). For $h/H_{m0\ deep}$ < 1.5 stability will increase gradually with decreasing relative depth up to 20%, if replacing $H_{1/3\ toe}$ by $H_{m0\ toe}$.

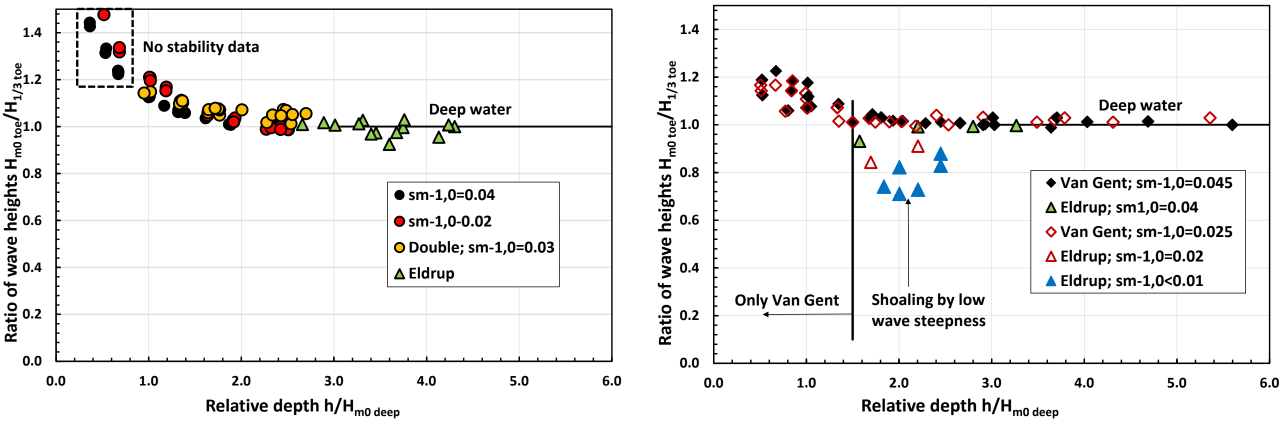

Figure 6: The ratio of spectral and significant wave height versus relative depth. Left graph for a foreshore 1:100 and right graph for a foreshore 1:30.

The wave height $H_{1/3\ toe}$ and $H_{2\%\ toe}$ are both statistical wave heights from the time domain and the ratio $H_{2\%\ toe}/H_{1/3\ toe}$ is given in Figure 7, again with in the left graph the data for a foreshore slope of 1:100 and right 1:30. A Rayleigh distribution of the wave heights gives the ratio 1.4 and this value has been given by a horizontal line. This is valid for linear narrow banded spectra and is a good approximation in deep water. When waves enter the area of shallower water the waves become nonlinear and also the highest waves will break and cause a deviation from the Rayleigh distribution. The breaking will decrease the ratio $H_{2\%\ toe}/H_{1/3\ toe}$, but when the breaking cause significant infragravity waves the ratio increases again. Battjes and Groenendijk (2000) have developed a method to predict $H_{1/3}$ as well as $H_{2\%}$ on a straight sloping foreshore at a certain depth and with a certain given spectral wave height H_{m0} at that location. The limit that is reached by Battjes and Groenendijk (2000) for shallow water is $H_{2\%\ toe}/H_{1/3\ toe} = 1.21$, given in the graphs by a dashed horizontal line. Note that the validity of this method (based on the data used) is for relative depths of about $h/H_{m0\ deep}$ > 1.5, also given in the graphs of Figure 7.

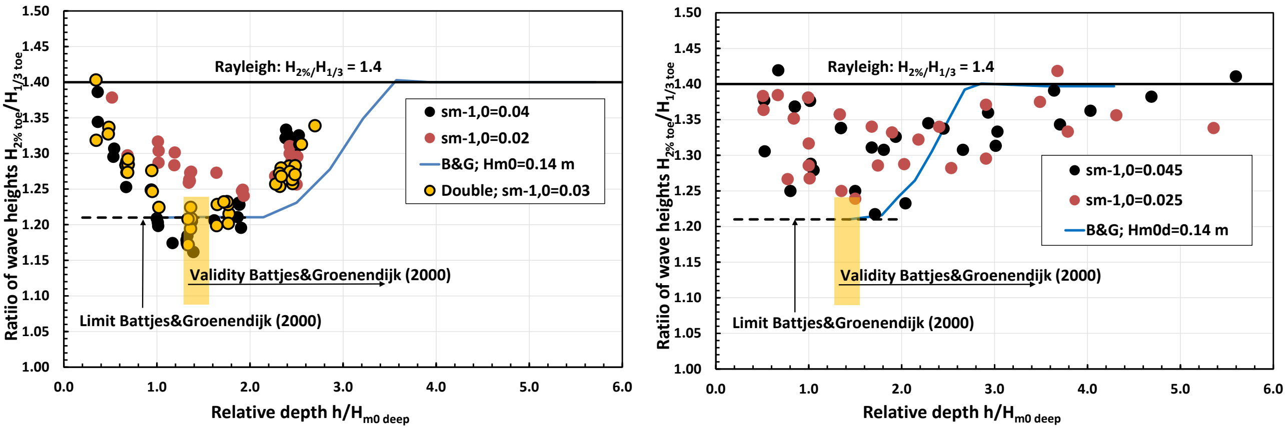

Figure 7: The ratio of two percent and significant wave height versus relative depth. Left graph for a foreshore 1:100 and right graph for a foreshore 1:30. The prediction method of Battjes and Groenendijk (2000), is not valid for small relative depths.

Figure 7 shows indeed that by decreasing relative depth the ratio $H_{2\%\ toe}/H_{1/3\ toe}$ decreases from 1.4 to around 1.21 at a relative depth of $h/H_{m0\ deep}$ = 1.5. This is most clear for the foreshore slope of 1:100. But for further decrease of the relative depth, the ratio $H_{2\%\ toe}/H_{1/3\ toe}$ increases again and even up to a Rayleigh distribution. The trend was already noted by Smith et al. (2002) for their tests on the foreshore of 1:100. This behaviour could not be described by Battjes and Groenendijk (2000) as they had no data in this area. Extrapolation of the method to low relative depths may thus lead to a significant underprediction of $H_{2\%\ toe}$. The data of Smith et al. (2002), being the data on the 1:100 foreshore (left graph) have also been described by Goda (2010b), where he concludes that the model distribution of Battjes and Groenendijk (2000) cannot deal with the actual process of wave height distribution transformation such that the distribution returns to the Rayleigh near the shoreline.

The 1:30 foreshore in the right graph of Figure 7 shows much more scatter. In the area with relative depth $h/H_{mo\ deep}$ = 1.5, where the ratio $H_{2\%\ toe}/H_{1/3\ toe}$ should become around 1.21 the scatter is very significant. All ratios larger than 1.3 are sea states tested on a permeable core (structures 4 and 5), where the data points lower than this value belong to sea states tested for the impermeable core tests (structures 6 and 7). There is no physical explanation for this difference.

Above analysis leads to a significant consequence in using $H_{2\%\ toe}$ in a stability formula like the Modified Van der Meer formula. It is quite easy to *measure* $H_{2\%}$ in a physical model, but is it also possible to *predict*? Spectral models often give $H_{m0\ toe}$ and then Battjes and Groenendijk (2000) is widely used to calculate $H_{1/3}$ as well as $H_{2\%}$. This is correct as long as the relative depth $h/H_{m0\ deep}$ > 1.5. But the method *underpredicts* significantly in shallower water. This means that we have no method to correctly apply the Modified Van der Meer formula for $h/H_{m0\ deep}$ < 1.5, unless model tests or numerical models have given the $H_{2\%\ toe}$. The Modified Van der Meer formula was developed by Van Gent et al. (2003) to describe stability of rock slopes in shallow water, but it is difficult to apply for $h/H_{m0\ deep}$ < 1.5, as the $H_{2\%\ toe}$ cannot be predicted in a reliable and simple way. And if the method of Battjes and Groenendijk (2000) is used in this area for design, it may lead to a significant under design of the armour layer.

2.5 Change in spectral wave periods

Hofland et al. (2017) describe the change in the spectral wave period $T_{m-1,0}$, when entering into shallow water and even up to extremely shallow water, where the toe of the structure may even be emerged. All the data they gathered were on tests with wave overtopping and then sometimes the toe of the structure may be emerged. The latter will hardly be the case for stability as there is no necessity to apply a rock armour slope if the toe of the structure is emerged and the waves become very small. This is a clear difference in stability of structures and wave overtopping at coastal structures.

Most of the data used by Hofland et al. (2017) were in very shallow water ($h/H_{m0\ deep}$ = 0.3-1.0) with foreshore slopes 1:35, 1:100 and extremely shallow water ($h/H_{m0\ deep}$ < 0.3) with foreshore slopes 1:35, 1:100, 1:250 and only limited data on gentle foreshore slopes of 1:100 and 1:250 were available for larger relative water depths $h/H_{m0\ deep}$ > 0.7. Data with a foreshore 1:30 is not expected to deviate significantly from the foreshore 1:35 data. For extremely shallow water the ratio $T_{m-1,0\ toe}/T_{m-1,0\ deep}$ may reach values up to 9, showing that the waves are fully dominated by infragravity waves. Hofland et al. (2017) developed a formula (Equation 4 in their paper) to describe the change in spectral period, where also the foreshore slope was included in the relative depth. They use the cotangent of the foreshore slope, m, to describe the slope angle. This formula will be compared with the data on foreshores of 1:100 and 1:30 by Van Gent et al. (2003).

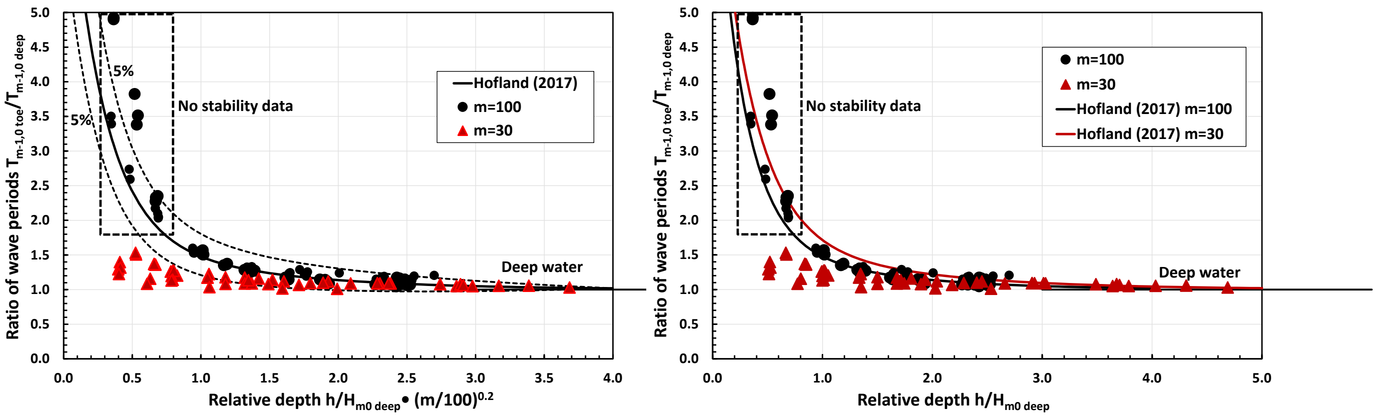

Figure 8: The change in spectral wave period over shallow foreshores according to Hofland et al. (2017,) but now with the present data on foreshore slopes with *m*=100 and 30. Left: with the influence of foreshore slope according to Hofland et al. (2017 – Equation 4) included in the x-axis. Right: without influence of the foreshore in the x-axis.

