Issue 26(2), 2026
ISSN: 1567-7141
Special Issue: hEART Symposium 2024
DOI: 10.59490/ejtir.2026.26.2.7798

TU Delft OPEN Publishing
Research Article

Optimizing the freeway toll rates for freight transportation following truck ban policy on central business district

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Keywords
toll optimization
multiclass traffic assignment model
passenger vehicles and trucks
solution algorithm

Publishing history
Submitted: 31 August 2024
Revised dates: 25 January 2025, 8 June 2025
Accepted: 7 September 2025
Published: 4 May 2026

Cite as
Yang, B. (2026). Optimizing the freeway toll rates for freight transportation following truck ban policy on central business district. *European Journal of Transport and Infrastructure Research, 26*(2). https://doi.org/10.59490/ejtir.2026.26.2.7798

Abstract

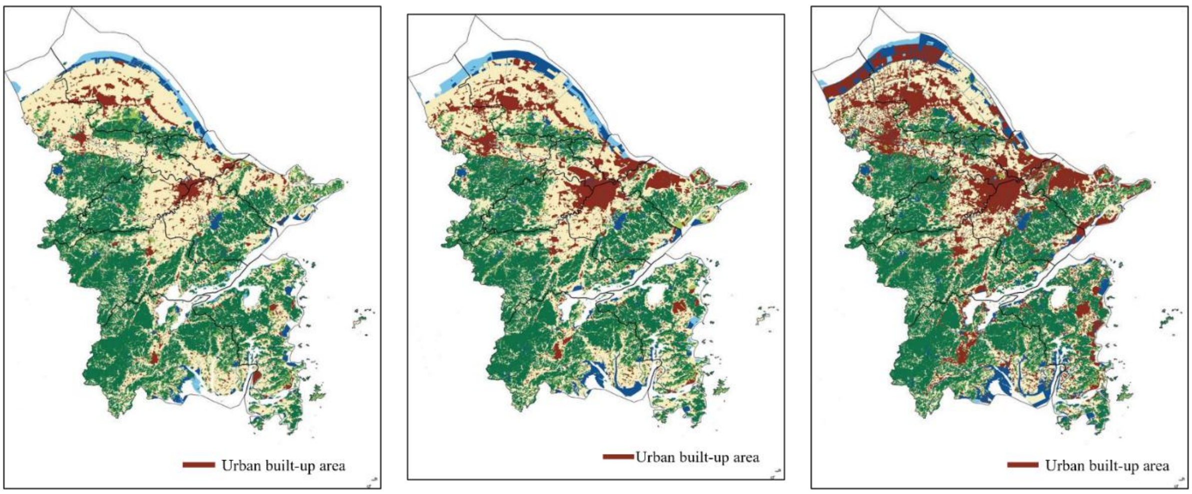
The truck ban policy on freeways in central business districts (CBD) is extensively used in China nowadays to improve traffic safety and reduce traffic congestion. However, this policy will drastically impact freight transportation, especially when the freeway truck volume is high. To mitigate the negative effects and encourage truck drivers to use alternative freeways, this study proposes a freeway tolling problem for different types of trucks to reduce travel costs following the truck ban policy in CBD. It is formulated as a bi-level optimization problem. The upper-level problem optimizes the freeway toll rates for different types of trucks to reduce the total travel cost (TTC) of the network. The lower-level problem is a multiclass traffic assignment model to characterize the equilibrium flow mixed with passenger vehicles and different types of trucks following the toll strategy. The bi-level problem is solved using a line search algorithm developed based on a feasible direction method. Application of the proposed method in Ningbo, China, finds that the proposed solution algorithm can efficiently solve the bi-level problem and converges only after 11 iterations. Compared to the initial state where the truck ban is not implemented, the optimal tolling strategy can effectively reduce the TTC of the network by 8.5%, with an increase of only 1.15% for all trucks. This indicates that the proposed method can effectively nudge truck drivers to use alternative routes with a minor rise in travel costs. Therefore, it can help traffic managers design better strategies to avoid the resistance of truck users following the truck ban policy in CBD.

