JOURNAL OF COASTAL AND HYDRAULIC STRUCTURES

Vol. 1, 2021, 8

Rock Armour Slope Stability under Wave Attack; the Van der Meer Formula revisited

Jentsje W. van der Meer[1]

[1]jm@vandermeerconsulting.nl, Van der Meer Consulting BV, Akkrum, The Netherlands; Emeritus IHE Delft, Delft, The Netherlands

This paper was submitted on 19 March 2021. It was accepted after double-blind review on 2 November 2021 and published online on 10 December 2021.

DOI: https://doi.org/10.48438/jchs.2021.0008

Cite as: "Van der Meer, J.W. (2021). Rock Armour Slope Stability under Wave Attack; the Van der Meer Formula revisited. Journal of Coastal and Hydraulic Structures, 1. https://doi.org/10.48438/jchs.2021.0008"

Abstract

The Van der Meer formula for rock slope stability under wave attack has now been in use in research and design for more than thirty years. This paper takes the original formula as a basis for exploration into a larger field of application as investigated by various authors, and includes improved insights into the parameters used. In the field of wave overtopping, the spectral period has become the leading wave period in preference to the mean or peak period. Here, the Van der Meer formula has been rewritten to include this spectral wave period, based on original data.

A guideline on how to integrate new research to the formula's application areas has been developed, including how to arrive at a modification to the formula. First the essence of the Van der Meer formula is given, viz a gradually developing damage curve according to a power function, and the relationship between damage development and storm duration. Each new research dataset should be validated for these relationships before any direct comparison is possible. Such a comparison can be done along graphs of damage versus the breaker parameter (including wave period and slope angle), including the original data of the Van der Meer formula. Possible modifications to the formula are best described by using multiplication coefficients for plunging and surging waves that directly show the influence on the original formula with deviation from the value 1.0.

Two significant investigations have then been re-analysed using these proposed guidelines: one exploring the influence of rock shape and one the influence of rock placing. The first research was performed with a significantly thinner armour layer, so it actually included two effects: rock shape and a thinner armour layer. The two effects have been separated and can be described by adjusted multiplication coefficients. Individually placed rock with three points of contact, or rock placed with a dense packing, increases stability significantly and may always be seen as a good measure. The damage curve, however, is not gradually developing, but shows a kind of brittle failure. Direct comparison with the Van der Meer formula is not possible then. A brittle failure needs a safety factor, resulting in similar stone sizes for design as from the formula.

Keywords

Coastal Structures, Stability, Rock Armour Slopes, Damage, Spectral Period, Rock Placing, Rock Shape

1 Introduction

1.1 History

The Van der Meer formula on rock slope stability against wave attack was originally published in Van der Meer (1987), Van der Meer (1988a), and also in the PhD thesis Van der Meer (1988b) where almost all relevant test data have been gathered. The work of Van der Meer was an extension of earlier work performed by Thompson and Shuttler (1975). Besides the PhD thesis, more information has been described in Delft Hydraulics' project reports M1983 (8 reports, in Dutch).

The Van der Meer formula has been given in major design manuals such as the Rock Manual (2007) and the Coastal Engineering Manual (CEM 2002). Since its publication, it has now been used in design and research for more than thirty years. This paper will take the original formula as a basis and will then explore into additional fields of application as investigated by various authors and will include improved insights into the parameters used. The formula developed in Van der Meer (1988b) is therefore repeated here:

$$\frac{H_s}{\Delta D_{n50}} = \begin{cases} 6.2\, P^{0.18} \left(\frac{S_d}{N^{0.5}}\right)^{0.2} \xi_m^{-0.5} & \text{for plunging waves} \quad (1) \\ 1.0\, P^{-0.13} \left(\frac{S_d}{N^{0.5}}\right)^{0.2} \cot\alpha^{0.5} \xi_m^{P} & \text{for surging waves} \quad (2) \end{cases}$$

where:

H_s	=	the significant wave height in the time domain $H_{1/3}$
Δ	=	the relative mass density $\Delta = \rho_r/\rho_w - 1$
ρ_r	=	the mass density of the armour rock
ρ_w	=	the mass density of the water
D_{n50}	=	the nominal diameter of the armour rock, $D_{n50} = (M_{50}/\rho_r)^{1/3}$
M_{50}	=	the median mass of the armour rock grading (50%-value by mass)
P	=	the notional permeability factor
S_d	=	the damage level determined from the erosion profile, $S_d = A_e/D_{n50}^2$
A_e	=	the area of the erosion profile
N	=	the number of waves (or storm duration)
ξ_m	=	the surf similarity or breaker parameter, $\xi_m = \tan\alpha/(s_{om})^{0.5}$
α	=	the slope angle
s_{om}	=	the notional wave steepness, $s_{om} = 2\pi H_s/(gT_m^2)$
g	=	the acceleration of gravity
T_m	=	the mean wave period measured in the time domain

The critical breaker parameter ξ_{mc} is given as:

$$\xi_{mc} = \left(\frac{6.2}{1.0}\; P^{0.31} \sqrt{\tan\alpha}\right)^{\frac{1}{P+0.5}} \quad (3)$$

For $\xi_m \leq \xi_{mc}$ plunging waves are found and Eq. 1 should be used, and for $\xi_m \geq \xi_{mc}$ Eq. 2 for surging waves should be used. For $\cot\alpha \geq 3$ one should use $\cot\alpha = 3$ to calculate the critical breaker parameter with Eq. 3. For a more in-depth description of the parameters the reader is respectfully referred to the Rock Manual (2007) and Van der Meer (1987, 1988a and 1988b). The original Van der Meer formula in general is applicable for:

- relatively deep water (no severe wave breaking over a foreshore);
- randomly- (bulk-) or individually placed rock in two full layers without specific care to orientation of the rock;
- equant/semi-round rock;
- narrow- and wide-graded armour rock, $D_{85}/D_{15} < 2.3$;
- slopes from 1:2 to 1:6 for an "impermeable core";
- slopes from 1:1.5 to 1:3 for a "permeable core";
- a slope of 1:2 for a "homogeneous structure";
- a wave steepness range of about 0.01 up to the physical maximum.

The formula is based on tests without a foreshore and without breaking waves, although a limited number of tests were performed on a foreshore of 1:30. Based on these tests it was concluded that in shallow water with limited wave breaking, because the largest waves may break more than the smaller ones, the damage was more accurately described by using the $H_{2\%}$ wave height instead of the significant wave height H_s. In Eqs. 1 and 2 this would mean that H_s is replaced by $H_{2\%}$ and that coefficients 6.2 and 1.0 are replaced by 8.7 and 1.4, because according to a Rayleigh distribution in deep water, $H_{2\%}/H_s$ is assumed to be 1.4. Just using Eqs. 1 and 2 would slightly overestimate damage for conditions with limited wave breaking, but this can also be viewed as a slightly conservative approach for design. The Van der Meer formula with Eqs. 1 and 2 is not valid for really shallow water and heavy wave breaking. The limit of application is about $H_{s,\,toe}/H_{s,\,deep} > 0.70$, where $H_{s,\,deep}$ is in a location before depth induced wave breaking occurs.

Various researchers have proposed modifications to the formula in order to include aspects that were not part of the original investigation with the intention of extending the application area. The Rock Manual (2007) describes the significant works of Latham *et al.* (1988) on the influence of armour stone shape and Stewart *et al.* (2002, 2003) on the influence of armour stone packing. In both cases the shape of the formula remained the same, but coefficients were adjusted and fitted to the new data.

Work by Smith *et al.* (2002) and Van Gent *et al*. (2003) resulted in a modified formula for shallow water. First the mean period T_m from the time domain was replaced by the spectral period $T_{m-1,0}$ and then a bulk fit was made on the new data, see Figure 5.42 in the Rock Manual (2007). It was noted that there were unexplained differences between the Van der Meer formula (Eqs. 1 and 2) and the new data, including within the area of application where they were expected to be similar. The results are given in the Rock Manual (2007) as the Modified Van der Meer formula for shallow water. A notable difference between the original and Modified Van der Meer formula is the use of different wave periods (T_m and $T_{m-1,0}$), which makes a direct comparison difficult. Recently Eldrup (2019) and Eldrup *et al.* (2019) investigated the estimation of the notional permeability factor and Eldrup and Andersen (2019) looked at the stability of rock armour layers in shallow water, specifically for very low wave steepnesses. They concluded that H_{m0} may be a better wave height to use than $H_{1/3}$ or $H_{2\%}$ because it is less influenced by the non-linearity of the waves.

Stability of rock slopes in (very) shallow water with heavy wave breaking is a field of research that has not yet fully been explored. One of the problems is that wave conditions may change drastically if the relative water depth becomes small. Further, if the wave heights in the flume become very small – perhaps only a few centimetres – the rock size of the armour slope will also have to be quite small in order to show relevant damage. This field of research is not further considered in this paper, which means that the focus is only on relatively deep water up to limited wave breaking, in line with the original Van der Meer formula.

In each investigation new data were analysed by the researchers in a specific way and mostly the methods differed between each other. The analyses often made use of the Van der Meer formula and results were to modify coefficients by fitting for a specific aspect or even include new parameters and leaving the main structure of the formula intact. Most of the investigations had a limited scope with respect to the original tests.

Van der Meer (1987) described the research and analysis that resulted in the Van der Meer formula. Van der Meer (1988a) described how the formula could be used in design of rock slopes, in a deterministic or probabilistic way. Now by hindsight we can conclude that a guideline is missing on how to compare later/new research with the existing formula, including the original data, and how to modify the formula in the most efficient and consistent way. This paper gives that guideline and then it has been applied in a re-analysis of a few significant existing investigations.

1.2 Objectives

One of the first objectives of this paper is to rewrite the Van der Meer formula into a version with the spectral wave period $T_{m-1,0}$. This is possible as spectra of the original research are available.

The Van der Meer formula has been based on a large dataset, covering a significant area of application, see the previous section. The formula was based on constant physical relationships that were found during analysis of the results. In this paper, the essence of the Van der Meer formula has been described with the relationship between wave height and damage, between damage and storm duration and the full working area of the formula by graphs.

Then various methods are described and analysed how to modify and/or extend the Van der Meer formula in a consistent way, if for specific aspects or research areas deviations are found. A preferred method is given. Finally, the substantial work of Latham *et al.* (1988) on rock shape and the work of Stewart *et al.* (2002 and 2003) on armour stone packing, has been re-analysed according to the new preferred method.

The work on shallow foreshores (Smith *et al.* (2002), Van Gent *et al.* (2003), Eldrup (2019) and Eldrup and Andersen (2019)) has not been considered in this paper for re-analysis, as it is still seen as a research area under investigation.

2 The spectral period in the Van der Meer formula

Holterman (1998) discovered that the spectral period $T_{m-1,0}$ could be a much more stable wave period for wave-structure-interaction than the peak period T_p or the mean period T_m (from the time domain). This is certainly the case if spectral shapes are not nicely single peaked, but bi-modal or flat. Flat spectra are often observed in areas where heavy wave breaking occurs and then a mean or peak period become less meaningful. In such a case a spectral period is preferred. Work of Van Gent and Smith (1999) and Van Gent (2001) proved that this was indeed the case for wave overtopping and since then this spectral period has been used for wave overtopping, like in EurOtop (2007, 2018). Work by Smith *et al.* (2002) and Van Gent *et al.* (2003) resulted in a modified stability formula for shallow water. First the mean period T_m from the time domain was replaced by the spectral period $T_{m-1,0}$ and then a fit was made on the new data, resulting in the Modified Van der Meer formula for shallow water.

The spectral period was not in use when the Van der Meer formula was developed. The choice at that time was to use the peak period or the mean period. The choice of the mean period was based on the conclusion that measured results for a very narrow and wide spectrum coincided best for this period, which had the positive effect that the spectral shape itself was no longer an aspect to be considered for stability of rock slopes, see Van der Meer (1988b). If the spectral period is to replace the mean period in the Van der Meer formula, the effect of spectral shape has to be validated again.

It is possible to rewrite the Van der Meer formula with the spectral period $T_{m-1,0}$, without refitting the data, because this spectral period can be derived from spectral graphs. In total 18 spectra are still available from the original work of Van der Meer (1988b). The work of Van der Meer (1988b) has more extensively been reported in the Delft Hydraulics' report M1983 (1988). In that report, graphs of six standard spectra are available and one graph is available in the PhD-thesis. By digitising the graphs of these spectra, it was possible to obtain the (average) relationship between T_p, T_m and $T_{m-1,0}$ for the three spectral shapes used during testing. Digitisation may have led to (small) deviations from the real spectra. Both T_m and T_p are available in the data set of Van der Meer (1988b, Appendix I). The tests with digitised spectra and derived spectral periods are given in Table 1.