The left graph of Figure 8 shows Equation 4 in Hofland et al. (2017) with the 90% confidence band and the data of Van Gent et al. (2003). The gentle foreshore of 1:100 fits fairly well, but this is not the case for the foreshore of 1:30 and relative depths $h/H_{m0\ deep}\cdot(m/100)^{0.2} < 1.5$, where m is the cotangent of the foreshore slope (m = 100 or 30). The data in the right graph shows that the gentle slope leads to a larger increase in wave period than the steep slope. This is opposite to the formula by Hofland et al. (2017). The deviations for the data on a 1:30 foreshore can have a significant influence on the stability if the energy period is predicted by Hofland et al. (2017). Data points located in the plunging domain with the measured period may be located in the surging domain when $T_{m-1,0}$ is predicted by Hofland et al. (2017). The foreshore slope of 1:100 gives ratios of $T_{m-1,0\ toe}/T_{m-1,0\ deep}$ up to 5, where it remains limited to about 1.5 for the steeper foreshore slope of 1:30. The data for the foreshore of 1:100 that give ratios larger than 1.5 (the dashed rectangle in the graphs), however, have been measured during calibration, but did not give reliable stability results with any damage, as the wave heights became too small. From Figure 8 it can be concluded that for decreasing relative depth $h/H_{m0\ deep} < 1.5$ the spectral period $T_{m-1,0\ toe}$ may increase up to 50% with respect to the period in deep water.

3 Rock stability at shallow foreshores

3.1 Summary of recent developments at deep water

Van der Meer (2021) revisited the original Van der Meer formula and rewrote it to include the spectral wave period $T_{m-1,0}$ instead of the mean period T_m, and he included two coefficients c_{pl} and c_{su} to enable a possible modification of new research data. This rewritten Van der Meer formula is given by:

$$\frac{H_s}{\Delta D_{n50}} = \begin{cases} 6.49 c_{pl}\, P^{0.18} \left(\frac{S_d}{\sqrt{N}}\right)^{0.2} \xi_{m-1,0}^{-0.5} & \text{for plunging waves: } \xi_{m-1,0} \le \xi_{m-1,0c} \quad (1) \\ 0.97 c_{su}\, P^{-0.13} \left(\frac{S_d}{\sqrt{N}}\right)^{0.2} \sqrt{cot\alpha}\, \xi_{m-1,0}^{P} & \text{for surging waves: } \xi_{m-1,0} > \xi_{m-1,0c} \quad (2) \end{cases}$$

Where the parameters used have been explained in the notation.

The critical breaker parameter $\xi_{m-1,0c}$ is:

$$\xi_{m-1,0c} = \left(\frac{6.49 c_{pl}}{0.97 c_{su}} P^{0.31} \sqrt{tan\alpha}\right)^{\frac{1}{P+0.5}} \quad (3)$$

For a more in-depth description of the parameters, the reader is respectfully referred to the Rock Manual (2007) and Van der Meer (1987, 1988a and 1988b).

3.2 Existing formulae proposed for shallow water

The modification to the original Van der Meer formula by Van Gent et al. (2003) was a replacement of the mean period T_m by the spectral period $T_{m-1.0}$ and a refit of the formula on their data leading to other coefficients. They also preferred to use the 2% wave height $H_{2\%}$ over the significant wave height. The formula was named as the Modified Van der Meer formula for shallow water in the Rock Manual (2007) and can be described as follows:

$$\frac{H_{2\%}}{\Delta D_{n50}} = \begin{cases} 8.4\, P^{0.18} \left(\frac{S_d}{\sqrt{N}}\right)^{0.2} \xi_{m-1,0}^{-0.5} & \text{for plunging waves} \quad (4) \\ 1.3 P^{-0.13} \left(\frac{S_d}{\sqrt{N}}\right)^{0.2} \sqrt{cot\alpha}\, \xi_{m-1,0}^{P} & \text{for surging waves} \quad (5) \end{cases}$$

The critical breaker parameter $\xi_{m-1,0c}$ is:

$$\xi_{m-1,0c} = \left(\frac{8.4}{1.3} P^{0.31} \sqrt{tan\alpha}\right)^{\frac{1}{P+0.5}} \quad (6)$$

Van Gent et al. (2003) also came with an alternative and simpler formula to describe their tests on shallow water. They omitted the influence of the wave period as this was seen lost in the amount of scatter in the data and although there was a small influence of the ratio $H_{2\%}/H_s$, for the new formula they decided to omit this too. Therefore, H_s being $H_{1/3}$

was used for the new formula. Finally, they decided to replace the notional permeability factor P by the ratio of nominal diameters of the core material and the armour layer. The formula was given as follows:

$$\frac{S_d}{\sqrt{N}} = \left(0.57\frac{H_s}{\Delta D_{n50}}\sqrt{tan\alpha}\frac{1}{(1+D_{n50core}/D_{n50})^{2/3}}\right)^5 \tag{7}$$

Note that the relationships $S_d = f(H_s^5)$ and $S_d = f(\sqrt{N})$ are still assumed to be valid for shallow water conditions, not only in the Modified Van der Meer formula (Equations 4-6), but also for this alternative and simpler formula. Equation 7 can also be written as a direct function of the stability number:

$$\frac{H_s}{\Delta D_{n50}} = 1.75\sqrt{cot\alpha}\ (1 + D_{n50core}/D_{n50})^{2/3}\left(\frac{S_d}{\sqrt{N}}\right)^{0.2} \tag{8}$$

Eldrup and Lykke Andersen (2019a) performed tests with highly nonlinear wave conditions and also with very small wave steepness ($s_{m-1,0}$ < 0.01), showing a large deviation between $H_{1/3\ toe}$ and $H_{m0\ toe}$. They preferred to use the spectral significant wave height in fitting on their own data and the data by Van Gent (2017) and came to the following modification:

$$\frac{H_{m0}}{\Delta D_{n50}} = \begin{cases} 4.5\left(\frac{S_d}{\sqrt{N}}\right)^{0.2} 1.6^P\ \xi_{m-1.0}^{(0.4P-0.67)} & \text{for plunging waves} \quad (9) \\ 3.1\left(\frac{S_d}{\sqrt{N}}\right)^{0.2} P^{0.17}\min(\cot\alpha, 2)^{0.23} & \text{for surging waves} \quad (10) \end{cases}$$

A difference with the previous formulae is that they used cumulative damage (not rebuilding the armour after every test) and used other P-values for permeable core structures than P = 0.5. This all makes it a little difficult to compare directly with the other formula.

Equations 1-10 will all be considered when analysing the available data on stability for shallow foreshores.

3.3 Considerations on some design parameters

Testing rock armoured structures on stability may lead to very small as well as very large damage values of S_d. The practical range for design is between "start of damage", S_d = 2 or 3 (depending on the slope angle) and "under layer visible", S_d = 8 to 17 (again depending on the slope angle, see Rock Manual, 2007). The damage curve in any of the stability formulae given by Equations 1-10 is a power curve: $S_d = f(H_s^5)$. A power curve is starting in the origin and according to the formula damage should start for any wave height larger than zero. This is of course not correct and as it is also difficult to detect a very small erosion area, S_d = 0 is never used. S_d = 2 or 3 includes already some rearrangements of stones and a small erosion area. But S_d < 1 should be considered as a less reliable measure as well as a value that may substantially deviate percentage wise from a prediction formula. For this reason, damage values S_d < 1 have not been considered in this paper for further analysis.

Also, too large damage values should not be considered. If the damage is (much) larger than "filter layer visible" it means that a part of the filter layer has probably eroded and this is outside any design criterion and may influence damage development. Therefore, too large damage defined by more than 150% of the criterion for "underlayer visible" has not been considered further. The criterion "underlayer visible" depends on the slope angle and is S_d = 8 for slopes 1:1.5 and 1:2; S_d = 12 for 1:3 and S_d = 17 for 1:4. The maximum damages for the slopes mentioned were therefore respectively 12, 18 and 25.5. Tests with S_d-values out of range were neglected and were also not used for a final comparison, as the only result will be more scatter.

Waves develop over a shallow foreshore, and the wave height and period may change. Also the mean wave period T_m may decrease even for relative water depths outside the zone of depth-limited wave conditions. The mean period is not used anymore in the rewritten Van der Meer formula to describe the wave period influence (Equations 1-3), but this period is often used to calculate the storm duration by the number of waves, N, that attack the structure. Since stability is not significantly affected by minor fluctuations in the number of waves and it is not easy to calculate the mean period (or number of waves) in shallow water, it is proposed to keep the mean period *at deep water* to be used for calculation of the storm duration by the number of waves N. Then it remains a simple description of the storm duration. It may well be that other researchers have taken the number of waves in their analysis, that has been measured at the location of the structure. In that case one needs a good prediction method to estimate the change in number of waves at shallow foreshores before one can apply a formula that is based on this number.

Van Gent et al. (2003) tested a large number of double peaked or bi-modal spectra. It was shown by them that these tests gave similar results as the single peaked spectra, provided that the spectral period $T_{m-1,0}$ was used. This paper will regard those tests as similar to single peaked spectra and will not deviate between tests with single or double peaked spectra.

Two validation checks should be made before new research should be compared with the rewritten Van der Meer formula (Van der Meer, 2021). The first is the relationship $S_d = f(\sqrt{N})$, which can only be checked if tests have been performed with an intermediate stability check, say after 1000 and 3000 waves. The other relationship is $S_d = f(H_s^5)$, where actually the exponent 5 should remain between 4 and 6 (Van der Meer, 2021). This relation can be checked with stability results for a fixed wave period or wave steepness. But this is very difficult to achieve for very shallow water, as the wave periods or steepness may be similar in deep water, they will change over the foreshore. It is hardly possible to find tests with a series of wave heights and damages for a fixed period or steepness at the toe of the structure. At first instance one may assume that the relationship remains intact, but there is no validation that this is also the case for very shallow water.