1 Introduction

In the past 20 years, China has experienced a rapid urbanization process. The built-up area of most cities in China has grown very fast. Substantial suburban regions in the past have become urban areas with dense populations. As shown in Figure 1(a)-(c), the built-up area of Ningbo, China, in 2020 is over 20 times that of 2000, and the urban population increased from 1.42 million to 7.34 million. Due to rapid urbanization, the freeways that used to be located outside the central business districts (CBD) are now within the CBD. For example, Figure 2 shows the freeways of Ningbo in different years. The Hangyong freeway was built in the middle 1990s and used to be outside the CBD area (see Figure 2(a)). However, with the growth of the city, it is now in the middle of the CBD area (See Figure 2(b) and Figure 2(c)).

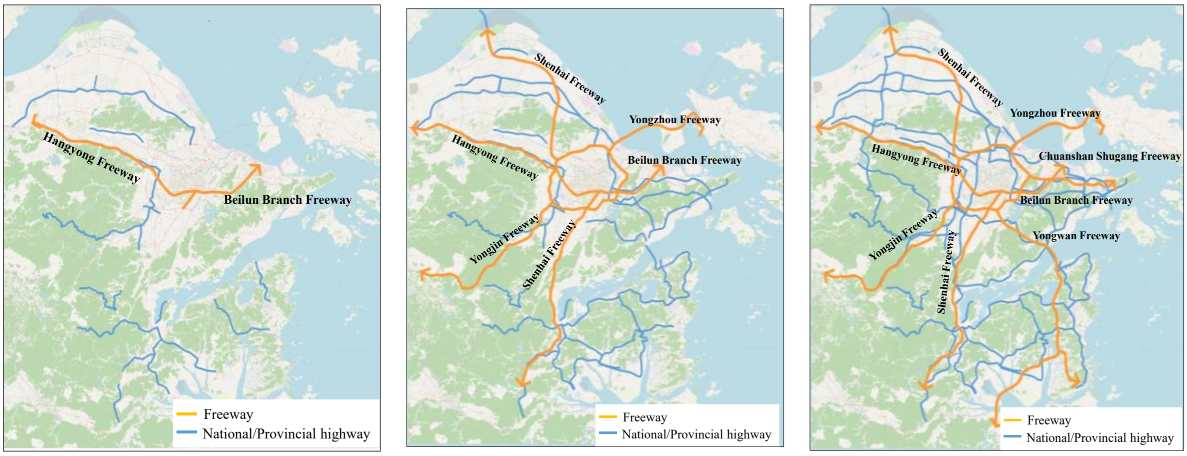
Note that the freeways contain lots of freight transportation, drastically affecting traffic safety because they can be used to ship dangerous goods, and trucks can generate large amounts of noise that significantly disturb the lives of the residents in the CBD area. Further, the mixed traffic flow with passenger vehicles and trucks will dramatically reduce travel efficiency and safety. To address this problem, many cities in China nowadays apply the truck policy in the CBD. That is, the trucks cannot access the freeways in the CBD area. To avoid the impacts of the truck ban policy on freight transportation, new freeway tolling strategies should be designed to encourage truck drivers to use other parallel freeways to arrive at their destinations. However, this is a non-trivial task as trucks and passenger vehicles interact, and the new freeway toll strategy significantly impacts both passenger and truck flows in the network. Thereby, it should be designed systematically to reduce the travel cost of the truck drivers while enhancing network performance.

In the literature, the tolling strategy is typically explored in the context of congestion pricing. The corresponding objective is to charge vehicles in certain areas to balance the traffic supply and demand to enhance network performance. The congestive pricing model can be divided into two categories, i.e., the first-best and the second-best toll pricing problem (Yang & Huang, 2005). For the first-best toll pricing problem, all users need to pay a toll to use any links in the network to reduce the TTC (Xu et al., 2013; Yang & Zhang, 2002), while the second-best toll pricing problem only charges users in a certain area to enhance the network performance determined by the traffic managers (Chiou, 2008). The second-best toll pricing problem is more practical (Han & Yang, 2008).