Note that tests 60, 164 and 171 appear twice in Table 1 because the data processing for the spectral shapes in Van der Meer (1998b) led to a little "smoother" spectra than from M1983 (1988). The average ratio for the standard spectrum of $T_{m-1,0}/T_m$ = 1.095. That value can be used to rewrite the Van der Meer formula into a version with the spectral period $T_{m-1,0}$. The formula for plunging waves, Eq. 1, becomes then:

$$\frac{H_s}{\Delta D_{n50}} = 6.49 P^{0.18} \left(\frac{S_d}{N^{0.5}}\right)^{0.2} \xi_{m-1,0}^{-0.5} \qquad \text{for plunging waves} \qquad (4)$$

Table 1: Spectral periods $T_{m-1,0}$ from digitised spectra for three spectral shapes, as used in Van der Meer (1988b).

Original graph	Test	Spectral shape	T_p (s) Measured	T_m (s) Measured	$T_{m-1,0}$ (s) Digitised	$T_p/T_{m-1,0}$ (-)	$T_{m-1,0}/T_m$ (-)
M1983 (1988) Fig. B1	25	Standard	2.53	2.17	2.41	1.05	1.11
M1983 (1988) Fig. B2	23	Standard	2.53	2.19	2.39	1.06	1.09
M1983 (1988) Fig. B3	22	Standard	2.56	2.21	2.42	1.06	1.10
M1983 (1988) Fig. B4	24	Standard	2.53	2.19	2.37	1.07	1.08
M1983 (1988) Fig. B5	21	Standard	2.53	2.18	2.38	1.06	1.09
M1983 (1988) Fig. B6	60	Standard	2.17	1.84	2.03	1.07	1.10
M1983 (1988) Fig. B7	26	Standard	3.17	2.65	2.89	1.10	1.09
Van der Meer (1988b) Fig. 2.6	60	Standard	2.17	1.84	2.00	1.08	1.09
					average	**1.068**	**1.095**
M1983 (1988) Fig. B8	195	Narrow	1.40	1.40	1.46	0.96	1.04
M1983 (1988) Fig. B9	164	Narrow	1.79	1.79	1.82	0.98	1.02
M1983 (1988) Fig. B10	161	Narrow	2.24	2.22	2.27	0.99	1.02
M1983 (1988) Fig. B11	186	Narrow	3.20	3.07	3.17	1.01	1.03
Van der Meer (1988b) Fig. 2.6	164	Narrow	1.79	1.79	1.79	1.00	1.00
M1983 (1988) Fig. B12	188	Wide	1.83	1.41	1.67	1.09	1.19
M1983 (1988) Fig. B13	171	Wide	2.67	1.81	2.32	1.15	1.28
M1983 (1988) Fig. B14	182	Wide	3.17	2.16	2.81	1.13	1.30
M1983 (1988) Fig. B15	176	Wide	4.25	2.85	3.79	1.12	1.33
Van der Meer (1988b) Fig. 2.6	171	Wide	2.67	1.81	2.19	1.22	1.21

where $\xi_{m-1,0}$ is the spectral breaker parameter $\xi_{m-1,0} = \tan\alpha/(s_{om-1,0})^{0.5}$ with $s_{om-1,0}$ being the notional wave steepness, $s_{om-1,0} = 2\pi H_s/(gT_{m-1,0}{}^2)$. The notional permeability factor, P, is also used as an exponent in the formula for surging waves, Eq. 2, which means that depending on this factor, the rewritten formula will vary a little. The coefficient 1.0 in Eq. 2 becomes 0.991, 0.956 and 0.947 for values P = 0.1, 0.5 and 0.6 respectively. They are all quite close and a value of 0.97 has been chosen as representative. This changes the formula for surging waves, Eq. 2, to:

$$\frac{H_s}{\Delta D_{n50}} = 0.97\, P^{-0.13} \left(\frac{S_d}{N^{0.5}}\right)^{0.2} \cot\alpha^{0.5}\, \xi_{m-1,0}^{P} \quad \text{for surging waves} \quad (5)$$

Van der Meer (1988b) tested an impermeable structure (P = 0.1) and a slope with cotα = 3 with three different spectral shapes (Fig. 2.6 in the reference). By comparing the stability results for the peak period as well as the mean period (Fig. 3.20 in the reference) it appeared that using the mean period gave results from a narrow and wide spectrum that were close to each other. For the peak period this was also the case for start of damage, but there were significant differences for much larger damage. This conclusion was used to develop the formula with the mean period T_m as a preferred period.

The tests with narrow and wide spectra are given in Appendix I of Van der Meer (1988b) by tests 158-197. Compared to the earlier tests with a standard spectrum, the structure was significantly less stable for the narrow and wide spectrum. As discussed in Van der Meer (1988b) this was thought to be caused by the shape of the armour stones that became more rounded/worn out after more than 150 tests. One test with the standard spectrum that had been performed earlier, was repeated during the period of testing the narrow and wide spectrum, and indeed gave similar stability as for these narrow and wide spectra and much lower stability than in the previous test. It is for this reason that the results of the narrow and wide spectrum were qualitatively compared, without comparing with the earlier test results, nor with a stability formula.

Table 1 also shows the spectral periods $T_{m-1,0}$ from digitised narrow and wide spectra, for all four periods tested. All three wave period measures, T_p, T_m and $T_{m-1,0}$ can be applied in stability graphs like Figures 1-3. Figure 1 shows that the results for a wide spectrum are significantly higher than for a narrow spectrum, in the range ξ_p = 2.5-4. All other data coincide well. The overall picture in Figure 2 with the mean period T_m is that all the data coincide fairly well with only minor deviations. This was the reason to select the mean period in developing the original Van der Meer formula.

Figure 3 is the graph with the spectral period $T_{m-1,0}$ and shows, as may be expected because $T_{m-1,0}$ is in between T_m and T_p, a result that falls in between the observations for the peak and mean periods in Figures 1 and 2 respectively. The overall picture is quite good with some deviations in the range $\xi_{m-1,0}$ = 3-4, but smaller than for the peak period. Based on visual fits it can be concluded that both the mean period and the new spectral period are able to neutralise the influence of spectral shape on rock slope stability and therefore the influence of spectral shape may be neglected when using Eqs. 4 and 5 with the spectral period $T_{m-1,0}$.

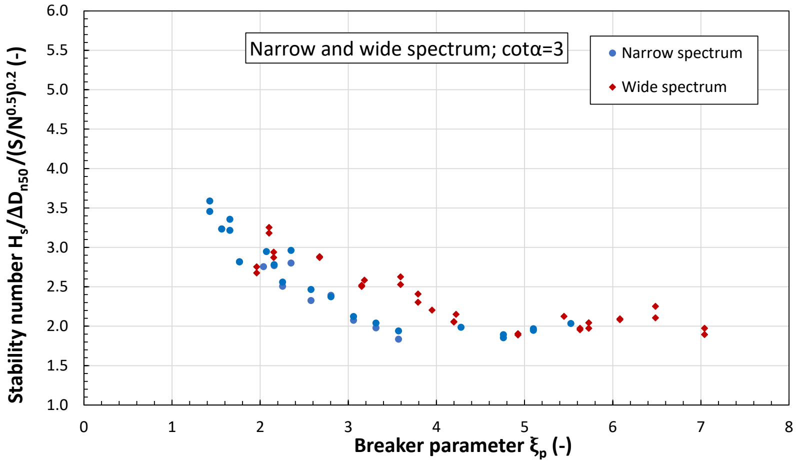

Figure 1: Comparing the stability results for a narrow and wide spectrum with the peak period T_p.

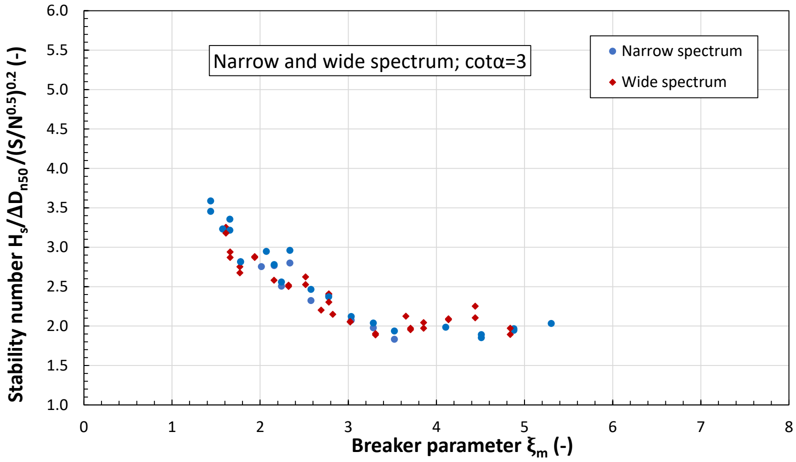

Figure 2: Comparing the stability results for a narrow and wide spectrum with the mean period T_m.

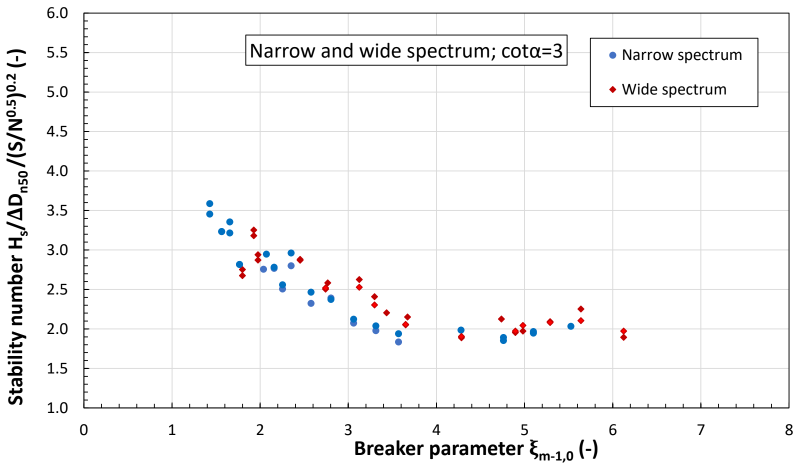

Figure 3: Comparing the stability results for a narrow and wide spectrum with the spectral period $T_{m-1,0}$.

3 The background of the Van der Meer formula

3.1 The essence of the formula

The combined data set of Thompson and Shuttler (1975) and Van der Meer (1988b) covered a significant range of rock slopes under a range of wave attack conditions. The slope angles ranged from cotα = 1.5 to 6 and the permeability of the structures from P = 0.1 (impermeable core, lower bound) up to P = 0.6 (homogeneous structure, upper bound), with a permeable structure with P = 0.5 in between. The Van der Meer formula (Eqs. 1 and 2 or 4 and 5) describe various physical relationships between parameters. For instance, the relationship between the stability number $H_s/\Delta D_{n50}$ and the wave period or the breaker parameter (ξ_m or $\xi_{m-1,0}$), which is often given by graphs like Figures 1-3. In such graphs the influence of the notional permeability factor can also be shown. Another relationship is the one between the significant wave height H_s and the damage S_d. Finally of interest is the relationship between damage S_d and the storm duration or number of waves N. All of them will be described here in detail.

An important relationship is given between $H_s/\Delta D_{n50}$ and $(S_d/N^{0.5})^{0.2}$. The exponent 0.2 gives the shape of a damage curve, where the damage S_d is a function of the wave height with an exponent of 1/0.2 = 5 or in general terms:

$$S_d = f(H_s^a) \tag{6}$$

The damage is gradually increasing according to a power curve with exponent "a", giving the shape of the damage curve. The value of 5 is of course important, but so also is the fact that damage is gradually increasing with the wave height. This shape of the damage curve and value of "a" was explicitly considered when developing the Van der Meer formula and was established for various parts of the data set:

For plunging waves:

- P = 0.1; exponent 0.22 (exponent in Eq. 6: a = 4.5)
- P = 0.5; exponent 0.17 (exponent in Eq. 6: a = 5.9)
- P = 0.6; exponent 0.19 (exponent in Eq. 6: a = 5.3)

For surging waves:

- P = 0.1; exponent 0.17 (exponent in Eq. 6: a = 5.9)
- P = 0.5; exponent 0.19 (exponent in Eq. 6: a = 5.3)
- P = 0.6; exponent 0.25 (exponent in Eq. 6: a = 4.0)

Overall, the exponent "a" was between 0.17 and 0.25, giving shape factors for the damage curve of S = f(H_s^4) up to S = f(H_s^6). An average value of 0.2 was chosen for the exponent to develop the stability formula. Other testing on stability of rock slopes should be validated against these values before the average exponent 0.2 is applied. But values of the exponent between 0.17 and 0.25 should not be considered as deviating significantly from the Van der Meer formula. And one should check for new research whether the damage is indeed gradually increasing according to a well-behaved curve.