Waves are always caused by wind and as such the steepness in design situations at deep water is always quite high, often close to the physical limit. Swell travelled long distance over the ocean and the wave steepness may become quite small, maybe as low as 0.01 or even a little smaller. It is of course possible that on a *shallow foreshore* the wave height reduces much, and the period increases and the wave steepness becomes much smaller than 0.01. This is even more the case if swell was generated at deep water. This is a true shallow water effect and then wave steepness may become very small and probably the data will deviate from the formulae for deeper water. That is one of the subjects of this paper.

3.4 Comparison of formulae

The rewritten Van der Meer formula (Equations 1-3) and the Modified Van der Meer formula for shallow water (Equations 4-6) are identical except for coefficients and the different use of H_s and $H_{2\%}$. This means that the modification factors c_{pl} and c_{su} values can be calculated for various ratios of $H_{2\%}/H_s$. A value of 1 means that the formulae are equal, a value smaller than 1 shows less stability and a larger value better stability. It also shows directly in percentage the difference in stability: a modification factor of 1.1 shows that the stability number is 10% larger than for the rewritten Van der Meer formula.

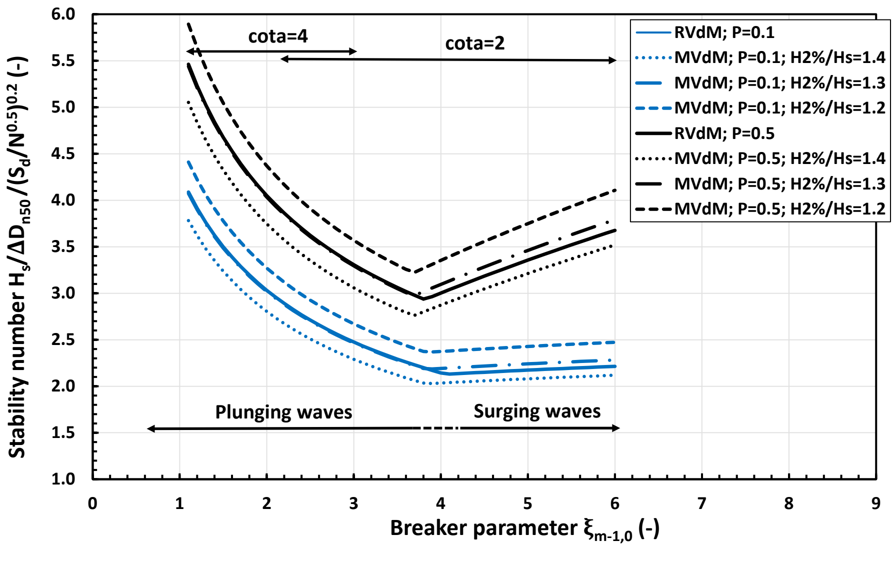

Figure 9: Direct comparison of the rewritten Van der Meer formula (Equations 1-3) with the Modified Van der Meer formula for shallow water (Equations 4-6) for three ratios of $H_{2\%}/H_s$; two slope angles of $cot\alpha = 2$ and 4; and for two permeabilities $P = 0.1$ and 0.5. The graph covers the full range in the dataset of Van Gent (2017). RVdM are Equations 1-3 and MVdM Equations 4-6.

Figure 9 directly compares the rewritten and Modified Van der Meer formulae and the tested range by Van Gent et al. (2003), meaning slope angles 1:2 and 1:4, both for notional permeability factors of P = 0.1 and 0.5. A division has been made between wave height ratios of $H_{2\%}/H_s$ = 1.4, 1.3 and 1.2. The left curve gives plunging waves and the right curve surging waves. A wave steepness range in deep water of $s_{m-1,0}$ = 0.05 (physical upper limit) and 0.007 (very long swell) has been assumed, which leads to breaker parameters in the graph of $\xi_{m-1,0}$ = 1.1-3.0 for a slope angle of 1:4 and $\xi_{m-1,0}$ = 2.2-6.0 for a slope angle of 1:2. If Rayleigh distributed waves are assumed with $H_{2\%}/H_s$ = 1.4, the Modified Van der Meer formula gives always less stability (c_{pl} = 0.92 and c_{su} = 0.96). With $H_{2\%}/H_s$ = 1.3, the formulae are quite close (c_{pl} = 1.00 and c_{su} = 1.03). Finally, for a low value of $H_{2\%}/H_s$ = 1.2, the Modified formula shows better stability (c_{pl} = 1.08 and c_{su} = 1.11). Based on the comparison in Figure 9 it seems that the difference in stability for deep and shallow water is not very significant. And also that for permeable structures and surging waves (the right upper curves in Figure 9) the stability will increase with increasing breaker parameter for shallow water. And this is questionable if for such shallow water conditions, the breaker parameter becomes so large or even larger than the application area.

The alternative and simpler formula developed by Van Gent et al. (2003) by Equation 7 or 8 can also be compared with the rewritten or Modified Van der Meer formula, also in graphs like Figure 9, if the formula is rewritten as follows:

$$\frac{H_s}{\Delta D_{n50}} / \left(\frac{S_d}{\sqrt{N}}\right)^{0.2} = 1.75\sqrt{cot\alpha}\,\left(1 + D_{n50core}/D_{n50}\right)^{2/3} \tag{11}$$

The left part of the equation is equal to the combined stability number as on the vertical axis in Figure 9. The right part of the equation will be a *horizontal* line in such a graph, depending on permeability (or size of rock) and slope angle.

The combined stability numbers for all the structures tested by Van Gent et al. (2003) are given in Table 1. Van Gent et al. (2003) proposed to use $H_{1/3}$ and not $H_{2\%}$ or the spectral wave height H_{m0} and therefore Equation 11 will only be used in graphs with $H_{1/3}$. Note that Equation 11 is proposed to be valid over the full range of tests, this is for the plunging as well as for the surging wave area in Figure 9.

Table 1: Characteristics of the seven structures in Van Gent (2017) with the last column giving the value for the simplified formula of Van Gent et al. 2003 (Equation 11) as used in the right graphs of Figures 13-16.

Structure	Permeability	P -	Foreshore 1:m	Slope $cot\alpha$	$D_{n50\text{-}armour}$ m	$D_{n50\text{-}core}$ m	$1.75cot\alpha^{0.5}(1+D_{n50core}/D_{n50})^{2/3}$ -
1	Permeable core	0.5	100	2	0.036	0.010	2.91
2	Permeable core	0.5	100	2	0.022	0.010	3.18
3	Permeable core	0.5	100	4	0.022	0.010	4.49
4	Permeable core	0.5	30	2	0.026	0.009	3.02
5	Permeable core	0.5	30	4	0.026	0.009	4.27
6	Impermeable core	0.1	30	2	0.026	0	2.47
7	Impermeable core	0.1	30	4	0.026	0	3.50

Also, the surging part of the modified formula by Eldrup and Lykke Andersen (2019a), Equation 10, can be described as a combined stability number and a function of slope angle and notional permeability factor, see Equation 12. Also, this formula becomes horizontal in a graph like Figure 9. Note that in this case, the formula is valid in the surging region (the right part of Figure 9) only, this in contrast to the alternative and simple formula by Van Gent et al. (2003), Equation 11.

$$\frac{H_{m0}}{\Delta D_{n50}} / \left(\frac{S_d}{\sqrt{N}}\right)^{0.2} = 3.1P^{0.17}\min(\cot\alpha, 2)^{0.23} \tag{12}$$

3.5 How to analyse and what to expect?

3.5.1 Check on validity of Van der Meer formula

Data on stability of rock slopes in shallow water are available from Van Gent et al. (2003) and Eldrup and Lykke Andersen (2019a). A first check on validity in order to compare with the Van der Meer formula is whether the relationship $S_d = f(\sqrt{N})$ is valid. Most of the tests of Van Gent et al. (2003) for structures 1 and 2 ($cot\alpha$ = 2; P = 0.5 and a foreshore slope of 1:100) were performed with about 1000 waves and then continued with another 2000 waves. All other tests were mainly performed with about 1000 waves only. The ratio between damage after 3000 and 1000 waves in the formula is $\sqrt{3000/1000} = 1.73$. Thompson and Shuttler (1975), which the square root function is based on, found a value of 1.81 based on 50 tests. Van der Meer (1987, 1988a and 1988b) found 1.64 based on 93 tests. Van Gent et al. (2003) had 13 tests where the damage was within range and the ratio of damage after 3000 and 1000 waves was 1.56. No relationship between this ratio and the relative water depth was identified, indicating that the ratio is not depending on depth. The value of 1.56 is the lowest value, but still close to 1.73, showing a significant increase in damage if the number of waves increases by another 2000 waves. The value is close enough to conclude that the formula that describes the increase in damage by increasing storm duration is still valid for shallow water. Eldrup and Lykke Andersen (2019a) performed only tests with 1000 waves and the relationship cannot be checked for this dataset.

As concluded before, the check on $S_d = f(H_s^5)$ cannot really be validated as wave periods or steepnesses change over the shallow foreshore and are not constant at the toe of the structures. It can only be assumed that the relationship is valid, until more data become available.

3.5.2 Effect of increasing breaker parameter

By wave breaking over the foreshore, the wave height will decrease and the wave period may increase due to infragravity waves. This all leads to an increase in breaker parameter $\xi_{m-1,0}$ and stability data that will shift to the right in a stability versus breaker parameter graph, like Figure 9. Both graphs in Figure 10 show data for an impermeable core, with the deep water data of Van der Meer (1987, 1988a and 1988b) as well as the rewritten Van der Meer formula (Equations 1-3) and for structure slopes of 1:3 and 1:4. Also data of Van Gent (2017) on a foreshore 1:30 with $P = 0.1$ and a structure slope of 1:4 has been added, but only for the tests outside the breaking zone ($h/H_{m0\ deep} > 1.5$). The data of Eldrup and Lykke Andersen (2019a) on a foreshore 1:100 with $P = 0.1$ and a structure slope of 1:3 have been added too. Note that here the relative depth was always $h/H_{m0\ deep} > 2.7$. All data in Figure 10 are outside the depth-limited wave conditions, or are data at deeper water. The different data follow the formula fairly well, although there is a significant scatter. The 5% lines are based on the original data of Van der Meer (1988) throughout this paper.