(a) 2000 *(b) 2010* *(c) 2020*

Figure 1. The built-up area of Ningbo, China in different years



(a) 2000 *(b) 2010* *(c) 2020*

Figure 2. The freeways and highways of Ningbo, China in different years

The congestion pricing problems are generally modelled as bi-level optimization programs, where the upper-level problem is formulated to enhance the designed system performance indicator, considering the physical constraints such as the capacity of links, ranges of the toll prices, etc., while the lower-level problem is the equilibrium network modelling problem incorporated to characterize the network equilibrium flow under the tolling strategy. The typical equilibrium model, including the user equilibrium (UE) model (Chiou, 2008), the system optimal model (Yang & Zhang, 2002), the logit-based stochastic user equilibrium (SUE) model (Yang, 1999), the multiclass traffic assignment (MTA) model, which considers, e.g., passenger vehicles and buses, etc.(Xu et al., 2013). Note that the bi-level optimization programs are generally non-convex and pose significant challenges in solution algorithm design. In the literature, these problems are often solved by heuristic algorithms, such as genetic algorithms (Jiang et al., 2011), simulated annealing (Liu & Ma, 2009), sensitivity analysis-based heuristic algorithms (Yang & Zhang, 2007), etc. However, these algorithms cannot guarantee convergence. Yang et al. (2003) proposed a solution algorithm based on marginal functions. Still, this algorithm can only guarantee local convergence, meaning it only converges when the starting point is close to the optimal solution. It is only suitable for solving certain types of congestion pricing problems.

The above-mentioned congestion pricing problems are insufficient to optimize the freeway toll rates for trucks following the truck ban in CBD. This is because, for port cities, substantial passenger demand and freight demand exist simultaneously. Their route choices and traffic impedances interact with each other. Further, freight transportation contains several types of trucks (e.g., light-duty trucks, heavy-duty trucks) whose travel efficiency and toll rates differ. The low-level traffic assignment models used in the above literature generally only consider homogeneous passenger transportation, the multimodal freight transportation is omitted. Therefore, they are not applicable to optimizing the freeway toll rates for different types of trucks to reduce the negative impacts of the truck ban policy on CBD. Further, the existing solution algorithms are mostly heuristic and converge very slowly or cannot guarantee convergence. A new fast convergence solution is needed to solve the pricing problem accurately to nudge the trucks better using alternative routes following the truck ban policy on CBD.

To address these problems, this study seeks to design a distance-based freeway pricing problem considering the mixed traffic flow with passenger vehicles and different types of trucks. The toll price is computed by the toll rates and the travelled freeway distances. It should be noted that the network we studied contains all types of roads, including city roads, freeways, and highways. But only freeways can charge fees, the other roads are toll-free. To reduce the impacts of truck ban policy on freight transportation, this study only controls the toll rates of different types of trucks on certain freeways. The toll rates for passenger vehicles remain fixed. Based on how trucks are charged for using freeways in China, they are divided into three types: light-duty trucks (LTD) with weights less than 4.5 tons, medium-duty trucks (MDT) with weights between 4.5 tons and 12 tons, and heavy-duty trucks (HDT) with weights larger than 12 tons.