The relationship $S_d/N^{0.5}$ gives the damage as a square root function of the number of waves (proportional to the test or storm duration):

$$S_d = f(N^{0.5}) \tag{7}$$

This relationship was based upon tests by Thompson and Shuttler (1975), who performed many tests with damage measurements after 1000, 2000, 3000, 4000 and 5000 waves. They also performed a few tests with more than 10,000 waves. The square root function is valid for $1000 \leq N \leq \approx 7000$. For $N < 1000$ one should take a linear relationship between S_d and N. For $N > 6000$ the relationship will become much flatter than the square root function and should tend to a limiting value quite close to 1.3 times the damage at N = 5000.

So, the square root function has a limited applicability and it is proposed that testing on stability of rock slopes should have damage measurements after at least two test durations in order to check the validity of the square root function for the tests. Validation would then be easy: for example, the ratio of damage after 3000 and 1000 waves is expected to be $(3000/1000)^{0.5}$ = 1.73.

For new research one should check first whether Eqs. 6 and 7 are valid. If so, it means that the stability number could be combined with $(S_d/N^{0.5})^{0.2}$, giving the parameter group for stability (or relative stability number):

$$\frac{H_s}{\Delta D_{n50}} / \left(\frac{S_d}{N^{0.5}}\right)^{0.2} \tag{8}$$

The remaining parameters to describe in a graph are then: the slope angle; the permeability of the structure, and the influence of the wave period or breaker parameter. Figure 4 shows this graph, which covers the whole application area of the Van der Meer formula, based on tests by Thompson and Shuttler (1975) and by Van der Meer (1988b). The graph shows the influence of the slope angle (gentle slopes on the left side, steep slopes on the right side) and the influence of the permeability of the structure as a function of the wave period or breaker parameter. It should be noted that a steep slope with cotα = 1.5 was only tested with a permeable core, and that a milder slope with cotα > 3 was not tested for a permeable core. The graph uses the spectral period $T_{m-1,0}$.

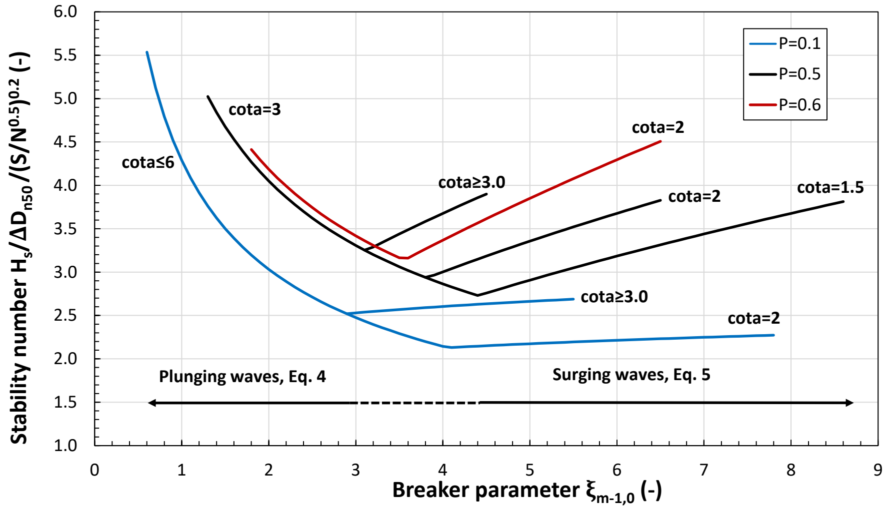

Figure 4: Application area of the Van der Meer formula in a $\xi_{m-1,0}$ - $H_s/\Delta D_{n50}$ / $(S_d/N^{0.5})^{0.2}$ graph.

The curves in Figure 4 cover the tested ranges and are not applied outside these ranges. The stability for plunging waves (the curves on the left side) decrease with decreasing permeability. The curves in this area are parallel (shifted vertically by the P-influence). The stability of permeable structures increases significantly in the surging waves region (right side of the graph) if the breaker parameter increases. This is caused by the parameter group $\xi_{m-1,0}^P$ in Eq. 5, where the exponent P gives a flat line if the value is small (P = 0.1 for an impermeable core) and an increasing line with a larger value of P (P = 0.5 or 0.6 for a permeable core or homogeneous structure). The values of P = 0.1, 0.5 and 0.6 were chosen as fixed values during the development of the formula in order to describe the different trends of stability under surging waves. A larger value of P gives a steeper curve.

The fixed values of P were based on the trend of measurements for surging waves, but they were also used to connect the three structures with different permeability into one formula. It is for this reason that P appears twice in the surging stability formula, Eq. 5.

In principle one may not extend the curves for application if no data are available unless caution is taken. This means that although the formula can calculate stabilities for, e.g., impermeable structures with a slope of 1:1.5, or permeable structures gentler than 1:3, or homogeneous structures other than 1:2, tests have not been performed for these structures.

3.2 Validation of the formula

Thompson and Shuttler (1975) performed their stability tests up to N = 5000 with damage measurements after each 1000 waves. The armour layer was reconstructed after each test. This procedure was followed by Van der Meer (1988b), although the total number of waves was N = 3000, with only one intermediate damage measurement at N = 1000. The armour layer was also reconstructed after each test.

This test procedure gave a damage graph (wave height versus damage for a fixed wave period), where the damage values are not dependent upon each other. That is different from a procedure where after each test, the armour slope is not reconstructed and where testing simply continues with a higher wave height. In order to find $\xi_{m-1,0}$ and $H_s/\Delta D_{n50}$-values for fixed damage levels for each test and test duration, average damage curves were constructed for each set of tests with the same wave period. These values for fixed damage levels are more reliable than a single test result alone, and enabled clear trends in $\xi_{m-1,0}$ - $H_s/\Delta D_{n50}$ graphs to be shown, and the Van der Meer formula to be developed accordingly.

But it is also possible to validate Eqs. 4 and 5, or Figure 4, directly with measured data instead of fixed damage levels. The validity of Eqs. 6 and 7 means that the parameter group in Eq. 8 can be used for the vertical axis, as in Figure 4. Figures 5-7 give these graphs for each structure type – impermeable, permeable, and homogeneous – separately. For the impermeable core data of Thompson and Shuttler (1975) the fixed damage data were used with an assumed relationship of $T_{m-1,0} = 1.1T_m$. The data and graphs are available on www.vdm-c.nl or DOI 10.5281/zenodo.5569052.

Some limitations were introduced, however, when producing Figures 5-7. The fixed damage levels, used for developing the Van der Meer formula, started with "start of damage", S_d = 2 or 3, depending on the slope angle and the highest damage levels were given by "underlayer visible", S_d = 8; 12; or 17, again depending on the slope angle. So, very small damage was not considered, nor damage that was much larger than the maximum allowable damage, i.e., damage far beyond "underlayer visible". In fact, damage S_d < 1 and damage that was 50% or more greater than the maximum allowable damage (underlayer visible) was not considered. Test results with damage values within this range are given in Figures 5-7, together with the Van der Meer formula, Eqs. 4 and 5.

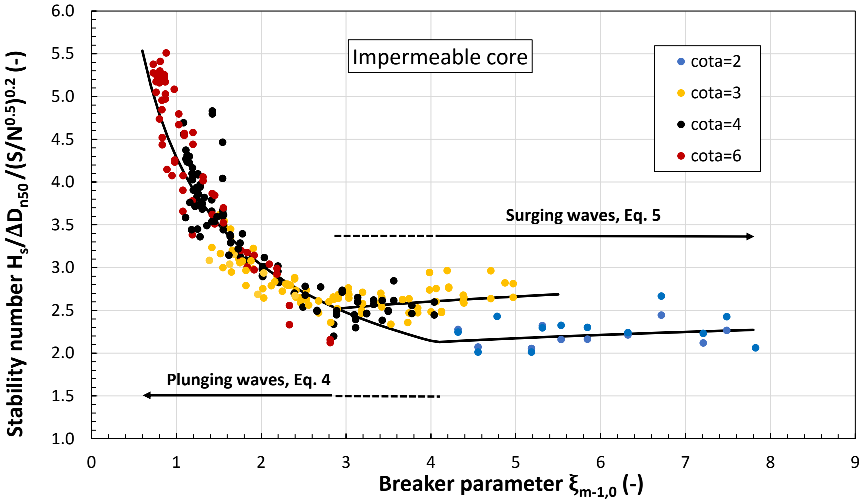

Figure 5: The $\xi_{m-1,0}$ - $H_s/\Delta D_{n50}$ / $(S_d/N^{0.5})^{0.2}$ graph for an impermeable core, P = 0.1, with measured test results of Van der Meer (1988b). Fixed damage levels of Thompson and Shuttler (1975) have been used, assuming $T_{m-1,0} = 1.1T_m$.

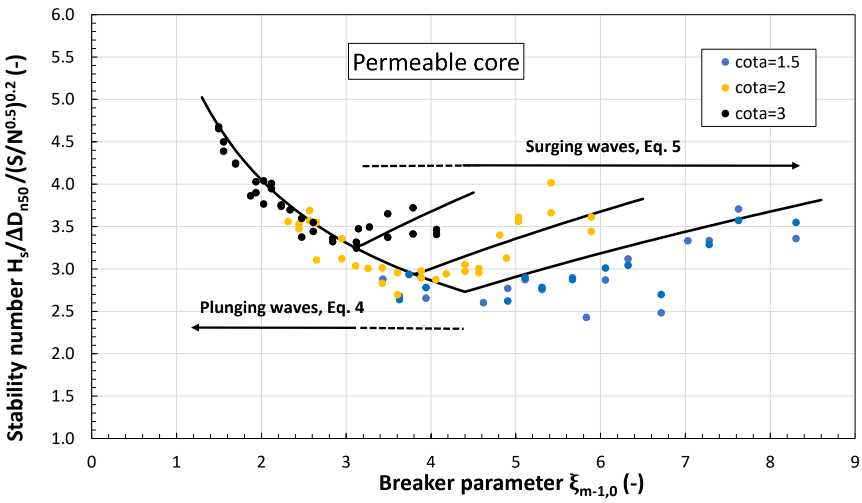

Figure 6: The $\xi_{m-1,0}$ - $H_s/\Delta D_{n50}$ / $(S_d/N^{0.5})^{0.2}$ graph for a permeable core, P = 0.5, with measured test results.

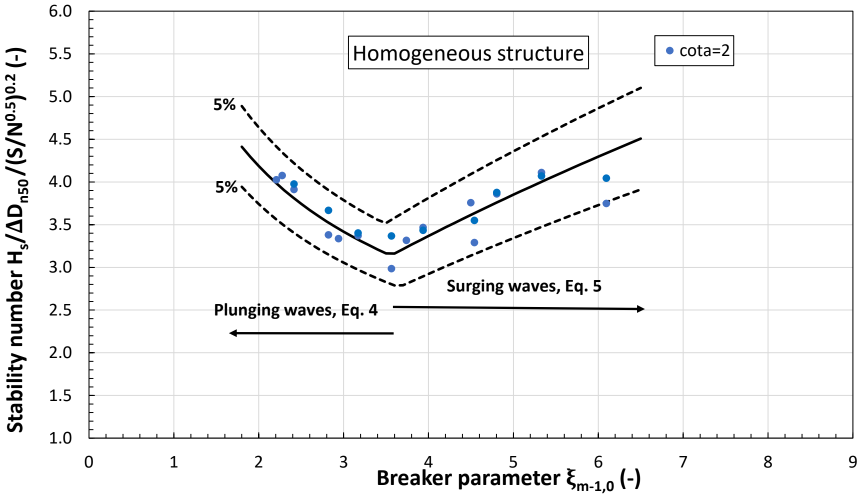

Figure 7: The $\xi_{m-1,0}$ - $H_s/\Delta D_{n50}$ / $(S_d/N^{0.5})^{0.2}$ graph for a homogeneous structure, P = 0.6, with measured test results and as an example the 5%-exceedance lines, giving the 90%-confidence band.