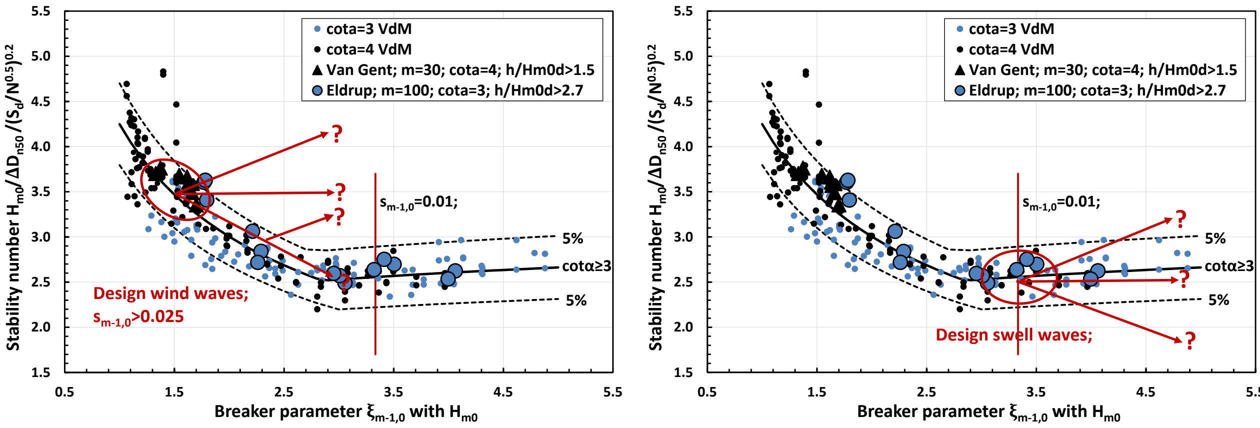

Figure 10: Breaking of wave heights over foreshores will cause an increase in the breaker parameter. The objective is to investigate whether stability under such conditions will follow the stability curve for deep water and where it will deviate. Left graph: steep wind waves in deep water are located at the left side of the graph. Right graph: long swell waves in deep water are located at the right side of the graphs (such data for shallow water are missing). The red arrows show a possible trend for shallow water data. Data with impermeable core.

The data of Van Gent (2017) with the largest water depth where $h/H_{m0\ deep}$ >1.5 are located at the left side of the graphs in Figure 10, as the deep water conditions had all waves with a fairly large wave steepness $s_{m-1,0} > 0.025$. These data could be considered as wind waves. The data of Eldrup and Lykke Andersen (2019a) also contained smaller wave steepness, like the original tests at deep water of Van der Meer. There are quite some data points in the graphs that have smaller wave steepness than 0.01 (right from the red line), which might be less interesting for real design swell conditions.

The question in the left graph of Figure 10 is: what will happen if the deep water wind wave conditions with $s_{m-1,0} > 0.025$ come into shallower water and the breaker parameter increases significantly? Will the data follow the formula, or deviate by a horizontal or even increasing trend (illustrated by the red arrows)? The same question can be asked for deep water swell conditions with $s_{m-1,0} \sim 0.01$ in the right graph of Figure 10. It is clear that if the breaker parameter increases significantly, the shallow water data will be located outside the application range for deep water. But will the trend shown by the red arrows become decreasing, horizontal or increasing?

For the wind waves with $s_{m-1,0} > 0.025$ it is possible to perform an analysis with the data of Van Gent (2017), but there are no data available with swell waves as deep water conditions. That area will be lacking data till new tests have been performed. It means also that any conclusion on stability in shallow water will be valid for wind wave conditions in deep water, not for swell wave conditions.

3.6 Stability outside the depth limited wave conditions ($h/H_{m0\ deep} > 1.5$)

Shoaling and slightly wave breaking may be present between h/H_{m0} = 2-5, where depth-limited breaking starts between h/H_{m0} = 1.5-2, see also Figure 2. For deeper water the waves will not be significantly influenced by breaking and infragravity waves are still small. Thus, a first hypothesis is that the rewritten Van der Meer formula (Equations 1-3) may at least be valid up to $h/H_{m0\ deep}$ = 1.5 (and not 3 as in the Rock Manual, 2007). Two situations will be analysed first: a permeable core with a 1:2 structure slope and an impermeable core with 1:3 and 1:4 structure slopes. The reason for these two situations is that Van Gent et al. (2003) as well as Eldrup and Lykke Andersen (2019a) have performed tests for these structures and that the impermeable data can be compared to the same stability curve since $cot\alpha \geq 3$. The other situations, permeable core with 1:4 slope by Van Gent et al. (2003) and impermeable core with a 1:2 slope by Van Gent et al. (2003) and Eldrup and Lykke Andersen (2019a) will be described a little further.

The graphs in the next six figures are given in pairs: in the left graph H_{m0} has been used and in the right graph $H_{1/3}$. Figure 11 shows the stability results outside the depth-limited wave conditions ($h/H_{m0\ deep} > 1.5$) for a 1:2 structure slope and a permeable core. The rewritten Van der Meer formulae in Equations 1-3 for deep water is given as a solid black line. Small red dots give the original data of Van der Meer (1987, 1988a and 1988b). The other data are from Van Gent (2017), Eldrup and Lykke Andersen (2019a) and Eldrup et al. (2019). One group of data points show a significantly smaller stability, indicated by the dashed oval in the graphs. These are the more or less deep water wave conditions ($h/H_{m0\ deep}$ = 2.5-3.7) on a 1:30 foreshore from Van Gent et al. (2003). It is unclear why these data deviate as it cannot be due to shallow water conditions. Similar data from Eldrup and Lykke Andersen (2019a) for a 1:30 slope (the two blue open diamonds on the left side of the graphs) show good agreement with the stability curve, as well as the data from both investigations for a foreshore of 1:100. There is more scatter, but overall, the data points follow the stability curve reasonably well.

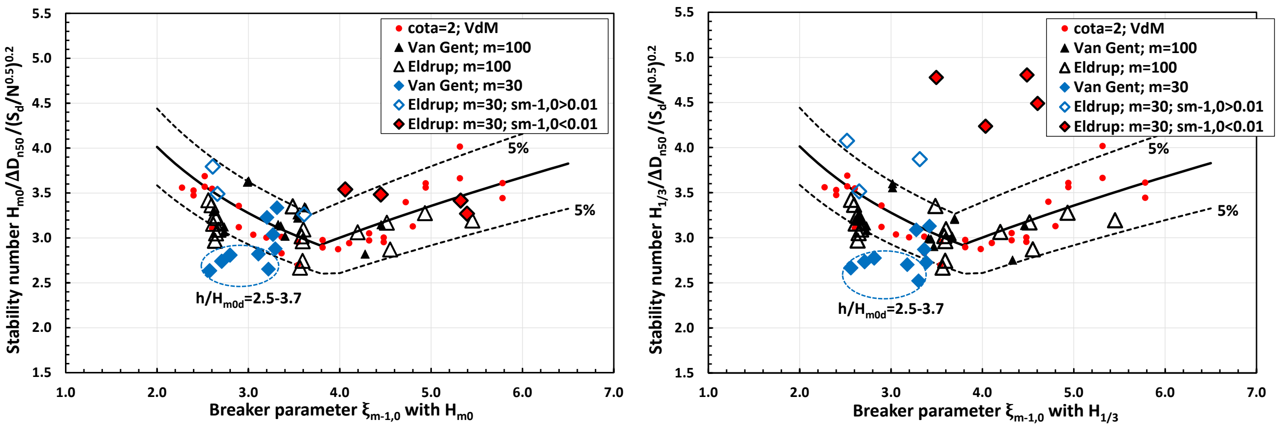

Figure 11: Stability for a permeable structure and slope angle 1:2 on foreshore slopes with m=30 and 100, and outside depth limited wave conditions with $h/H_{m0\ deep}$ >1.5. Solid black line is described by Eqs. 1-3. Left graph with the spectral wave height H_{m0}. Right graph with the significant wave height $H_{1/3}$, showing a significant deviation for the nonlinear shoaling waves with very low wave steepness (red diamonds in the graphs).

There is another exception: Eldrup and Lykke Andersen (2019a) performed tests with a very low wave steepness with $s_{m-1,0} < 0.01$ (the red diamonds in the graphs). The deviation from the stability curve is very large using $H_{1/3}$ (right graph) and although there is still a deviation with H_{m0} (left graph), it is much smaller. This was the reason that Eldrup and Lykke

Andersen (2019a) proposed to use H_{m0} instead of $H_{1/3}$ when considering rock stability for nonlinear waves in shallow water.

Figure 12 shows the stability data outside the depth-limited wave conditions for structure slopes of 1:3 and 1:4 and an impermeable core. The data of Eldrup and Lykke Andersen (2019a) cover the full wave steepness range and validate the Van der Meer formula (Equations 1-3). As two structure slopes have been tested, the original data of Van der Meer for deep water have been given as well for these two structure slopes (by small dots). For the fairly deep wave conditions of Eldrup and Lykke Andersen (2019a) with $h/H_{m0\ deep}$ > 2.7 there is no shoaling on the 1:100 foreshore and there is no significant difference between using H_{m0} or $H_{1/3}$. Also the data of the foreshore 1:30 of Van Gent et al. (2003) are in agreement with the other data, although a little above the stability curve. These data validate that the rewritten Van der Meer formula may at least be valid up to $h/H_{m0\ deep}$ > 1.5.

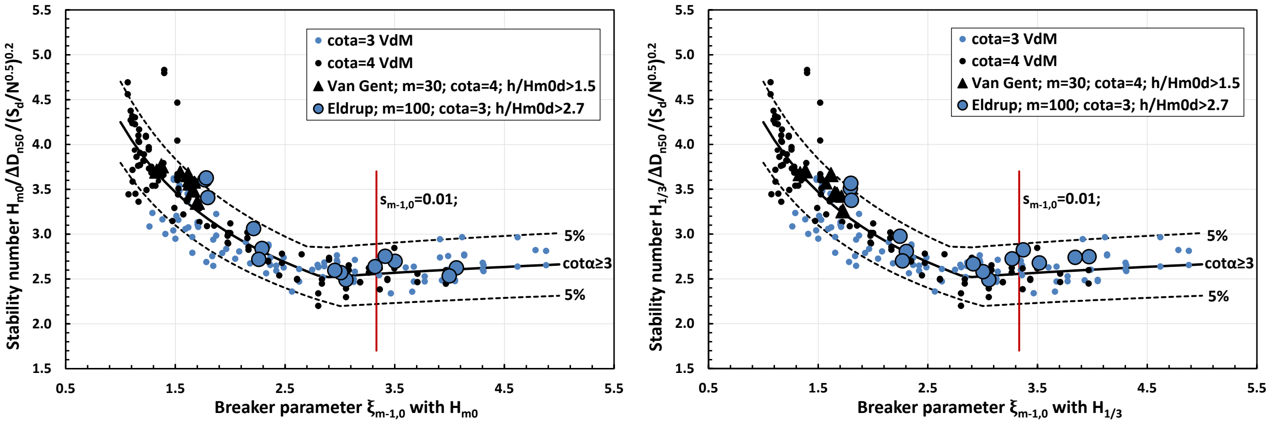

Figure 12: Stability for an impermeable structure and slope angles 1:3 and 1:4 on foreshore slopes with m=30 and 100, outside depth limited wave conditions with $h/H_{m0\ deep}$ >1.5. Solid black line is described by Eqs. 1-3. Left graph with the spectral wave height H_{m0}, right graph with the significant wave height $H_{1/3}$.