To model the impacts of the toll rates of different types of trucks on equilibrium network flow distribution, the link travel cost functions are designed to consider the impacts of gasoline consumption, capacity of mixed traffic, and toll rates simultaneously. The cross-nested logit (CNL) model is used to characterize the travellers' route choice behavior of the four vehicle types (i.e., passenger vehicle, LTD, MTD, and HTD), and a MTA model developed upon a variational inequality (VI) is formulated to model the equilibrium state of the mixed traffic flow. To solve the MTA model involving passenger vehicles and different types of trucks, a route-swapping-based algorithm is designed. It solves the MTA model based on an analytical model to reduce computational complexity. Following that, a bi-level programming problem is designed to find the optimal toll rates for different types of trucks following the truck ban policy in CBD. The upper-level problem optimizes the toll rates for different types of trucks to minimize the network flow's TTC while ensuring that all the trucks' TTCs after the truck ban policy are only increased by a certain percentage. The lower-level MTA model captures the impacts of toll rates on the redistribution of network flows. To solve the bi-level problem, a line search algorithm developed upon the norm-relaxed method of feasible direction (NRMFD) is formulated. It is globally convergent and can solve the bi-level problem very fast (Wang et al., 2022). Numerical application in optimizing the freeway toll prices for trucks in Ningbo, China, finds that the designed method can effectively find an optimal tolling strategy to reduce the negative impacts of truck ban policy on freight and passenger transportation.

The structure of this study is as follows. Section 2 presents the MTA model and its solution algorithm for the mixed traffic flow with passenger vehicles and different types of trucks, following the formulation of its link travel cost function. In Section 3, a freeway optimal tolling problem is formulated for different truck types, and a solution algorithm is presented to solve it. Section 4 applies the proposed model in optimizing the freeway toll rates for trucks in Ningbo following truck ban in CBD. Section 5 concludes the paper.

2 Multiclass traffic assignment model for the traffic flow mixed with passenger vehicles and different types of trucks

In this section, we will develop a multiclass traffic assignment model for the mixed traffic flow with passenger vehicles and different types of trucks. Let Z represent the set of all types of trucks, $\boldsymbol{Z} = \{LDT, MDT, HDT\}$. In the next section, we will first formulate the link travel cost functions for different types of vehicles and then present the traffic assignment model for the mixed traffic flow. The framework for developing the multiclass traffic assignment models can be found in Figure 3.

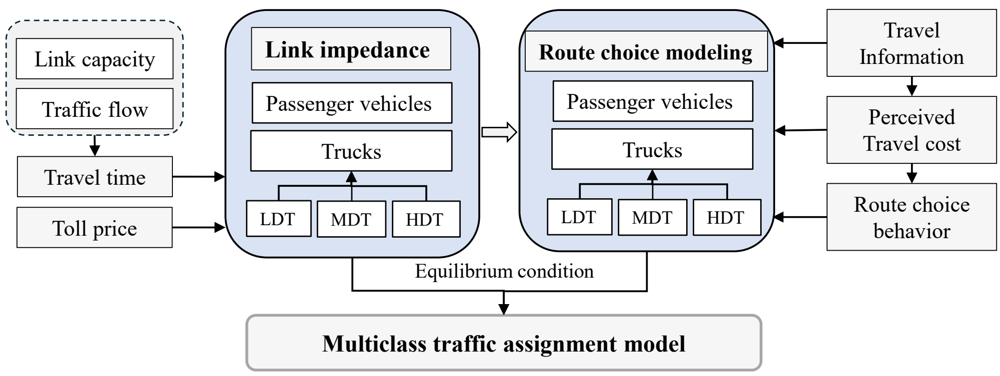


Figure 3. Framework for developing the multiclass traffic assignment models

2.1 Link travel cost functions for the mixed traffic flow

In a mixed traffic flow environment, the link travel cost for different types of vehicles is different because their speed limit, toll price, and gasoline consumption characteristics are different. Further, existing literature also found that due to the large time headway required for trucks, the ratio of trucks drastically impacts the link capacity. With these impact factors, the link travel time for passenger vehicles and trucks are formulated as

$$\bar{t}_{a,P} = \frac{L_a}{s_{a,P}}\left[1 + \left(\frac{v_{a,P} + \sum_{\forall z \in Z} PCU_z \cdot v_{a,z}}{Q_a}\right)^4\right], a \in \boldsymbol{L_P} \tag{1a}$$