It is convenient and pleasing to have one formula, with a part for plunging and another part for surging waves, that describes the stability of rock armoured structures for wide ranges of slope angles, permeabilities and wave conditions. A drawback could be that deviations may occur in specific areas of the ranges tested. In Figure 5 (impermeable core) and Figure 7 (homogeneous structure) the curves match closely with the trend of the data. But in Figure 6 (permeable core) the descending curve on the left side of the graph for plunging waves is a little on the high side with respect to the data and this is certainly the case for surging waves and a slope of 1:1.5 (the blue dots, compared to the curve). The curves for surging waves and gentler slope angles with cotα = 2 and 3, however, are in good agreement with the data. For a permeable structure with a steep slope of 1:1.5 a larger factor of safety is advised for design than for other geometries, or as an alternative, a smaller P-value can be used. Eldrup and Andersen (2019) suggest a value of P = 0.44 which gives a better fit for this slope angle. However, applying this value to slopes of 1:2 and 1:3 would give curves in the surging wave area outside the data. Thus, P = 0.44 should only be used for a slope of 1:1.5 and *not* for gentler slopes.

The reliability of a formula is important for design and Figures 5-7 clearly show that a certain scatter in results is present. This scatter is due to differences in random behaviour of rock slopes, the accuracy of measurement, and curve fitting. The reliability of the original Van der Meer formula was described by assuming the that the coefficients 6.2 and 1.0 in Eqs. 1 and 2 respectively were stochastic variables with a normal distribution described by standard deviations of σ = 0.4 and 0.08 respectively. This in turn gives coefficients of variation coefficients of 6.5% and 8.0% respectively. This can be applied directly to the formula rewritten with the spectral period, Eqs. 4 and 5, giving coefficients with mean values of 6.49 and 0.97 and associated standard deviations of σ = 0.42 and 0.078 respectively. The 5%-lines have been given as an example in Figure 7, showing the 90%-confidence band between these lines.

3.3 Artificial neural networks and machine learning techniques

Machine learning techniques may be applied to arrive at prediction methods that are more precise than a formula from curve fitting. In principle one should apply such techniques only if the database is large and the process complicated (meaning a lot of parameters are needed to describe the process). Good examples are the CLASH neural network (Van Gent *et al.*, 2007) and the EurOtop neural network (Zanuttigh *et al.*, 2016) to predict the wave overtopping discharge for a very wide range of structures. The databases contained around 10,000 and 13,000 tests respectively. For rock stability, the databases of Thompson and Shuttler (1975) and Van der Meer (1988b) taken together are large enough for the application of an artificial neural network.

An important item in developing machine learning techniques is knowledge about the database and the parameters that should be important to describe the process. The homogeneity of the data in the database is also a very important concern. Mase *et al.* (1995) and Kim and Park (2005) developed neural networks with the data of Thompson and Shuttler (1975) and Van der Meer (1988b) – the same data that was used to develop the Van der Meer formula, Eqs. 1 and 2. The authors of both papers used the full dataset as given in Van der Meer (1988b, Appendix I). Kim and Park (2005) mention that they used 641 data, which is indeed almost the whole dataset. Thus, this includes: the shallow water tests; tests with different mass density of the rock; large scale tests, and tests on low-crested structures with a part of the tests with the crest level well below the still water level. The full dataset also includes those tests with narrow and wide spectra which showed lower stability than expected, caused by the rounding of the stones after many tests. Thus, the full database is not homogeneous.

In Figures 5-7 limitations were set to the measured damage S_d. Very low damage S_d <1 was not considered because measurements were considered to be quite unreliable, and complete failure of the armour layer with more than 1.5 times the damage when the underlayer became visible, is fully out of the range of application and is therefore also not considered. These exceptions were not made when developing the neural network. Mase *et al.* (1995) and Kim and Park (2005) included both a parameter for spectral shape, but as given in section 2 of this paper, it should have been a parameter for rock shape. Both papers use the stability number $H_s/\Delta D_{n50}$ as an output, where actually the wave height and geometry of the structure are inputs. It is the damage S_d to a specific structure under specific wave conditions that should have been the output.

Mase *et al.* (1995) used "seven parameters of stability number, damage level, number of attacking waves, surf-similarity parameter, permeability parameter, the dimensionless water depth in front of the structure and the spectral shape." The overall conclusion was: "The agreement between the predicted stability numbers by the neural network and the measured ones is also good, but not better than the stability formula itself". Actually, this means that most of the scatter around the Van der Meer formula is caused by the difference in behaviour of rock under wave attack, and this type of scatter cannot be reduced significantly by a neural network.

Kim and Park (2005) conclude that "the degree of agreement between the measured stability numbers and the predicted ones is not so good." That may be indeed the case, because they applied the Van der Meer formula wrongly for the narrow and wide spectrum, they did not include a crest level influence for low-crested structures, and they included very low and very large (out of range) damage. They proceeded to include a spectral shape parameter with the result that the neural network became a little better than the formula. In both papers some knowledge about the database was missing, with the consequence that a non-homogeneous database was used for the application of the formula as well as for the training of the neural network. It does not mean that developing a machine learning technique on the existing stability data is not useful, but it should be done on a proper selection of the database and using the correct inputs and output.

4 Modifying or extending the formula by new research

Research on specific aspects that were not investigated by Van der Meer (1988b) may directly be compared with one of the graphs in Figures 5-7. Or, if a notional permeability factor has been investigated that is different from P = 0.1, 0.5 or 0.6, a curve may be added for that particular factor using Eqs. 4 and 5. Various investigations have looked at aspects not considered in the Van der Meer formula and sometimes showed deviations from this formula, and various methods have been used to modify the formula. Although P = 0.1 is considered as a lower boundary, Latham *et al.* (1988) describe tests where the armour layer was only 1.6D_{n50} thick and not 2-2.2D_{n50} as in the original research. Certainly, for an impermeable core the wave energy has to be dissipated in the armour layer and a thinner armour layer may give larger wave forces as well as less stability by less interlocking of stones. Latham *et al.* (1988) indeed showed that all tests with different rock shapes showed less stability than the Van der Meer formula. One could then assume that P = 0.1 is not the lower bound, but that a value of P = 0.07 or 0.05 would be better. Figure 8 shows the results if the low-boundary P-value is changed to 0.07 or 0.05. For the plunging wave region, the stability indeed decreases, but there is hardly any effect in the surging wave region in which the curves are almost on top of each other. The only difference is that the critical breaker parameter shifts to the left. If test results show that stability is reduced in the whole area of plunging and surging waves, modifying the P-value does not lead to a good result. It can be concluded that decreasing the notional permeability factor P below 0.1 is probably not a good idea and that P = 0.1 can be considered as a real lower boundary and another way to explain the new data and incorporate it into the method should be found.

One alternative could be to increase or decrease the coefficients 6.2 and 1.0 in Eqs. 1 and 2, or coefficients 6.49 and 0.97 in Eqs. 4 and 5 (in case of using the spectral period $T_{m-1,0}$). This could be with a constant percentage, leading to a constant change over the full range of the breaker parameter. Some results, however, show that the influence of a specific parameter is different in the plunging and surging regimes. For example, Latham *et al.* (1988) showed that the deviation from the curve in the surging waves regime was larger than in the plunging one.

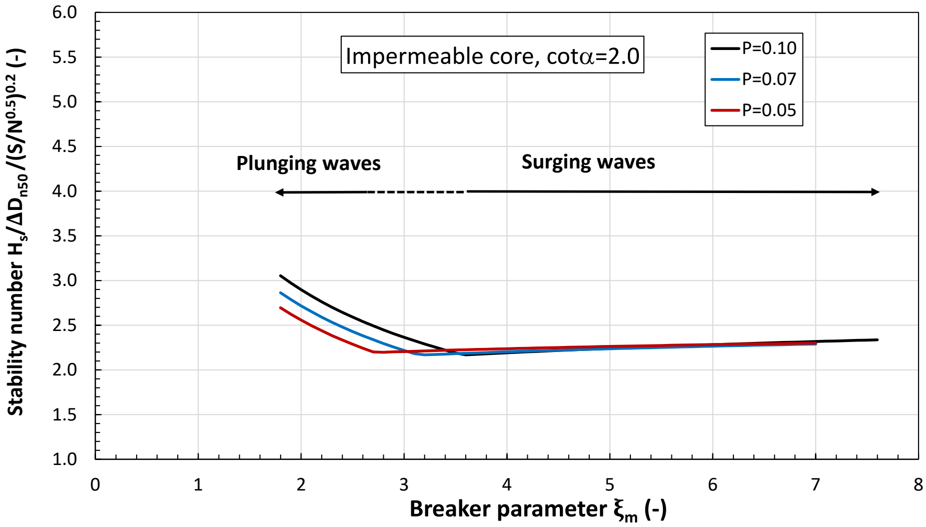

Figure 8: Modifying the Van der Meer formula with lower *P*-values than the boundary value $P = 0.1$.

Coefficients c_{pl} and c_s or c_{su} are sometimes used to *replace* the coefficient 6.2 in the plunging wave formula (Eq. 1) and 1.0 in the surging wave formula (Eq. 2), respectively – see for example Latham *et al.* (1988) and Stewart *et al.* (2002). Confusingly, coefficients are also used as a *multiplication factor* on the original values (6.2 and 1.0) and then the values are always around 1. An example is Table 5.30 in the Rock Manual (2007), whose values are actually based on Latham *et al.* (1988). There are advantages in using a multiplication factor instead of a replacement:

- the multiplication factor shows immediately if the stability increases (c_{pl} and $c_{su} > 1$) and by what percentage (directly with the nominal diameter), or decreases (c_{pl} and $c_{su} < 1$);
- by comparing c_{pl} with c_{su} it also becomes clear what the difference in influence is in the plunging and the surging wave regime; a larger deviation from 1 gives a larger influence;
- the stability formula itself has the same main coefficients (as 6.2 and 1.0 in Eqs. 1 and 2, respectively);
- the same multiplication factors can be used for a slightly modified formula, as when using the spectral period in Eqs. 4 and 5, and as when using $H_{2\%}$ instead of H_s to incorporate the influence of a non-Rayleigh distribution of the waves.

On this basis, c_{pl} and c_{su} are chosen as multiplication factors, giving the following formulae:

Modifying the original Van der Meer formula

$$\frac{H_s}{\Delta D_{n50}} = \begin{cases} 6.2 c_{pl} P^{0.18} \left(\frac{S_d}{N^{0.5}}\right)^{0.2} \xi_m^{-0.5} & \text{for plunging waves} \quad (9) \\ 1.0 c_{su} P^{-0.13} \left(\frac{S_d}{N^{0.5}}\right)^{0.2} \cot\alpha^{0.5}\, \xi_m^{P} & \text{for surging waves} \quad (10) \end{cases}$$

Modifying the original $H_{2\%}$ version of the Van der Meer formula

$$\frac{H_{2\%}}{\Delta D_{n50}} = \begin{cases} 8.68 c_{pl} P^{0.18} \left(\frac{S_d}{N^{0.5}}\right)^{0.2} \xi_m^{-0.5} & \text{for plunging waves} \quad (11) \\ 1.4\, c_{su} P^{-0.13} \left(\frac{S_d}{N^{0.5}}\right)^{0.2} \cot\alpha^{0.5}\, \xi_m^{P} & \text{for surging waves} \quad (12) \end{cases}$$

Modifying the original spectral period $T_{m-1,0}$ version of the Van der Meer formula

$$\frac{H_s}{\Delta D_{n50}} = \begin{cases} 6.49 c_{pl} P^{0.18} \left(\frac{S_d}{N^{0.5}}\right)^{0.2} \xi_{m-1,0}^{-0.5} & \text{for plunging waves} \quad (13) \\ 0.97 c_{su} \, P^{-0.13} \left(\frac{S_d}{N^{0.5}}\right)^{0.2} \cot\alpha^{0.5} \, \xi_{m-1,0}^{P} & \text{for surging waves} \quad (14) \end{cases}$$

The c_{pl}-values are equal in Eqs. 9, 11 and 13 and the c_{su}-values are equal in Eqs. 10, 12 and 14. The critical breaker parameter ξ_{mc} may shift with the use of c_{pl} and c_{su} and also with the use of a different wave period. For Eqs. 9-12 the critical breaker parameter becomes:

$$\xi_{mc} = \left(\frac{6.2 c_{pl}}{1.0 c_s} P^{0.31} \sqrt{\tan\alpha}\right)^{\frac{1}{P+0.5}} \quad (15)$$

For Eqs. 13 and 14 this is:

$$\xi_{m-1,0c} = \left(\frac{6.49 c_{pl}}{0.97 c_s} P^{0.31} \sqrt{\tan\alpha}\right)^{\frac{1}{P+0.5}} \quad (16)$$

The final results of Latham *et al.* (1988) – given in Box 5.30 in the Rock Manual (2007) – are used here as an example. Box 5.30 gives c_{pl} = 0.95 for semi-round and very round stones with c_{su} = 1.0 for semi-round and c_{su} = 0.8 for very round stones. These stone shapes show stability less than or sometimes equal to "standard" stone shapes that can be described as equant and fresh. Tabular rock is shown to be a little more stable with c_{pl} = 1.1 and c_{su} = 1.3. Values are given with respect to Eqs. 9 and 10. Applying those values gives Figure 9, where the values used are given in the legend. With this method of different multiplication factors c_{pl} and c_{su}, it is very clear that the critical breaker parameter ξ_{mc} can be changed significantly – in the graph, this ranges between ξ_m = 2.7 and 4.8. Applying this method gives the largest freedom, but if the tests do not cover the whole range of applicability of the Van der Meer formula one should also check what happens for this wider range or, in the case of presenting graphs like Figures 5-7, making a clear distinction between the tested range and possible extrapolations into an applicable, but not tested, zone.