3.7 Stability in shallow water

3.7.1 Stability for spectral and significant wave heights

Data for stability in shallow water including $h/H_{m0\ deep}$ < 1.5 is only available from the investigation of Van Gent et al. (2003). The structures that were tested have been summarized in Table 1 and actually consist of four situations. These are a permeable structure with P = 0.5 and a structure slope with $cot\alpha$ = 2 (structures 1, 2 and 4 in Table 1) as well as a structure slope with $cot\alpha$ = 4 (structures 3 and 5 in Table 1) and an impermeable structure with P = 0.1 and structure slopes with $cot\alpha$ = 2 as well as 4 (structures 6 and 7 respectively in Table 1). For the permeable structures, two foreshore slopes have been investigated, a foreshore slope 1:100 and 1:30. For the impermeable structure only data for a foreshore slope of 1:30 is available.

Figures 13-16 give the four situations within the left graph using H_{m0} and in the right graph $H_{1/3}$ is used. The graphs are similar to Figures 11 and 12, except that the shallow water data have been added too. Shallow water data with $h/H_{m0\ deep}$ = 1-1.5 are given by open symbols, the shallower data with $h/H_{m0\ deep}$ < 1 have symbols filled with yellow. The modified stability formula for surging waves by Eldrup and Lykke Andersen (2019a), Equation 12, uses the H_{m0} and is given in the left graphs as a horizontal line. Actually, this formula does not make a difference between a structure slope of 1:2 and 1:4 for the surging region and the combined stability number becomes 3.23 for a permeable core with P = 0.5 and 2.46 for an impermeable core with P = 0.1. The right graph of Figures 13-16 gives the alternative simple formula of Van Gent et al. (2003) as in Equation 11 and is a little different for each structure tested. Now the horizontal line should be valid for the whole region of breaker parameter, not only for surging waves.

Figure 13 gives the data for a permeable structure with P = 0.5 and a structure slope of 1:2. The data for $h/H_{m0\ deep}$ > 1.5 have been discussed above with Figure 11, as well as the deviation of the data points for a 1:30 foreshore (the dashed oval in the graphs). If these specific points are discarded, all other data points with $h/H_{m0\ deep}$ > 1.5 are within the 90% confidence band. This validates that Equation 1 is at least valid up to a relative water depth of $h/H_{m0\ deep}$ > 1.5. The shallow water data for H_{m0} (left graph) are always higher in the graph than for $H_{1/3}$. Which is expected as the ratio $H_{m0}/H_{1/3}$ increases with decreasing relative water depth, see also Figure 6. There is quite a large scatter for the shallow water data with $h/H_{m0\ deep}$ < 1.5, with 36% (9 points) outside the 90% confidence band for H_{m0} and 40% (10 points) for $H_{1/3}$. The shallow water data with H_{m0} seem to follow the trend of Equations 1-3 and are sometimes even more stable.

Equation 12 in the left graph does not show the trend of the data and is lower than the average of the data. This is because tests by Van Gent et al. (2003) were only included in the analysis by Eldrup and Lykke Andersen (2019a) if second-order wavemaker theory was found valid according to the definition given by Eldrup and Lykke Andersen (2019b). This means that some of the tests might have unwanted free harmonics generated. However, it is not known what influence those free waves have on the stability of the structure. This actually meant that only three of the data points by Van Gent et al. (2003) in Figure 13 with $\xi_{m-1,0}$ > 4 contributed to establishing Equation 12. This choice also meant that many of the data points with $h/H_{m0\ deep}$ < 1.5 were not included in their analysis. However, the influence of the unwanted free waves is expected to be minor compared to the scatter observed for the data and thus it was decided to include them in the present paper. Equation 11 in the right graph gives horizontal lines that more or less give the average of all shallow water data, as it was fitted on these data, but the scatter is very significant.

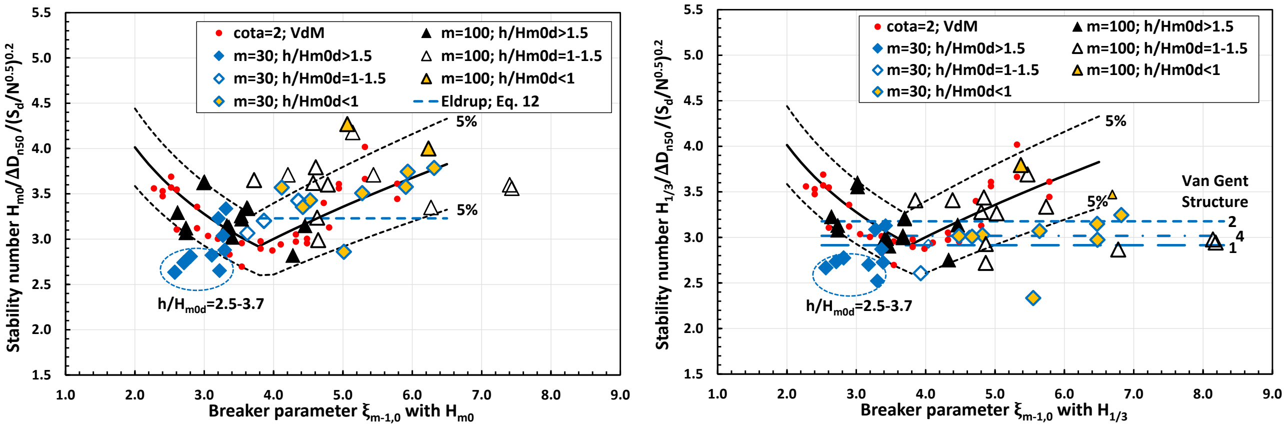

Figure 13: Stability for a permeable core (P=0.5) and $cot\alpha$ = 2 in shallow water. The solid line gives the rewritten Van der Meer formula (Equations 1-3), the horizontal line in the left graph gives Equation 12 of Eldrup and Lykke Andersen (2019a) and the lines in the right graph give Equation 11, the simplified formula by Van Gent et al. (2003). Left graph with the spectral wave height, H_{m0}, right graph with the significant wave height, $H_{1/3}$.

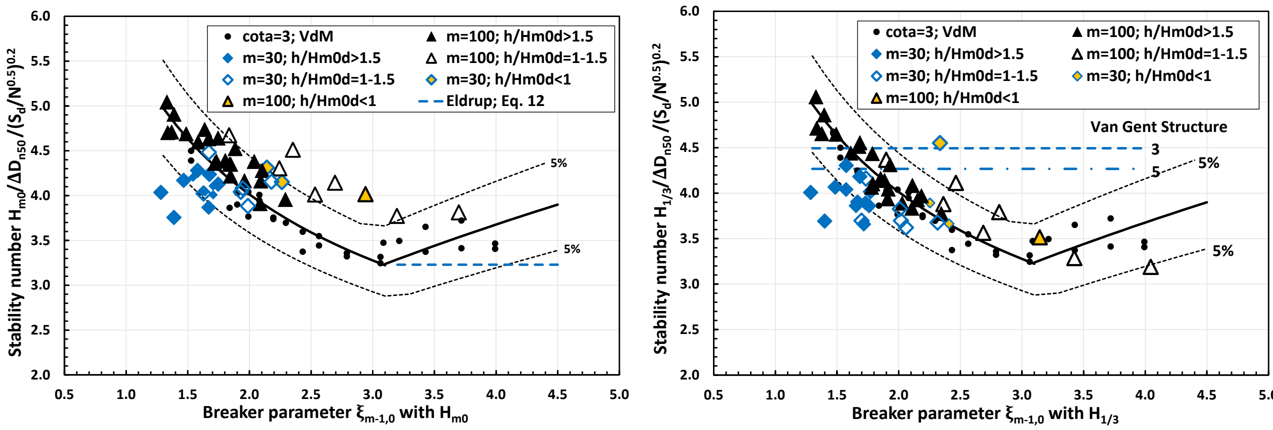

Figure 14: Stability for a permeable core (P=0.5) and $cot\alpha$ = 4 in shallow and very shallow water. The solid line gives the rewritten Van der Meer formula (Equations 1-3), the horizontal line in the left graph gives Equation 12 of Eldrup and Lykke Andersen (2019) and the lines in the right graph give Equation 11, the simplified formula by Van Gent et al. (2003). Left graph with the spectral wave height, H_{m0}, right graph with the significant wave height, $H_{1/3}$.

Figure 14 gives the data for a permeable structure with P = 0.5 and a structure slope of 1:4. The data outside the depth-limited wave conditions for $h/H_{m0\ deep}$ > 1.5 (solid symbols) match quite well with Equation 1 for plunging waves, as 12% of the data for H_{m0} and 15% of the data for $H_{1/3}$ is outside the 90% confidence band. This validates again that Equation 1 is at least valid up to a relative water depth of $h/H_{m0\ deep}$ > 1.5. The shallow water data for H_{m0} (left graph) are again higher in the graph than for $H_{1/3}$, as expected. The scatter for the shallow water data with $h/H_{m0\ deep}$ < 1.5 is given by 37% of the points for H_{m0} outside the 90% confidence band and 25% for $H_{1/3}$. The shallow water data with H_{m0} show a better stability than Equation 1, where the data with $H_{1/3}$ follow more the trend of Equation 1. Equation 12 in the left graph is lower than the average of the data in the area of application, but as for Figure 13 none of the Van Gent et al. (2003) data were used in the surging domain. Equation 11 in the right graph gives horizontal lines that give better stability than the average of all shallow water data, even though it was fitted on these data.

Figure 15 gives the data for an impermeable structure with P = 0.1 and a structure slope of 1:2. Also data of Eldrup et al. (2019) for a 1:2 slope has been added (foreshore slope 1:100), which all had relative depths of $h/H_{m0\ deep}$ > 3. The data outside the depth-limited wave conditions for $h/H_{m0\ deep}$ > 1.5 (solid symbols) match as average with Equation 1 for plunging waves, validating that Equation 1 is at least valid up to a relative water depth of $h/H_{m0\ deep}$ > 1.5. The scatter, however, is significant: 50% of all the data is outside the 90% confidence band, for H_{m0} as well as $H_{1/3}$. The data of Eldrup et al. (2019) for surging waves are consistently a little larger than Equation 2 and the original data of Van der Meer. The shallow water data for H_{m0} (left graph) are always higher in the graph than for $H_{1/3}$. There is not much scatter for the shallow water data in the left graph with $h/H_{m0\ deep}$ < 1.5 (only one point just outside the 90% confidence band) and they give a trend similar to the surging wave formula (Equation 2), but with worse stability. In the right graph for $H_{1/3}$ five of the nine points are outside the 90% confidence band. Equation 12 in the left graph shows the more or less horizontal trend of the data and was fitted to Eldrup et al. (2019) data and Van Gent et al. (2003) data with $\xi_{m-1,0} < 4$ because the applicability range given by Eldrup and Lykke Andersen (2019b) for second-order wavemaker theory was exceeded. Equation 11 in the right graph gives a horizontal line that is a little higher than the average of all shallow water data, although it was fitted on these data, but the line is significantly higher than the shallow water data with $h/H_{m0\ deep}$ < 1.5.