$$\bar{t}_{a,T} = \frac{L_a}{s_{a,T}}\left[1 + \left(\frac{v_{a,P} + \sum_{\forall z \in Z} PCU_z \cdot v_{a,z}}{Q_a}\right)^4\right], a \in \boldsymbol{L_T} \tag{1b}$$

where $\bar{t}_{a,P}$ and $\bar{t}_{a,T}$ are the travel times of link a for passenger vehicles and trucks, respectively. L_a is the length of the link a; $v_{a,P}$ and $v_{a,z}$ are traffic flow for passenger vehicles and type z trucks. $s_{a,P}$ and $s_{a,T}$ are the speed limit for passenger vehicles and trucks, respectively. The two-speed limits are the same on most roads in the city. But they are different on freeways, which is 120 km/h and 100 km/h for passenger vehicles and all types of trucks, respectively, in China. PCU_z is passenger car unit (PCU) for truck type $z, z \in \boldsymbol{Z}$. The PCU for the LDT, MDT, and HDT are generally 1.5, 2.5, and 3.5, respectively. $\boldsymbol{L_P}$ and $\boldsymbol{L_T}$ are the sets of links for passenger vehicles and trucks, respectively. Note that they are generally unequal because some links are not allowed for trucks to access (e.g., urban expressways).

Q_a is the capacity of link a. As the high ratio of trucks in the flow will reduce the capacity, it is formulated as

$$Q_a = n_a \cdot C_a \cdot D_T \tag{2}$$

where n_a is the number of lanes on link a. C_a is the capacity of a lane on the road a. Note that the capacity of different types of roads (such as urban expressways, trunk roads, highways, etc.) is also different because their speed limit is different. These values can be found in China's urban road design criteria. According to Singh and Santhakumar (2021), D_T is formulated as

$$D_T = 1.58 \cdot (r_T)^2 + 0.3743 \cdot R_T - 0.009295 \tag{3}$$

where r_T is the total ratio of trucks in the mixed traffic flow. Eq. (3) is already calibrated using field data.

In addition to time, gasoline consumption is also one of the major impact factors for travellers' route choices, especially for truck drivers. According to Zhang et al. (2014), the gasoline consumption rate (per vehicle-mile) of a vehicle is an exponential function of the average traffic speed, formulated as

$$E_{a,P} = \vartheta_1 \left(\frac{L_a}{\bar{t}_{a,C}}\right)^{-\vartheta_2} L_a, \tag{4}$$

$$E_{a,z} = \vartheta_{3,z} \left(\frac{L_a}{\bar{t}_{a,T}}\right)^{-\vartheta_{4,z}} L_a, z \in \boldsymbol{Z} \tag{5}$$

where $E_{a,P}$ and $E_{a,z}, z \in \boldsymbol{Z}$ are gasoline consumption rates (per vehicle-mile) for passenger vehicles and type z trucks, respectively. L_a is the length of link a, $L_a/\bar{t}_{a,P}$ and $L_a/\bar{t}_{a,T}$ are the average travel speeds of a passenger vehicle and a truck on link a, respectively. $\vartheta_1, \vartheta_{3,z} > 0$ and 0<$\vartheta_2 < 1, 0 < \vartheta_{4,z} < 1$ are positive coefficients that need to be estimated.

Based on the above discussion, the link travel cost functions for passenger vehicles and trucks are formulated as follows

$$t_{a,P} = \bar{t}_{a,P} \cdot VOT_P + \eta \cdot E_{a,P} + \tau_{a,P} \cdot L_a \tag{6a}$$

$$t_{a,z} = \bar{t}_{a,T} \cdot VOT_z + \eta \cdot E_{a,z} + \tau_{a,z} \cdot L_a, z \in \boldsymbol{Z} \tag{6b}$$

where VOT_P and VOT_z are the values of time for passenger vehicle users and type z truck users. η is the gasoline price per unit volume. $\tau_{a,P}$ and $\tau_{a,z}$ are toll rates per kilometer for passenger vehicles and type z trucks, respectively, using the link a. It should be noted that the toll rates for different types of trucks are different, and only freeways can charge vehicles in China. The other roads are toll-free.