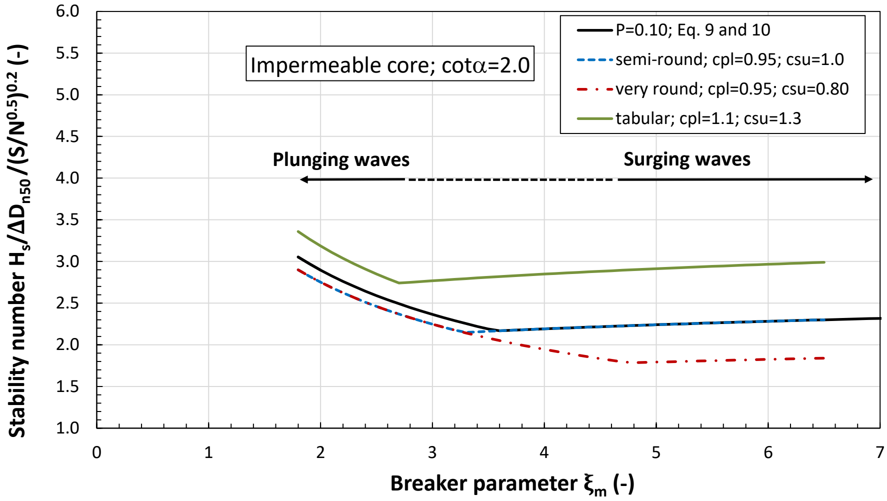

Figure 9: Modifying the Van der Meer formula independently for the plunging and surging waves regime, showing the influence of stone shape. Based on Latham et al. (1988) in Box 5.30 in the Rock Manual (2007).

Research into areas that are not covered by the Van der Meer formula may lead to modifications to that formula, but may also lead to a new formula if modification is not possible. Whether a modification is possible depends upon whether the structure of the Van der Meer formula is still valid. A first check is the validity of the shape of the damage curve (Eq. 6). That should be gradually increasing with a shape factor between about 4 and 6 (5 is used in Eq. 6). A second check is on the damage increase with increasing test duration, Eq. 7. It is therefore proposed that damage in a given test should always be measured at least for two durations (or number of waves). Only if Eq. 7 can be validated is a modification of the formula possible. If both Eqs. 6 and 7 have been validated, one can make a direct comparison with the Van der Meer formula in graphs like Figures 5-7. Modifications may then possibly be described by Eqs. 9-14, taking into account the limitations with respect to S_d.

The damage S_d is normally derived by profiling the rock slope and measuring the eroded area. This is only possible if a profiler (nowadays often a laser profiler) is available. If not, usually the number of displaced stones out of a coloured band is counted. This is very common for reporting damage to concrete armour layers, where a damage profile is not really of interest, see Van der Meer (1999) and the Rock Manual (2007, section 5.2.2.3). The damage number N_{od} is then "the number of displaced armour units within a strip of breakwater slope of width D_n, where the nominal diameter of the armour unit D_n is defined as the equivalent cube size of the unit concerned" (Rock Manual, 2007). One can follow the same procedure for rock armour, but using D_{n50} as representative width.

The problem now is how to relate a profiled damage using S_d to a counted damage of displaced rock N_{od}. For some of the original Van der Meer (1988b) tests, both methods were applied and the relationship is given in Figure 10 (retrieved from Delft Hydraulics-M1983 (1988)). Tests with slope angles of cotα = 2, 3 and 4 were available with a uniform grading D_{85}/D_{15} = 1.25, and riprap with a wide grading D_{85}/D_{15} = 2.25, where D_{85} and D_{15} are the 85% and 15% sieve sizes of the rock. The coloured bands were 4.5D_{n50} wide. There seems to be a distinct difference between a uniform and a wide grading, which may possibly be caused by the fact that stones much smaller than D_{n50} are present in the wide grading. For uniform and wide gradings the following relationships were found:

$$S_d = 1.32N_{od} + 0.6 \quad \text{(Uniform grading; } D_{85}/D_{15} = 1.25\text{)} \quad (17)$$

$$S_d = 1.32N_{od} - 0.6 \quad \text{(Wide grading; } D_{85}/D_{15} = 2.25\text{)} \quad (18)$$

The profiled damage for a uniform grading is always larger than the damage by counting because profiling also takes account of the porosity of the rock layer (left graph of Figure 10). Small damage cannot be determined from profiling as re-arrangement of stones and small settlement or compaction does not give a consistent erosion profile, nor a small number of displaced stones. In general, this is also the case for the wide grading in the right graph of Figure 10, but for small damage many more stones were counted than for a uniform grading. The reason might be that the wide grading has stones much smaller than the uniform grading. Every displaced stone is counted, but the volume or erosion surface is of course much smaller for these smaller stones than for a uniform grading, where every stone has more or less the size D_{n50}. For this reason, start of damage is S_d = 2 and not zero. S_d-values smaller than 2 and up to S_d = 1 may still be accurate, but damages smaller S_d < 1 are significantly less reliable. Although profiling a damaged rock slope would give the most accurate measure of the damage S_d, Eqs. 17 and 18 give a fair prediction if only the number of displaced stones is available.

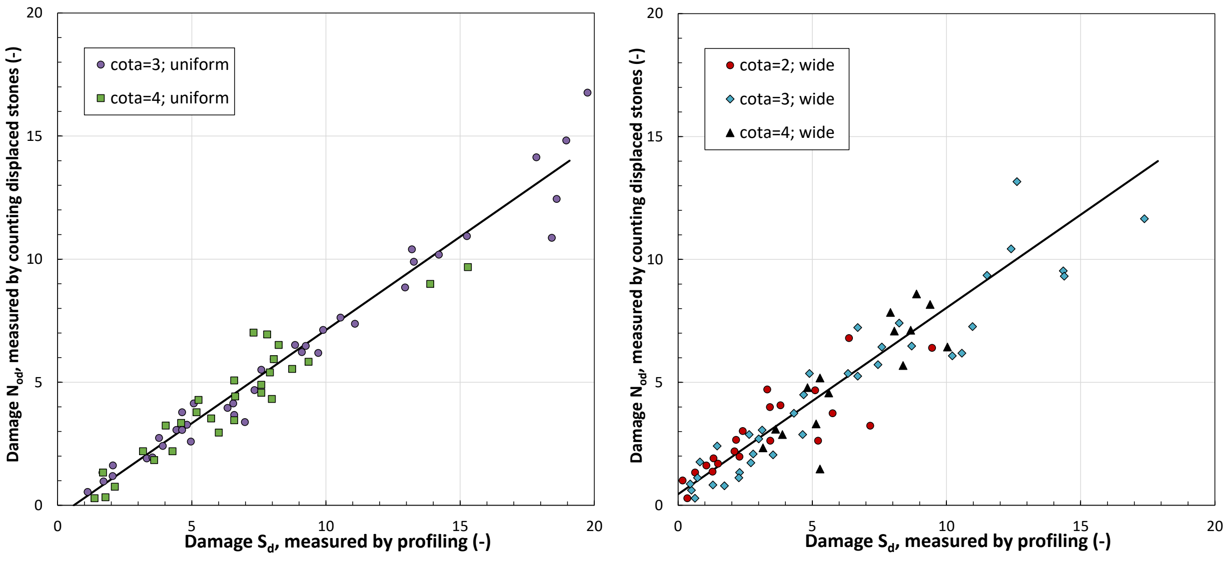

Figure 10: Relationship between the damage S_d from profiling the erosion area and counting the number of displaced stones out of a coloured band 4.5D_{n50} wide (from original data in Van der Meer, 1988b). Left graph: a uniform grading with D_{85}/D_{15} = 1.25; right graph: a wide grading (riprap) with D_{85}/D_{15} = 2.25.

5 Re-analysis on stone shape and packing

5.1 Introduction

Two substantial investigations on rock slope stability have been performed at HR Wallingford after the development of the Van der Meer formula. The first investigation has been described in two reference reports: Bradbury *et al.* (1988) with all measured data and a further analysis in Latham *et al.* (1988) on rock shape and modifying the *P*-parameter in the Van der Meer formula. Final results were summarised in the Rock Manual (2007) as Box 5.30. Rock slopes with an impermeable core and a slope with cotα = 2.0 were tested with different stone shapes: fresh, equant, semi-round, very round and tabular. Bradbury *et al* (1988) will be referenced here as SR 150, being HR Wallingford project number of the investigation.

The second investigation has been described in Stewart *et al.* (2002, 2003), being HR Wallingford investigation SR 621. The main objectives were twofold: to investigate the relationships between rock armour placing, void porosity and rock shape, and to investigate their influence on hydraulic stability. As in SR 150, a slope with cotα = 2.0 was tested. The work will be referenced as SR 621.

Both investigations showed problems in comparing the tests results with the Van der Meer formula. In SR 150 the average stability in the tests was lower than in the investigations of Thompson and Shuttler (1975) and Van der Meer (1988b), with distinct differences by stone shape. In SR 621 the scatter in results, when compared with the Van der Meer formula, was significantly larger than expected and this had influence on finding reliable conclusions for the investigation.

Both investigations will be reanalysed here, according to the method described in section 4. The Van der Meer (1988b) considerations on spectral shape will also be considered because the stone changed from semi-round/equant rock into very round/smooth rock due to the many tests that had been performed with the same rock grading. This resulted in a lower stability (see Section 2).

5.2 Re-analysis of SR 150 on stone shape

Bradbury *et al*. (1988) and Latham *et al.* (1988), or SR 150, give a final proposal for application in the Rock Manual (2007), Table 5.30. For semi-round, very round and tabular rock specific values are given for modifying the coefficients in the Van der Meer formula, see also Figure 9. The fresh and equant rock were supposed to be similar to the tests for the Van der Meer formula.

The results of the testing, however, showed that all structures were significantly less stable than expected. The main reason noted was the difference in armour layer thickness. Permeability is important and, in this case, with an impermeable core, the lack of permeability is a significant issue. The tested armour layer had a thickness of 1.6D_{n50}, where Thompson and Shuttler (1975) and Van der Meer (1988) had a layer thickness of 2.2D_{n50}, or 2.0D_{50}, where D_{50} is the sieve size. The analysis did not come to quantitative conclusions on the influence of the layer thickness, but by assuming that fresh and equant rock would be similar to the Thompson and Shuttler and Van der Meer tests, modifications were proposed for semi-round and very round rock (less stable) and tabular rock (more stable), see Table 5.30 in the Rock Manual (2007).

In case of a permeable core, the armour layer thickness may not have a large influence on stability because there is still quite some permeability underneath the armour layer to dissipate wave energy. But for an impermeable core, there is no significant permeability beneath the armour layer and all wave energy should be dissipated in the armour layer. This may have a significant effect on stability.

The first check is on the validity of Eq. 6 – a gradually increasing damage curve. SR 150 reports that in average the shape of the damage curves could be described by an exponent of 4.0 instead of the 5.0 in Eq. 6. This is a small and acceptable deviation – see also Section 3.1 where values between 4 and 6 are not considered as a deviation. The second check is on the damage development with respect to the test duration, Eq. 7. The damage was measured after 1000 and 3000 waves and the average increase in damage was reported as a factor 1.77 which is very close to the expected value of $(3000/1000)^{0.5}$ = 1.73.