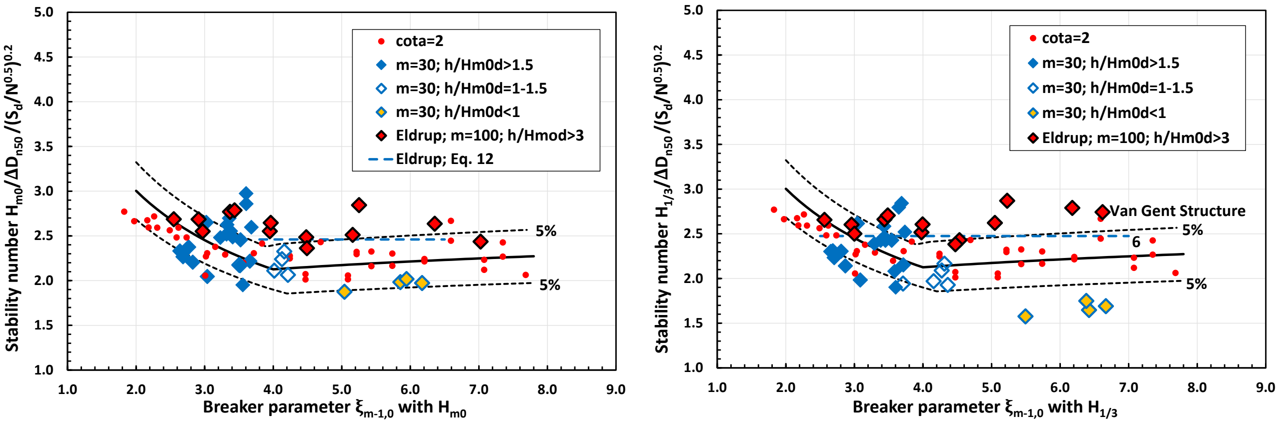

Figure 15: Stability for an impermeable core (P=0.1) and $cot\alpha$ = 2 in shallow and very shallow water. The solid line gives the rewritten Van der Meer formula (Equations 1-3), the horizontal line in the left graph gives Equation 12 of Eldrup and Lykke Andersen (2019a) and the line in the right graph gives Equation 11, the simplified formula by Van Gent et al. (2003). Left graph with the spectral wave height, H_{m0}, right graph with the significant wave height, $H_{1/3}$.

Figure 16 gives the data for an impermeable structure with P = 0.1 and a structure slope of 1:4. The data for $h/H_{m0\ deep}$ > 1.5 have been discussed above with Figure 12 and all shallow water data fall within the 90% confidence band. The shallow water data for H_{m0} (left graph) are always higher in the graph than for $H_{1/3}$ and four points (25%) are above the 90% confidence band There is not much scatter for the shallow water data with $h/H_{m0\ deep}$ < 1.5 and the shallow water data with $H_{1/3}$ seem to follow the trend of Equations 1-3 quite well, with only one point just outside the 90% confidence band. The stability is a little better for these data if H_{m0} is used. Equation 12 in the left graph has actually no data to compare with. Equation 11 in the right graph gives a horizontal line that is higher than the average of all shallow water data, although it was fitted on these data.

Overall, it may be concluded that data outside the depth-limited wave conditions are quite well described by the rewritten Van der Meer formula (Equation 1-3) and that this formula is at least valid up to $h/H_{m0\ deep}$ > 1.5. For shallow water conditions with $h/H_{m0\ deep}$ < 1.5 the picture is quite diverse. Sometimes very large scatter is found (Figure 13) and sometimes the data follow the trend of Equation 1 for plunging waves, certainly for the gentler slope of 1:4 and the use of $H_{1/3}$ (Figures 14 and 16, right graph).

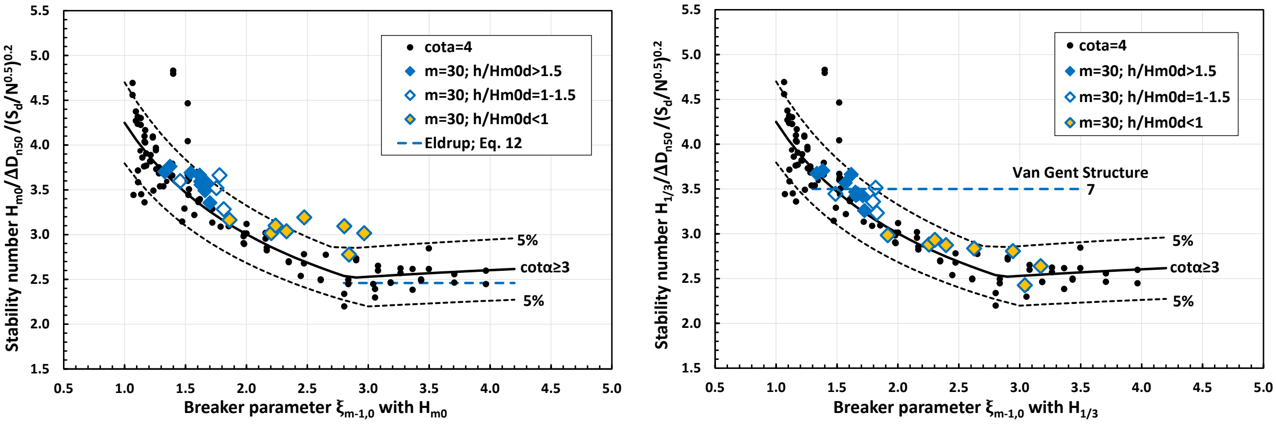

Figure 16: Stability for an impermeable core (P=0.1) and $cot\alpha$ = 4 in shallow and very shallow water. The solid line gives the rewritten Van der Meer formula (Equations 1-3), the horizontal line in the left graph gives Equation 12 of Eldrup and Lykke Andersen (2019a) and the line in the right graph gives Equation 11, the simplified formula by Van Gent et al. (2003). Left graph with the spectral wave height, H_{m0}, right graph with the significant wave height, $H_{1/3}$.

For $h/H_{m0\ deep}$ < 1 trends seem to become more or less horizontal, but the stability may be larger or smaller than the surging wave formula (Equation 2) and is not consistent. It is clear that for shallow water with $h/H_{m0\ deep}$ < 1, results deviate from the rewritten original *and* Modified Van der Meer formula (Equations 1-3 and 4-6 respectively). It means that we actually do not have a good prediction method for shallow water with $h/H_{m0\ deep}$ < 1 and only a limited prediction with significant scatter by the stability formula (Equations 1-3) for the intermediate zone with $h/H_{m0\ deep}$ = 1-1.5. Eldrup and Lykke Andersen (2019a) gives no influence of the wave steepness in the surging domain, cf. Equation 12. This tendency is followed for some datasets in shallow water, especially for the steeper slopes. However, the deep water tests by van der Meer (1988b) show a clear increasing trend for the permeable structures. Although Equation 11 of Van Gent et al. (2003) was fitted on the data in the graphs, it often does not give the average of the data points.

3.7.2 Stability for the 2% wave height

The Modified Van der Meer formula by Van Gent et al. (2003) has in their work been presented as a bulk analysis in a kind of stability curve with $S_d/\sqrt{N}$ on the vertical axis. In such a curve the influence of the wave period or breaker parameter is lost, which is why the combined stability number is given in this paper versus the breaker parameter. Equations 4-6 use the wave height $H_{2\%}$ and in Figures 17 and 18 the data with this wave height is given for the four structures that were tested. Now only the data of Van Gent et al. (2003) have been given and no original data of the deep water tests of Van der Meer (1987, 1988a and 1988b), as the $H_{2\%}$ has not been tabulated there.

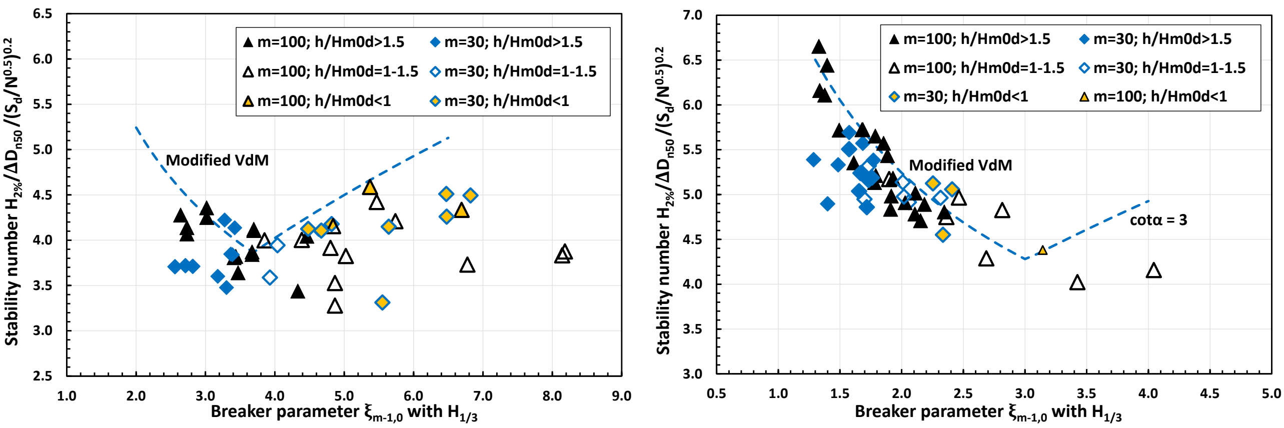

Figure 17: Stability with $H_{2\%}$, compared with the Modified Van der Meer formula as proposed by Van Gent et al. (2003) (Equations 4-6). Data for a permeable core with slope angles $cot\alpha$ = 2 (left graph) and 4 (right graph).

A permeable structure with a structure slope of 1:2 (left graph of Figure 17) shows that the plunging wave formula on the left side is a fit through the data, although a little too high. The surging curve on the right side is completely above the data and does not represent the shallow water data well. A permeable structure with a structure slope of 1:4 (right graph of Figure 17) shows that the plunging wave formula on the left side is also a reasonable fit of the data, although again, it is a little too high. The surging curve on the right side is again too high with respect to the data.

An impermeable structure with a structure slope of 1:2 (left graph of Figure 18) shows that the plunging wave formula on the left side is a good fit of the data. The surging curve on the rights side is completely above the shallow water data. An impermeable structure with a structure slope of 1:4 (right graph of Figure 18) shows that the plunging wave formula on the left side is a good fit of the data. The surging curve on the right side is more or less a fair estimation of the average of the data points in this area. Note that the formula was fitted on these data.