On the basis of the above validations, a direct comparison can be made with the Van der Meer formula through a graph like Figure 5 and concentrating on an impermeable slope with cotα = 2. Figure 11 shows this graph for fresh, equant and semi-round rock that seem to have more or less similar stability. The data were retrieved from Table 4 of SR 150. The data in the graph are given with solid symbols if $1 \leq S_d \leq 12$ and with open symbols if the data are outside this range. There is quite some scatter, but also a clear trend can be observed. The results for breaker parameters $\xi_m < \approx 2.5$ are in line with the formula. For $\xi_m > \approx 2.5$ the average result is clearly lower than the formula. This is not due to the difference in rock shape, as the average line of all tests was taken, so can only be due to the reduced armour layer thickness of $1.6D_{n50}$. As long as there are really plunging waves with limited run-up and run-down, the layer thickness has no influence ($\xi_m < \approx 2.5$), but as soon as the waves become more surging the larger run-up and run-down forces destabilise the armour rock earlier than with a thicker armour layer.

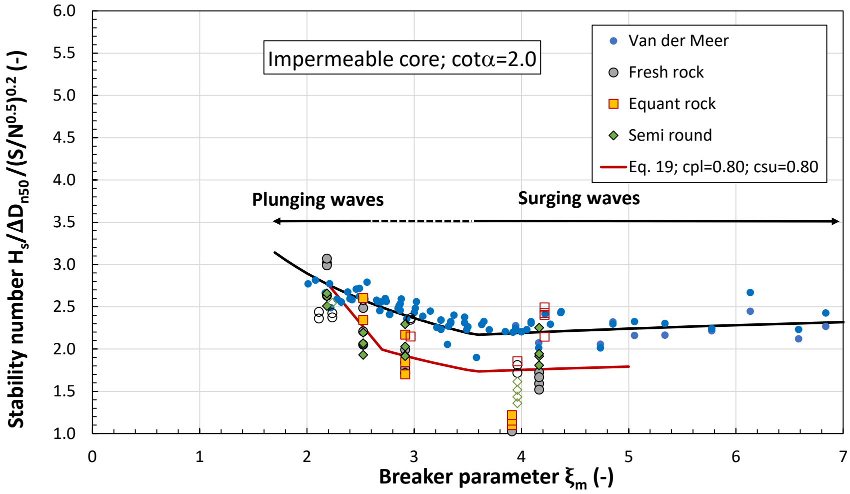

Figure 11: Test results from SR 150 for fresh, equant and semi-round rock, showing similar stability. The closed symbols show $1 < S_d < 12$, the open symbols show $S_d < 1$ or $S_d > 12$.

Modification of the Van der Meer formula can be done with Eqs. 9 and 10 and fitting the multiplication factors c_{pl} and c_{su}. The overall trend, due to *a thinner layer thickness of* $1.6D_{n50}$ and cotα = 2 with an impermeable core, can be written as follows (Eq. 19):

- No influence if $\xi_m \leq 2.2$; $c_{pl} = c_{su} = 1.0$
- $c_{pl} = 0.80$ and $c_{su} = 0.80$ if $\xi_m \geq 2.7$ (19)
- A linear interpolation between $2.2 < \xi_m < 2.7$ to avoid a sharp discontinuity

As described in Section 2, Van der Meer (1988b) also performed tests where rock shape had an influence on the stability of the armour layer. When testing a 1:3 rock slope with impermeable core with different spectral shapes, it appeared that stabilities against waves of a very narrow spectrum and of a very wide spectrum were significantly lower than against waves from a Pierson Moskowitz spectrum which was used for all other tests. It was discovered that due to the large amount of testing with the same armour stones (more than 150 tests with reconstruction of the slope after each test), the stone shape had become more rounded and smoother/worn out.

Because this is a data set with rounded/smooth/worn out armour stone and also with another slope angle, it is interesting to analyse the results in more depth and compare them with the results of SR 150. In Figure 12 the results with the narrow and wide spectrum (Van der Meer tests 158-197) are shown, together with the results of a 1:2 and 1:3 slope and the stability formula (Eqs. 1 and 2). For $\xi_m < \approx 2.5$ again there is no influence and the results for the narrow and wide spectra data points are situated close to the curve, not showing an influence of stone shape. For $\xi_m > \approx 2.5$ the stability drastically decreases and the results for the slope with cotα = 3 becomes – strikingly – even lower than for a slope with cotα = 2 in the surging waves region. The overall trend, due to *rounded/smooth stones* and cotα = 3 with an impermeable core can be written as follows, again using Eqs. 9 and 10:

- No influence if $\xi_m < 2.2$; $c_{pl} = c_{su} = 1.0$
- $c_{pl} = 0.90$ and $c_{su} = 0.75$ if $\xi_m \geq 2.7$ (20)
- A linear interpolation between $2.2 < \xi_m < 2.7$

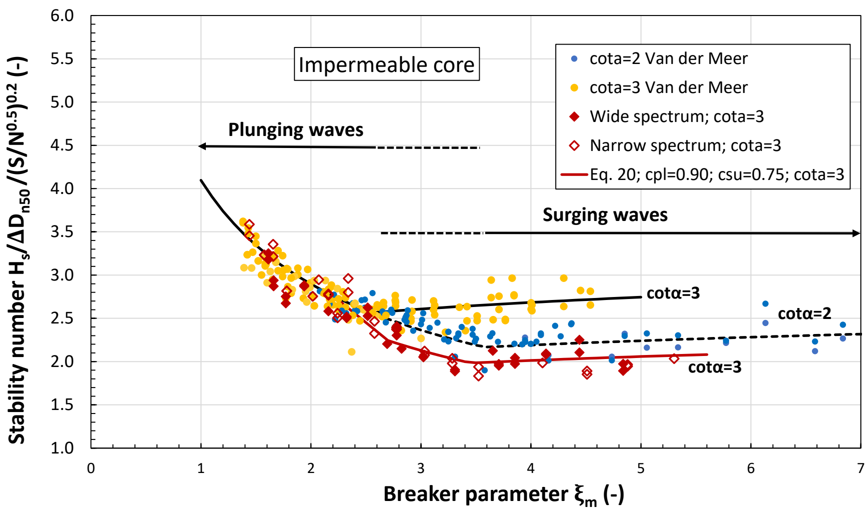

Figure 12: Tests of Van der Meer (1988b) with rounded/smooth armour rock (performed with a wide and narrow spectrum).

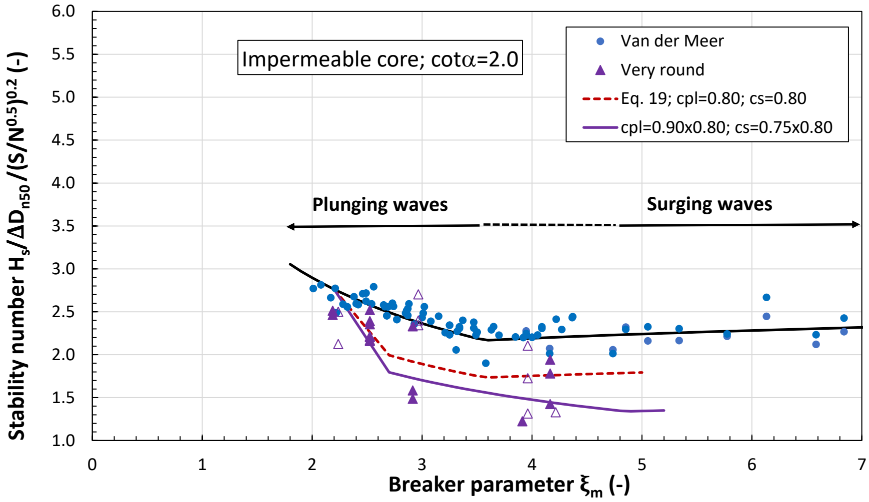

Figure 13: Results with very round rock in SR 150. The trends given show the influence of a thinner armour layer (red dashed line) and the influence of the thinner layer combined with a lower stability for very rounded rock (purple line).

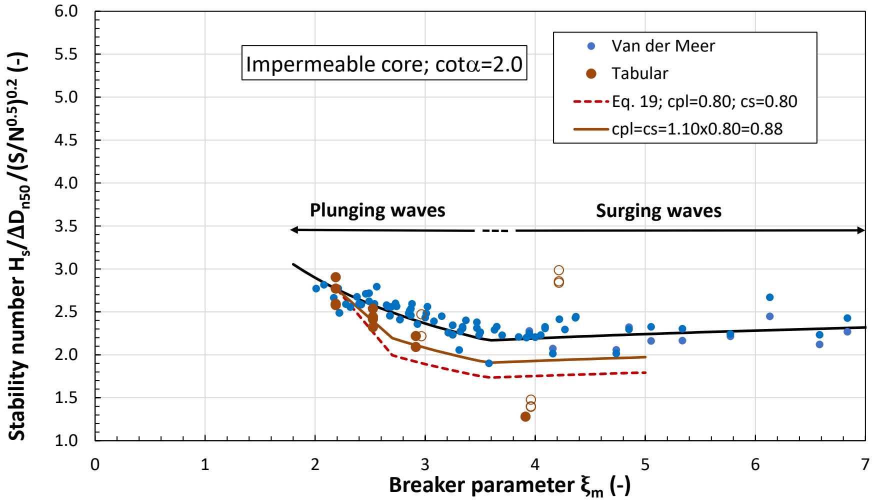

Figure 14: Results with tabular rock in SR 150. The trends given show the influence of a thinner armour layer (red dashed line) and the influence of a thinner layer combined with a better stability for tabular rock (brown line).

Results in Eq. 20 can be compared with the very round rock in SR 150, but now for a slope of cotα = 2.0. Figure 13 gives the results. The scatter is significant but within this scatter, the modifications above for a slope of cotα = 3.0 do also apply for a slope of cotα = 2.0. Note that the critical breaker parameter ξ_{mc} shifts significantly to the right compared to the tests with a slope of cotα = 3.0.

For the more stable tabular rock (Figure 14) the values of $c_{pl} = c_{su}$ = 1.10 have been chosen. The combined values for the influence of layer thickness as well as rock shape become $c_{pl} = c_{su}$ = 1.10x0.80 = 0.88.

In summary, based upon a comparison of the SR 150 data and the tests of Van der Meer with rounded/smooth/worn out rock, the following outcomes are derived:

- Conclusions for an *impermeable core* and slope angles with cotα = 2 and 3 are valid, but for a more permeable structure, conclusions on a thinner armour layer are not validated. The thinner armour layer findings should therefore only be considered valid for P = 0.1;
- Fresh, equant and semi-round rock do not show significant differences from the Van der Meer formula;
- For clearly plunging waves with ξ_m < 2.2 (or $\xi_{m-1,0}$ < 2.5) *rock shape has no influence* on stability and the Van der Meer formula can directly be applied. Additionally, a *thinner armour layer* has no influence on stability in the plunging waves region;
- For ξ_m > 2.7 (or $\xi_{m-1,0}$ > 3.0), the following multiplication factors can be applied:
    - For the influence of a *thinner armour layer* with a thickness of 1.6D_{n50}, c_{pl} = 0.80 and c_{su} = 0.80. Eq. 19;
    - For the influence of *very rounded rock* c_{pl} = 0.90 and c_{su} = 0.75. Eq. 20;
    - For the influence of *tabular* rock $c_{pl} = c_{su}$ = 1.1;
- A linear interpolation may be taken between 2.2 < ξ_m < 2.7;
- In case of two influences, the respective values of c_{pl} and c_{su} must be multiplied.

5.3 Re-analysis of SR 621 on rock placing and packing

Another very extensive investigation has been described in Stewart *et al.* (2002, 2003), being HR Wallingford investigation SR 621. The main objective was to investigate the influence of the geometry (rock placing, void porosity and rock shape) at "real" structures and at model scale and the hydraulic performance of such armour layers formed by individually placed stones. This study also used a slope with cotα = 2.0. The placement methods were described by SR 621 as follows:

"In the standard placement approach the rocks were placed with a minimum of orientation control. Each rock was placed with the orientation that it had naturally adopted in the stockpile. The only placement criterion that had to be satisfied was that the rock should have a minimum of three points of contact in the layer in which it was placed. Only if three points of contact could not be achieved with the rock's natural orientation, was the rock rotated. In the dense packing approach, greater orientation control was applied. The rocks were rotated until the orientation that was likely to produce the maximum number of points of contact in the layer (and the minimum volume of voids) was achieved. Individual rocks were removed and replaced several times if necessary."

It should be noted that the standard placement is a placement with care (three points of contact) and as individual stones. This is different from the bulk random placement as applied by Thompson and Shuttler (1975) and Van der Meer (1988b). The method of placement was applied to the whole armour layer, from toe to crest. In reality this will be more difficult as specific placement of rock under water is not easy, if not impossible (for dense packing). The specific placement can be started above low tide and if design conditions include surge, such a condition may then indeed find an armour layer that has also been constructed according to specifications below the design or surge level.

In total, 12 structures with a double layer of armour rock were tested comprising 10 structures with a permeable core with P = 0.5 (135 tests) and 2 structures with an impermeable core with P =0.1 (29 tests). Damage was not measured by profiling, but by counting displaced armour stones from a video. By including the measured porosity of the armour layer, the counted stones were converted to a damage value S_d, which is similar to that in Eqs. 17 and 18.

The main conclusion of the investigation is given by SR 621 as follows:

"It was shown that the stability of individually placed layers is, in most cases, at least as good as that of bulk-placed layers. When particular care is taken to pack the individual rocks tightly the stability of the layer can far exceed the stability of a bulk placed layer, in some cases withstanding twice the wave height for a given level of damage. The performance of such layers is, however, very sensitive to the degree of workmanship involved in their construction. For this reason, the findings of this phase of the study should be applied with caution."

The study also concluded that there was significant scatter – much larger than in Van der Meer (1988b). This scatter did influence the analysis as well as the final conclusions of the research. For re-analysis of the results in this paper, the stability data were taken from Figures 5.4–5.34 in SR 621. The first check is on the relationship between wave height and damage, Eq. 6, where a gradual increase in damage is assumed. For this reason, *damage graphs* have been composed, where the measured damage is directly related to the significant wave height that caused this damage. Figures 15 and 16 show these graphs for a permeable and impermeable core, respectively. The data points are divided into the four wave periods that were tested and the left graph gives the results of a standard placement and the right graph the dense packing. In order to check a gradual increase of damage Eqs. 1 and 2 have been used to calculate the damage curves for the four wave periods used. The same colour of data points and curve belong to the same wave period.

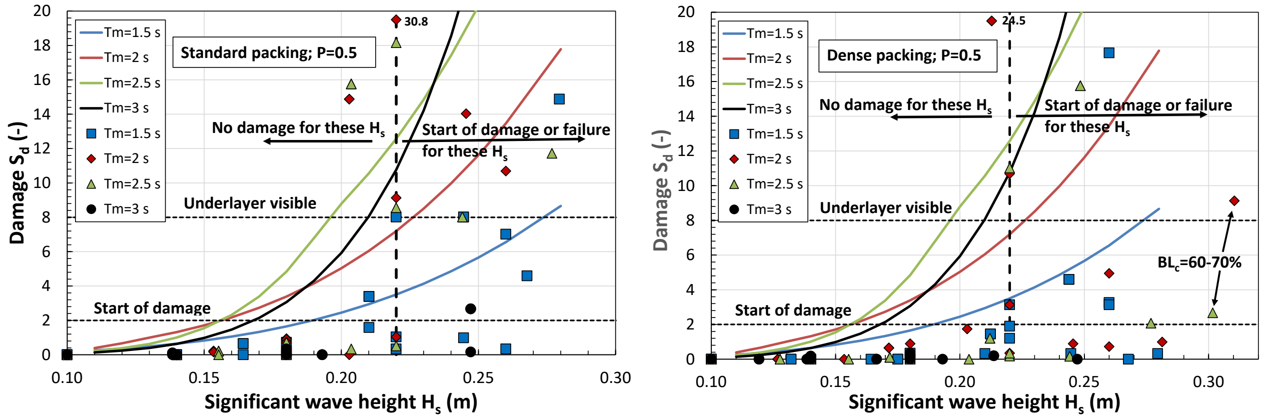

Figure 15: Damage graphs on stability for SR 621 for a permeable core with, left, the standard placement and right, the dense packing. The curves show the Van der Meer formula. The results show a brittle failure.

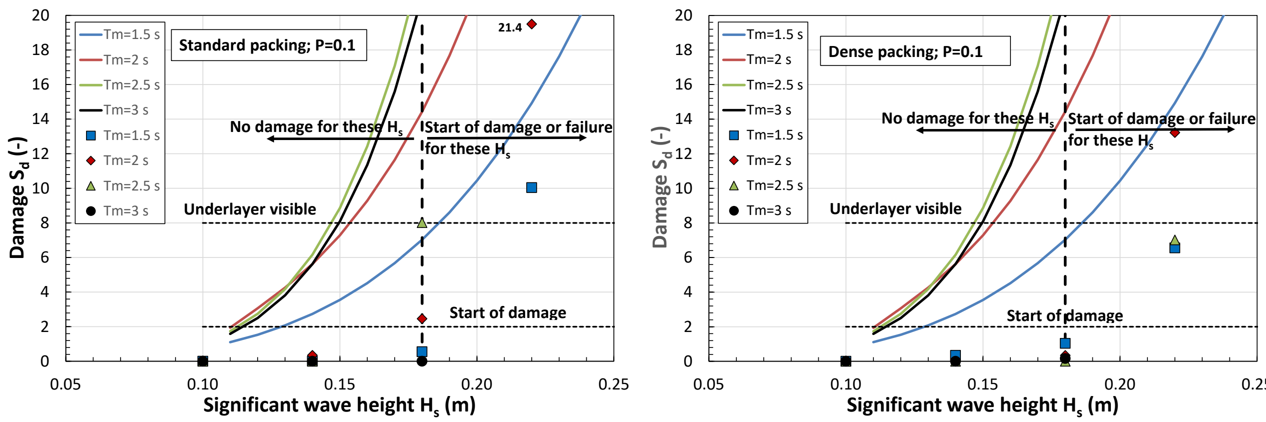

Figure 16: Damage graphs on stability for SR 621 for an impermeable core, with left the standard placement and right the dense packing. The curves show the Van der Meer formula.

Two conclusions are clear from Figures 15 and 16: the vast majority of tests gave hardly any damage even up to large wave heights. And secondly, the data do not show a gradually increasing damage development – here, larger wave heights may show smaller damage than smaller wave heights. Compare for example the damages for a wave period of 2 s in Figure 15 and for wave heights larger than 0.2 m. The graphs also show a dashed line indicating the thresholds for start of damage S_d = 2 and *underlayer visible* S_d = 8. Both figures show in total 164 test results; 90 test data showed S_d = 0 and

128 tests showed $S_d < 2$. Large damage, $S_d > 8$ was observed for 17 tests, which means that only 19 tests (12% of tests) fall within the "design range" for this slope, $2 \leq S_d \leq 8$.

Zero or small damage was observed up to very large wave heights, where the Van der Meer formula predicts failure. But if damage was able to develop, it was often very large damage, far beyond "filter layer visible". The scatter is also striking, for example for the permeable structure with standard packing (left graph of Figure 15), for a wave height of H_s = 0.22 m and a wave period of T_m = 2 s, damage ranging from S_d = 1 up to S_d = 30.8 is seen. Another example is found in the right graph of Figure 15: S_d = 1 is found for a wave period of 2 s and wave height of 0.28 m, alongside a damage of S_d = 24.5 for a much smaller wave height of 0.22 m. Finally, for permeable structures, start of damage S_d = 2 is never reached for wave heights up to 0.20 m. For impermeable structures this is 0.18 m.

This all shows that the standard placement of individual stones as well as a dense packing approach results in a so-called "brittle failure" in which no damage is seen to quite a large wave height, moving quite suddenly to failure under a wave height only a little larger. This is in complete contrast to the Van der Meer formula and so predicting this damage with that formula is simply not possible.

Brittle failure is often observed for one-layer systems with concrete armour units like Accropode™, Coreloc™, Xbloc™ and Cubipod™. These armour layers have units that interlock very well and due to this interlocking, the stability is very good, without damage up to a large wave height. But if damage develops for such a large wave height, the armour layer may also fail completely. A brittle failure has the advantage that the structure is stable for wave heights where other structures would already show significant damage or worse. The results in Figures 15 and 16 show this clearly. The disadvantage is that complete failure may occur without warning. This disadvantage, however, can be turned into a welcome advantage by applying a safety factor for design. A safety factor around 1.5 is often applied for the *design* wave height, where the basic wave height as basis for this calculation is taken as the value where damage would start. Single layer concrete units show start of damage roughly between stability numbers $H_s/\Delta D_n$ = 3.7-4.2. The design of the unit mass is then based on stability numbers $H_s/\Delta D_n$ = 2.5-2.8, showing a safety factor of 1.5. The advantage is then that for design conditions, no damage is expected and this can even be expected well into overload conditions.

If such a safety factor of 1.5 were to be taken into account for the standard and dense packing, would that mean that armour could be designed at a lower mass than with the Van der Meer formula? From Figures 15 and 16 a wave height can be determined where large damage/failure occurs and where zero or hardly any damage occurs for smaller wave heights. These wave heights can then be divided by the safety factor of 1.5 and compared with the design wave heights using the Van der Meer formula. If we assume S_d = 2-3 as design values for the design wave height and apply the Van der Meer formula, the design wave heights for a permeable core become H_s = 0.16-0.18 m, depending on the wave periods (in Figure 15, the intersection with the "start of damage line" with the damage curves). For an impermeable core this would be H_s = 0.11-0.13 m (Figure 16).

Figures 15 and 16 give all test results together. The actual SR 621 tests identified differences in stone shape using the *blockiness coefficient* BL_c % (= 100 × volume of stone / volume of bounding box). A larger blockiness coefficient indicates a more cubical or box-like stone shape. Gradings with blockiness coefficients of BL_c = 40-50%, 50-60% and 60-70% were tested. The results for the first two shapes were similar, but the most blocky shapes with BL_c = 60-70% were significantly more stable, especially for the dense packing cases. See for example Figure 15, where all damages for this grading were smaller than S_d = 2 for a wave height up to H_s = 0.30 m. The two data points with H_s > 0.30 m give larger damage and belong to the tests with BL_c = 60-70%. The individual graphs for each structure are not given here but were used in the following procedure to arrive at the wave height where failure starts for each tested structure.

Table 2: Application of brittle test results for design with a safety factor of 1.5.

Description	Design H_s (m); Eqs. 1 and 2	Failure H_s (m)	Failure H_s/1.5 (m)	Effect on design
Standard; $BL_c \leq$ 50-60%	0.16-0.18	0.205; Fig. 15	0.14	No
Standard; BL_c = 60-70%	0.16-0.18	0.245; Fig. 15	0.16	No
Dense; $BL_c \leq$ 50-60%	0.16-0.18	0.22; Fig. 15	0.15	No
Dense; BL_c = 60-70%	0.16-0.18	0.31; Fig. 15	0.20	Yes, +10% in D_{n50}
P=0.1; standard; BL_c = 50-60%	0.11-0.13	0.18; Fig. 16	0.12	No
P=0.1; dense; BL_c = 50-60%	0.11-0.13	0.22; Fig. 16	0.15	Yes, +10% in D_{n50}

Table 2 shows the outcome of the procedure. The first column gives the tested structure: a standard or dense packing; a permeable or impermeable core, and with or without very blocky stones (BL_c =60-70%). The second column gives the design wave heights as discussed above, assuming the Van der Meer formula and design values of S_d = 2-3. The third column gives the wave height that shows start of failure, derived from the graphs for individual structures. The fourth column shows the "brittle failure design wave height", dividing the failure wave height by the safety factor 1.5. The last column shows the effect on design, comparing the fourth column with the second one.

If the brittle design wave height is similar to, or smaller than the design wave height according to the stability formula, it is not possible, or not safe, to design for a smaller rock diameter or mass. That is the case for all standard packings. There are only two situations tested where this procedure would lead to a reduction in size of rock: a permeable core with a dense packing and blocky stones (BL_c = 60-70%), and an impermeable core with dense packing. The latter option is not advised however, because the increase in stability was not found for a permeable core and BL_c = 50-60%. This means actually that only a dense packing with blocky stones maybe designed with a 10% smaller diameter than according to the Van der Meer formula (or using $c_{pl} = c_{su}$ = 1.1).

Another way might be to follow the same procedure as for single armour layers, determining a stability number, regardless of the wave period and then applying a safety factor. Based on Figure 15 it can be assumed that start of damage (and failure) for a permeable core with P = 0.5 starts around H_s = 0.22 m. This is given as a vertical line in Figure 15. With the rock grading tested, this results in a stability number of $H_s/\Delta D_{n50}$ = 2.5. With a safety factor of 1.5 the design value becomes $H_s/\Delta D_{n50}$ = 1.7. For the impermeable core with P = 0.1 a threshold wave height can be determined from Figure 16 as H_s = 0.18 m. This leads to a stability number of $H_s/\Delta D_{n50}$ = 2.0 and a design value of $H_s/\Delta D_{n50}$ = 1.4. The expectation is then that no damage will occur under design conditions and even an overload condition should not show an increase of damage. This is of course a large advantage over randomly, bulk placed rock armour with gradually increasing damage with wave height, even if the size of the rock was similar. The safety factor of 1.5 is a safe value. It is left to the designer to determine whether a smaller safety factor would be appropriate, leading effectively to a smaller stone size.