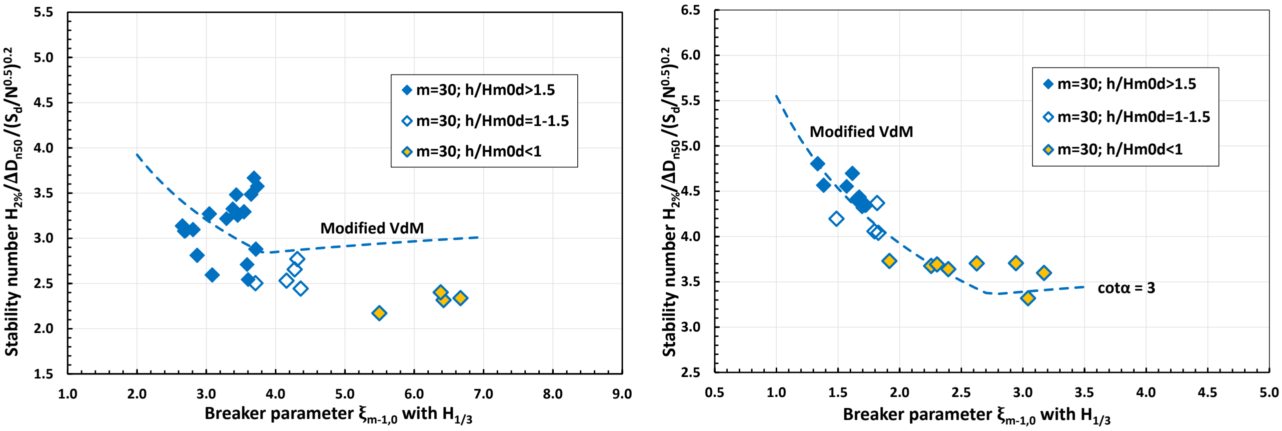

Figure 18: Stability with $H_{2\%}$, compared with the Modified Van der Meer formula as proposed by Van Gent et al. (2003) (Equations 4-6). Data for an impermeable core with slope angles $cot\alpha$ = 2 (left graph) and 4 (right graph).

Overall, it can be concluded that the Modified Van der Meer formula as developed by Van Gent et al. (2003) is reasonable for the plunging wave formula (Equation 4), but often it overpredicts the stability significantly for the surging wave part (Equation 5).

4 Discussion

Above analysis shows that the rewritten Van der Meer formula (Eqs. 1-3) describes stability of rock armoured slopes in shallow water at least up to $h/H_{m0\ deep}$ > 1.5, with a preference for using H_{m0} if nonlinear waves are present. For shallower water, where the waves are constantly breaking and where infragravity waves may come into play, it seems that the trends in the given graphs become more or less horizontal with considerable scatter. This means that under these conditions the wave period (and therefore the breaker parameter) have no influence anymore or the wave period to be used is not the energy period. A conclusion that was also drawn by Van Gent et al. (2003) who came with the simple alternative formula (Eqs. 7, 8 or 11), although fitted on all the data. Eldrup and Lykke Andersen (2019a) came to the same conclusion and suggested a formula for surging waves on a horizontal trend (Eqs. 10 or 12). None of these formulae describe the average of the data points in this shallow water area with $h/H_{m0\ deep}$ < 1.5 in a good way.

In order to give some guidance on stability in shallow water with $h/H_{m0\ deep}$ < 1.5 the average of the combined stability numbers $H_s/\Delta D_{n50}\ /(S_d/\sqrt{N})^{0.2}$ was calculated for each slope angle and permeability and for H_{m0} as well as $H_{1/3}$. Also, the standard deviation of each structure was calculated. The results are given in Table 2.

The averages in Table 2 are proposed as a first design guidance for stability in shallow water with a wave steepness in deep water $s_{m-1,0}$ > 0.025. One can of course interpolate between slope angles and permeabilities, but extrapolation should be done with great care. In any case the significant uncertainty should be taken into account in design. Table 2 shows that in many cases a standard deviation around 0.33. It is proposed to use at least this value when designing a structure in shallow water with $h/H_{m0\ deep}$ < 1.5. The given values are based on foreshore slopes of 1:30 and 1:100.

Table 2: Average and standard deviation of the combined stability numbers $H_s/\Delta D_{n50}\ /(S_d/\sqrt{N})^{0.2}$ for shallow water with $h/H_{m0\ deep}$ < 1.5 and for various slope angles and permeabilities, based on Van Gent (2017). The averages could be used for a first design guidance in shallow water.

Slope angle $cot\alpha$	2	4	2	4
Permeability factor P	0.1	0.1	0.5	0.5
	$H_s/\Delta D_{n50}\ /(S_d/\sqrt{N})^{0.2}$			
Mean for $H_s = H_{m0}$	2.08	3.21	3.55	4.20
St. Dev. for $H_s = H_{m0}$	0.14	0.27	0.34	0.33
Mean for $H_s = H_{1/3}$	1.86	2.99	3.12	3.79
St. Dev. for $H_s = H_{1/3}$	0.21	0.33	0.33	0.35

5 Conclusions

In conclusion, the analysis of rock armour stability has yielded several findings and implications for the prediction of rock armour stability of rubble mounds in shallow water. The data sets considered contained both sea and swell waves in shallow water, but for the swell waves no data was available for depth limited conditions ($h/H_{m0\ deep}$ < 1.5). The analysis was performed with damage data, S_d, that were in design range. Too small and too large damages were not considered. The number of waves N was taken from the deeper water condition.

The original Van der Meer formula rewritten with the energy period ($T_{m-1,0}$) was found to be valid for wave conditions where $h/H_{m0\ deep}$ > 1.5 no matter if $H_{1/3}$ or H_{m0} was used. The only exception is for low steepness swell waves on fairly steep slopes (non-linear waves), where the prediction is only reliable when using H_{m0} (Eldrup and Lykke Andersen (2019a)). The use of $H_{2\%}$ yielded the least accurate results, even when using the Modified Van der Meer formula as developed by Van Gent et al. (2003). The application limit $h/H_{m0\ deep}$ > 1.5 corresponds to the transition of waves being in depth-limited conditions. The limit also corresponds to a local wave height larger than approximately 70% of the offshore wave height. In shallow water conditions with 1 < $h/H_{m0\ deep}$ < 1.5, the formula provided slightly less reliable predictions. In very shallow water where $h/H_{m0\ deep}$ < 1, the results deviated substantially from the formula, showing less stability (mainly by using $H_{1/3}$) as well as more stability (mainly by using H_{m0}) and often showing a more horizontal trend. For practical use, it seems reasonable to use H_{m0} over $H_{1/3}$. It overcomes the deviation with nonlinear long waves on fairly steep foreshores, as well as it gives a conservative design approach as data with H_{m0} are in line or give better stability than calculated with the rewritten Van der Meer formula.

The simple alternative formula of Van Gent (2003), Eq. 7, 8 or 11, does not match with the data with $h/H_{m0\ deep}$ < 1.5, mainly because the formula was fitted on all data. Also, the formula for surging waves by Eldrup and Lykke Andersen (2019a), Eq. 10 or 12 does not match the data of Van Gent et al. (2003). The reason for this was that only few data by Van Gent et al. (2003) from the surging domain was used as for the other data the waves were generated outside the applicability range for second-order wavemaker theory given by Eldrup and Lykke Andersen (2019b).

In summary, for shallow water conditions with $h/H_{m0\ deep}$ < 1.5, there is currently no reliable method to describe the stability of rock slopes under wave attack. The data in the surging wave area show different trends (more or less horizontal in the given stability graphs) as well as showing better or less stability than the formula. While Van Gent (2017) dataset could potentially be used to develop an alternative method, it is recommended to conduct new tests with lower wave steepness (swell design conditions) shoaling into shallow water in order to cover the full range of design conditions before attempting developing an extended formula. In the meantime, two design methods are proposed to come to a preliminary design for shallow water with $h/H_{m0\ deep}$ < 1.5:

- use the rewritten Van der Meer formula with H_{m0} as a conservative approach;
- use fixed combined stability numbers $H_s/\Delta D_{n50}\ /(S_d/\sqrt{N})^{0.2}$ for $h/H_{m0\ deep}$ < 1.5, $cot\alpha$ = 2 and 4 and P = 0.1 and 0.5, which are available with given reliability (standard deviation) in Table 2, with the restriction that the deep water wave steepness $s_{m-1,0}$ should be larger than 0.025.

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Author contributions (CRediT)

Van der Meer: Conceptualization, Data curation, Formal Analysis, Writing – original draft, Writing – review & editing. Lykke Andersen: Analysis, review drafts. Eldrup: Analysis, review drafts.

Data access statement

The data on the original and rewritten Van der Meer formula can be found at: https://doi.org/10.5281/zenodo.5569052. The file https://www.researchgate.net/publication/320559323_Data-Stability_of_rock_slopes_with_shallow_foreshores gives the data of Van Gent (2017). Other data acquired in the study will be made available on request.

Declaration of interests

The authors report no conflict of interest.

Notation

Name	Symbol	Unit
Area of the erosion profile	A_e	m^2
Modification factor in stability equation for plunging waves in Equation 1 for new research; in application of original formula, c_{pl} = 1	c_{pl}	-
Modification factor in stability equation for surging wavesin Equation 2 for new research; in application of original formula, c_{su} = 1	c_{su}	-
Nominal diameter of the armour rock, $D_{n50} = (M_{50}/\rho_r)^{1/3}$	D_{n50}	m
Nominal diameter of the core	$D_{n50core}$	m
Gravitational acceleration	g	m/s^2
Water depth	h	m
Water depth at the toe of the structure	h_{toe}	m
Two percent wave height in the time domain	$H_{2\%}$	m
Two percent wave height in the time domain at the toe of the structure	$H_{2\%\ toe}$	m
Spectral significant wave height	H_{m0}	m
Spectral significant wave height in deep water	$H_{m0\ deep}$	m
Spectral significant wave height at the toe of the structure	$H_{m0\ toe}$	m
Significant wave height	H_s	m
Significant wave height at the toe of the structure	$H_{s\ toe}$	m
Significant wave height in the time domain	$H_{1/3}$	m
Significant wave height in the time domain at the toe of the structure	$H_{1/3\ toe}$	m
Median mass of the armour rock grading (50%-value by sample mass)	M_{50}	kg
Cotangent of the foreshore slope	m	-
Spectral moment, $m_n = \int_0^\infty f^n S(f) df$, n = −1, 0	m_n	m^2
Number of waves (or storm duration)	N	-

Damage number by counting of displaced stones, related to a width of D_{n50}	N_{od}	-
Notional permeability factor	P	-
Notional wave steepness, $s_{m-1,0} = 2\pi H_s/(gT_{m-1,0}^2)$	$s_{m-1,0}$	-
Damage level determined from the erosion profile, $S_d = A_e/D_{n50}^2$	S_d	-
Mean wave period measured in the time domain	T_m	s
Spectral wave period, $T_{m-1,0} = m_{-1}/m_0$	$T_{m-1,0}$	s
Spectral wave period at deep water	$T_{m-1,0\ deep}$	s
Spectral wave period at the toe of the structure	$T_{m-1,0\ toe}$	s
Peak wave period	T_p	s
Cotangent of the structure slope angle	$cot\alpha$	-
Relative mass density $\rho_r/\rho_w - 1$	Δ	-
Surf similarity or breaker parameter, $\xi_m = tan\alpha/(s_{om})^{0.5}$	ξ_m	-
Surf similarity or breaker parameter, $\xi_{m-1,0} = tan\alpha/(s_{m-1,0})^{0.5}$	$\xi_{m-1,0}$	-
Critical surf similarity or breaker parameter, Equations 3 and 6	$\xi_{m-1,0c}$	-
Mass density of the armour rock	ρ_r	kg/m^3
Mass density of the water	ρ_w	kg/m^3

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Appendix A. Correction in Van Gent (2017) dataset

Figure A-1 shows similar graphs as in Figure 3, but now for the waves measured at a foreshore slope of 1:30, where the data are solely taken from Van Gent (2017). The general trend of start of depth induced wave breaking for roughly $h/H_{m0\ deep} < 2$ is also similar as for the foreshore 1:100. It seems that the smaller wave steepness (red data points) gives larger breaking wave heights than for larger wave steepness (black data points), for the same relative depth. This may be caused by the steeper foreshore slope, cf. Figure 2.