The authors of SR 621 conclude that the findings of the study should be applied with caution. The present work finds that the c_{pl} and c_{su}-values that were proposed should not be used. Nevertheless, the conclusion to be drawn from SR 621 can be phrased in a much more positive way: a standard placement with care (three points of contact) or a dense packing will always perform better, or much better than randomly bulk placed rock armour, as long as the size of the grading is the same.

6 Conclusions

It is possible to rewrite the Van der Meer formula with the spectral wave period $T_{m-1,0}$ instead of the mean period T_m. It is confirmed that with the modified Eqs. 4 and 5, there continues to be no influence of spectral shape when using this $T_{m-1,0}$ wave period. The full valid application area of the Van der Meer formula is given in Figure 4.

Research outside the valid application area of the Van der Meer formula should first be checked against specific relationships that are included in the formula. These include a gradual development of damage according to a power curve with an exponent between 4 and 6 (Eq. 6), and an increase in damage with storm duration through a square root function (Eq. 7). If so validated, then a direct comparison can be made with the original data as well as the formula itself in graphs with damage as a function of the breaker parameter (Figures 5-7). The data and graphs are available on www.vdm-c.nl and DOI 10.5281/zenodo.5569052.

Modifying or extending the Van der Meer formula is best achieved by multiplication coefficients c_{pl} and c_{su} in the formula, see Eqs. 9-16. The deviation from 1.0 shows directly the influence on stability: a structure with a smaller value is less stable and with a larger value, more stable. The influence of the investigated aspect may differ in the plunging and surging regimes, resulting in different values for c_{pl} and c_{su} for each regime.

A method is given to calculate the damage S_d if the actual damage was measured by counting displaced stones from the armour layer (Eqs. 17 and 18).

Artificial neural networks and machine learning techniques may be useful, but only if a large and homogeneous database is used for training and with the correct input and output parameters, based on an understanding of the driving physical processes and engineering science.

Re-analysis of the SR 150 investigation (Latham *et al.*, 1988) showed that for assessment of stability, the influence of a thin 1.6D_{n50} armour layer could be separated from the influence of rock shape. For an impermeable core and a slope with cotα = 2, a thinner armour layer is less stable than a 2.0D_{n50} thick armour layer, but that there is no influence in the plunging wave regime for $\xi_m < 2.2$ (or $\xi_{m-1,0} < 2.5$). Very round rock is less stable and it appeared that tabular rock is slightly more stable than the standard shapes of "fresh, equant and semi-round rock". All influences are described by values of c_{pl} and c_{su}.

Specific standard placement of individual stones with three points of contact and dense placement with even more care very greatly influence the stability of a rock armour layer. Re-analysis of the SR 621 investigation (Stewart *et al.*, 2002, 2003) shows that the behaviour of such rock slopes can be described by "brittle failure". The structure is stable and shows hardly any damage up to a very large wave height, but if damage occurs for such a wave height, the full structure may fail. Such a behaviour does not underpin Eq. 6 or Eq. 7 and cannot directly be compared with the Van der Meer formula. By applying a safety factor of 1.5 on the start of failure wave height observed in the SR 621 tests, the required stone size for design will be more or less equal to applying the Van der Meer formula. The behaviour of the structure will however be much better, with zero or small damage expected under design conditions and even under overload conditions.

Stability of rock slopes at shallow or very shallow foreshores with heavy wave breaking has not been considered as it is seen as a field of research that has not yet fully been explored. In addition to validation of the fixed relationships in the Van der Meer formula (Eqs. 6 and 7), the change in wave conditions over the foreshore will be important, and a methodology to measure/define these in a consistent way is a further challenge.

Acknowledgements

The original work of Van der Meer (1988b) was funded by the Rijkswaterstaat, the Dutch Public Works Department. Thomas Lykke Andersen and Mads Røge Eldrup are acknowledged for their participation in preparing this paper. John-Paul Latham and Terry Stewart are acknowledged for the extensive description of their work which made it possible to perform a thorough re-analysis. They are also acknowledged for general review of the paper and in particular section 5.

Notation

Name	Symbol	Unit
Area of the erosion profile	A_e	m^2
Blockiness coefficient = 100% x rock volume/*XYZ*	BL_c	-
Coefficient in stability equation for plunging waves	c_{pl}	-
Coefficient in stability equation for surging waves	c_{su}	-
Sieve diameter, 15% of mass not exceeded	D_{15}	m
Sieve diameter, 50% of mass not exceeded	D_{50}	m
Sieve diameter, 85% of mass not exceeded	D_{85}	m
Nominal diameter of concrete units, $D_n = (M/\rho_r)^{1/3}$	D_n	m
Nominal diameter of the armour rock, $D_{n50} = (M_{50}/\rho_r)^{1/3}$	D_{n50}	m
Gravitational acceleration	g	m/s^2
Water depth	h	m

Two percent wave height in the time domain	$H_{2\%}$	m
Significant wave height in the time domain $H_{1/3}$	H_s	m
Significant wave height in deep water	$H_{s,\ deep}$	m
Significant wave height at the toe of the structure	$H_{s,\ toe}$	m
Mass of the concrete units	M	kg
Median mass of the armour rock grading (50%-value by mass)	M_{50}	kg
Number of waves (or storm duration)	N	-
Damage number by counting of displaced stones, related to a width of D_{n50}	N_{od}	-
Notional permeability factor	P	-
Notional wave steepness, $s_{om} = 2\pi H_s/(gT_m^2)$	s_{om}	-
Notional wave steepness, $s_{om-1,0} = 2\pi H_s/(gT_{m-1,0}^2)$	$s_{om-1,0}$	-
Damage level determined from the erosion profile, $S_d = A_e/D_{n50}^2$	S_d	-
Mean wave period measured in the time domain	T_m	s
Spectral wave period	$T_{m-1,0}$	s
Peak wave period	T_p	s
The dimensions of the smallest box that can enclose a rock	X, Y, Z	m
Slope angle	$\cot\alpha$	-
Relative mass density $\Delta = \rho_r/\rho_w - 1$	Δ	-
Surf similarity or breaker parameter, $\xi_m = \tan\alpha/(s_{om})^{0.5}$	ξ_m	-
Surf similarity or breaker parameter, $\xi_{m-1,0} = \tan\alpha/(s_{om-1,0})^{0.5}$	$\xi_{m-1,0}$	-
Critical surf similarity or breaker parameter, Eqs. 3 and 15	ξ_{mc}	-
Critical surf similarity or breaker parameter, Eq. 16	$\xi_{m-1,0c}$	-
Mass density of the armour rock	ρ_r	kg/m^3
Mass density of the water	ρ_w	kg/m^3
Standard deviation	σ	-

References

Bradbury, A.P., N.W.H. Allsop, J-P. Latham, M. Manion and A.B. Poole (1988). Rock armour for rubble mound breakwaters, sea walls and revetments: Recent progress. HR Wallingford Report SR 150.

CEM (2002). Coastal Engineering Manual (CEM), Engineer Manual 1110-2-1100. U.S. Army Corps of Engineers, Washington, D.C. (6 volumes).

Delft Hydraulics-M1983 (1988). Taluds van losgestorte materialen. (Slopes of loose materials – in Dutch). Statische stabiliteit van stortsteentaluds onder golfaanval. Ontwerpformules. Verslag modelonderzoek, deel I. (Static stability of rock slopes under wave attack. Report on model investigation, Part I).

Eldrup, M.R. (2019). Wave overtopping and stability performance of rubble mound breakwaters exposed to shallow water waves. PhD-thesis Aalborg University, Denmark.

Eldrup, M.R. and T.L. Andersen (2019). Extension of shallow water rock armour stability to nonlinear waves. Coastal Engineering 153 (2019) 103536. https://doi.org/10.1016/j.coastaleng.2019.103536.

Eldrup, M.R., T.L. Andersen and H.F. Burcharth (2019). Stability of rubble mound breakwaters – A study of the notional permeability factor, based on physical model tests. Water 2019, 11, 934; doi:10.3390/w11050934.

EurOtop (2007). Wave overtopping of sea defences and related structures: assessment manual. Pullen, T., N.W.H. Allsop, T. Bruce, A. Kortenhaus, H. Schüttrumpf and J..W. van der Meer, www.overtopping-manual.com.

EurOtop (2018). Manual on wave overtopping of sea defences and related structures. An overtopping manual largely based on European research, but for worldwide application. Van der Meer, J.W., Allsop, N.W.H., Bruce, T., De Rouck, J., Kortenhaus, A., Pullen, T., Schüttrumpf, H., Troch, P. and Zanuttigh, B., www.overtopping-manual.com.

Holterman, S. (1998). 'Golfoploop op dijken met ondiepe voorlanden (Wave run-up at dikes with shallow foreshores). MSc thesis, Delft University of Technology, Delft, The Netherlands (in Dutch).

Kim, D.H. and W.S. Park (2005). Neural network for design and reliability analysis of rubble mound breakwaters. In: Elsevier, Ocean Engineering 32 (2005) 1332–1349.

Latham, J-P., M. Manion, A.B. Poole, A.P. Bradbury and N.W.H. Allsop (1988). The influence of armourstone shape and rounding on the stability of breakwater armour layers. Coastal Engineering Research Group, Queen Mary & Westfield College, University of London, Research Report 1.

Mase, H., Sakamoto, M., Sakai, T. (1995). Neural network for stability analysis of rubble-mound breakwaters. In: Proc. ASCE, Journal of WPC and OE. 121 (6), 294–299.

Rock Manual (2007). The Rock Manual. The use of rock in hydraulic engineering. CUR, CIRIA, CETMEF.

Smith, G., I. Wallast and M.R.A. van Gent (2002). Rock slope stability with shallow foreshores. In: ASCE, Proc. ICCE 2002, pp. 1524-1536.

Stewart, T.P., S.D. Newberry, J.D. Simm and J-P Latham (2002). Hydraulic performance of tightly packed rock armour – results from random wave model tests of armour stability and overtopping. In: ASCE, Proc ICCE 2002, pp 1449-1461.

Stewart, T.P., S.D. Newberry, J-P Latham and J.D. Simm (2003). Packing and voids for rock armour in breakwaters. HR Wallingford Report SR 621.

Thompson, D.M. and R.M. Shuttler (1975). Riprap design for wind-wave attack. A laboratory study in random waves. HR Wallingford Report EX707.

Van der Meer, J.W. (1987). Stability of breakwater armour layers - Design formulae. In: Journal of Coastal Engineering, 11, pp. 219-239.

Van der Meer, J.W. (1988a). Deterministic and probabilistic design of breakwater armour layers. In: Proc. ASCE, Journal of WPC and OE, Vol. 114, No. 1.

Van der Meer, J.W. (1988b). Rock slopes and gravel beaches under wave attack. Doctoral thesis, Delft University of Technology. Also Delft Hydraulics Publication no. 396. www.vdm-c.nl.

Van der Meer, J.W. (1999). Design of concrete armour layers. ASCE, Proc. Coastal Structures '99, Santander, Spain. Losada (ed.), Balkema, Rotterdam, pp. 213-221.

Van Gent, M. and G. Smith (1999). Physical model investigations on coastal structures with shallow foreshores: 2D model tests with single and double peaked wave energy spectra. Delft Hydraulics Report H 3608.

Van Gent, M. R. (2001). Wave runup on dikes with shallow foreshores. In: Journal of Waterway, Port, Coastal, and Ocean Engineering, 127(5):254–262.

Van Gent, M. R., Smale, A. J., and Kuiper, C. (2003). Stability of rock slopes with shallow foreshores. In: ASCE, Proc. Coastal Structures 2003, pages 100–112.

Van Gent, M.R.A., H.F.P. van den Boogaard, B. Pozueta and J.R. Medina (2007). Neural network modelling of wave overtopping at coastal structures. In: Coastal Engineering 54, 586–593.

Zanuttigh, B., S. Formentin and J.W. van der Meer (2016). Prediction of extreme and tolerable wave overtopping discharges through an advanced neural network. In: Ocean Engineering 127 (2016) 7-22; http://dx.doi.org/10.1016/j.oceaneng.2016.09.032.