But the graphs show significant deviations in the breaking zone area between tests that were performed for a permeable structure (P = 0.5) and an impermeable structure (P = 0.1). That is unexpected as similar wave conditions were generated in deep water and the wave heights in shallow water were measured *without the structure being present*. From a physical point of view there cannot be a difference if wave conditions in deep water are similar as well as the foreshore slope and the shallow location where the waves were measured. The data come from Van Gent (2017), and values were verified by the same relative depth ratios being given in Van Gent et al. (2003).

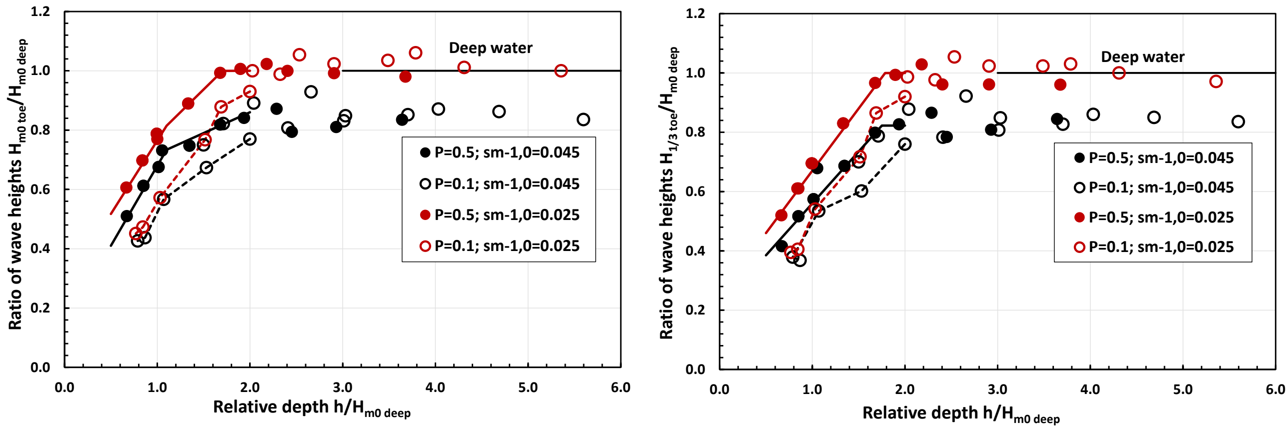

Figure A-1: Wave breaking over the 1:30 foreshore given as the ratio of wave heights in shallow and deep water versus the relative depth. Left graph with the spectral wave height H_{m0} and right graph with the significant wave height $H_{1/3}$. The graphs show an unexpected difference between similar test conditions in the wave breaking zone.

To investigate this in depth a number of tests have been selected from Van Gent (2017) where the foreshore (1:30) was the same, the water depth was the same and the deep water wave conditions (H_{m0}, $T_{m-1,0}$ and the peak period T_p) were similar. Table A-1 gives the summary of tests for four different test series. In the right half of the table the wave conditions at the toe of the structure are given, as well as the damage S_d that was measured at the rock slope (now of course with the structure present in the flume).

Table 1 shows that the wave heights at the toe for an impermeable structure are about 30% smaller than for similar conditions with stability tests on a permeable structure. The permeability of the structure as well as the slope angle of the structure have an influence on stability. A permeable structure is more stable than an impermeable structure. Also a more gentle structure slope will show better stability than a steep slope. The last column in Table A-1 gives the measured damage. Indeed, a gentler structure slope gives less damage for the same wave height and period. The damages for the 1:4 structure slope in the last two test series, are more or less similar for the permeable and impermeable core. That can only be physically explained if indeed the wave heights at the structure for the impermeable core are smaller than for the permeable core. The conclusion of this analysis must be that the *measured wave heights at the toe* of the structure might be correct.

But what can cause this difference in wave heights at the same local water depth and with the same deep water wave conditions? The only plausible explanation is that a different water depth was used during the tests compared to the one given in the spreadsheet and giving consequently a wrong local water depth. Tests were often performed with 0.05 m increments in water level. The difference in wave height at the toe between the two structure types is roughly 0.03 m (a little more than half of 0.05 m). If we assume that tests with an impermeable core were actually performed with a 0.05 m lower water level than given in the spreadsheet, the wave heights would become quite similar. This has been done for tests of Structures 6 and 7 in Van Gent (2017) with a single peaked spectrum and $h/H_{m0\ deep} < 1.5$. The new graphs are given in Figure A-2. Now the data of the two structures (open and closed symbols) match very well, due to a shift to the left of the corrected data of the impermeable core, by having a smaller relative depth $h/H_{m0\ deep}$.

Table A-1: Examples of tests from Van Gent (2017) with unexpected differences in wave heights in shallow water for a foreshore slope of 1:30 with similar conditions in deep water. Also the damage to the rock armoured slope is given as a reference.

Structure	No	*cotα*	*P*	h_{deep}	h_{toe}	Deep water H_{m0}	Deep water T_{m-10}	Deep water T_p	At the toe H_{m0}	At the toe T_{m-10}	Ratio H_{m0} P=0.1 to P=0.5	Damage S_d
-	-	-	-	m	m	m	s	s	m	s	%	-
4	21	2	0.5	0.525	0.125	0.147	1.451	1.572	0.090	1.977		1.96
5	21	4	0.5	0.525	0.125	0.147	1.451	1.572	0.090	1.977		0.55
6	18	2	0.1	0.525	0.125	0.144	1.448	1.589	0.063	2.025	70.0	7.09
4	22	2	0.5	0.525	0.125	0.149	1.988	2.476	0.104	2.724		5.04
5	22	4	0.5	0.525	0.125	0.149	1.988	2.476	0.104	2.724		0.62
6	19	2	0.1	0.525	0.125	0.148	2.021	2.471	0.070	2.614	67.3	10.34
5	23	4	0.5	0.600	0.200	0.190	1.657	1.828	0.139	1.994		5.49
7	8	4	0.1	0.600	0.200	0.187	1.676	1.840	0.106	1.940	76.3	6.96
5	24	4	0.5	0.600	0.200	0.200	2.247	2.810	0.154	2.690		12.57
7	9	4	0.1	0.600	0.200	0.194	2.290	2.866	0.111	2.484	72.1	12.20

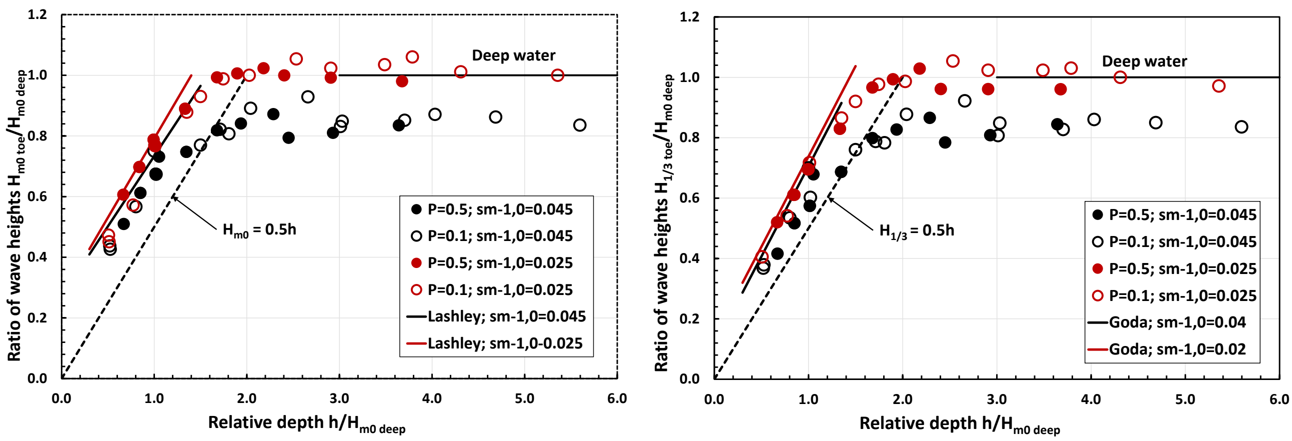

Figure A-2: Wave breaking over the 1:30 foreshore given as the ratio of wave heights in shallow and deep water versus the relative depth. Left graph with the spectral wave height H_{m0} and right graph with the significant wave height $H_{1/3}$. The water level of tests with an impermeable core and $h/H_{m0\ deep}$ < 1.5 has been decreased by 0.05 m. The graphs show now agreement in wave height development.

The question remains whether an incorrectly tabled water level is only present for a relative water depth $h/H_{m0\ deep}$ <1.5, or for all tests of Structures 6 and 7. For larger values of $h/H_{m0\ deep}$ >1.5 the data are on a horizontal trend and a change in relative water depth does not change this trend. It is also unclear what the situation was for these structures and double peaked spectra. Therefore, tests with double peaked spectra have not been used for the 1:30 foreshore to describe the behaviour of waves over the foreshore. The *stability* formulae only contain the wave parameters and not the depth. As the analysis shows that the wave parameters most likely are correct, all data will be used in the stability analysis.

Above analysis of inconsistency of data seems to be an administrative mistake in creating a data spreadsheet and is not a comment or criticism of test performance. As such, the complete analysis as described above was submitted to the authors of Smith et al. (2002) and Van Gent et al. (2003) for reply or comment. They replied that they neither question nor confirm our analysis, but note that the water level or local water depth was not a parameter in their damage analysis. It means that there is no further clarification on the water levels that were used for testing for these two structure types.