2.2 *Multiclass traffic assignment model for the mixed traffic flow*

Note that travellers using different types of vehicles all have limited information on traffic conditions. To model this fact, other than using the UE model, the CNL model will be used to model the route choice behavior of all types of travellers.

Let $\boldsymbol{W}_P$ and $\boldsymbol{W}_T$ be the set of OD pairs for passenger vehicles and trucks, respectively. Let $\boldsymbol{R}_P^w$ and $\boldsymbol{R}_T^w$ be the set of routes connecting OD pair $w \in W_P$ and $w \in W_T$, respectively. Denote $f_{k,P}^w$ and $f_{k,z}^w, z \in \boldsymbol{Z}$ as the passenger vehicle flow and type z truck flow on path k for OD pair w, respectively. Let $\mathbf{f}_P$ and $\mathbf{f}_z, z \in \boldsymbol{Z}$ be the set of all path flows for passenger vehicles and type z trucks, respectively. Let q_P^w and $q_z^w, z \in \boldsymbol{Z}$ be the demand of passenger vehicles and trucks with type z for OD pair w, respectively. $\mathbf{q}_P$ and $\mathbf{q}_z, z \in \boldsymbol{Z}$ be the vector of demands of passenger vehicles and type z trucks for all OD pairs. Δ_P and Λ_P are the link-path and OD-path matrices, respectively, for passenger vehicles. Δ_z and Λ_z are the link-path and OD-path matrices, respectively, for type z trucks, $z \in \boldsymbol{Z}$.

In the literature, many models have been proposed to characterize the equilibrium network flow of vehicles, among which the UE model and the logit-based SUE model are perhaps the best-used ones. However, these models are built upon strong assumptions. The UE model suffers from the perfect assumption of travellers' information, and the logit-based SUE model has the inherent drawback of not distinguishing the overlapped routes due to the assumption of Independence of Irrelevant Alternatives. To address this problem, this study uses the CNL model to characterize the network flows mixed with passenger vehicles and different types of trucks. It relaxes the strong assumption of the UE problem and overcomes the route overlap issue simultaneously. The practical application shows that it can characterize the network flows with high accuracy (Ramming, 2002).

For simplicity, we only use the passenger vehicles to demonstrate the details of the CNL model. The CNL model introduces an inclusion coefficient ($\alpha_{m,k}^w, m \in \boldsymbol{\Gamma}_P$) for each path $k \in R_P^w$ and link m to denote the overlapping degree of this path with other paths in nest $m \in \boldsymbol{\Gamma}_P$. Let $A_P^w(k)$ be the probability for a passenger vehicle user to choose path k between OD pair w. Then we have

$$A_P^w(k) = \sum_{m \in \boldsymbol{\Gamma}_P} A_P^w(k|m) A_P^w(m) \tag{7a}$$

where $A_P^w(m)$ is the probability for a passenger vehicle user to choose link m. $A_P^w(k|m)$ is the probability for a passenger vehicle user to choose path k given that they choose link m.

$$A_P^w(k|m) = \frac{\left[\alpha_{m,k}^w \exp\left(-\theta c_{k,P}^w\right)\right]^{1/u}}{\sum_{l \in \boldsymbol{R}_P^w} \left[\alpha_{m,l}^w \exp\left(-\theta c_{l,P}^w\right)\right]^{1/u}} \tag{7b}$$

$$A_P^w(m) = \frac{\left(\sum_{k \in \boldsymbol{R}_P^w} \left[\alpha_{m,k}^w \exp\left(-\theta c_{k,P}^w\right)\right]^{1/u}\right)^u}{\sum_{b \in \boldsymbol{\Gamma}_H} \left(\sum_{l \in \boldsymbol{R}_P^w} \left[\alpha_{b,l}^w \exp\left(-\theta c_{l,P}^w\right)\right]^{1/u}\right)^u} \tag{7c}$$

where $c_{k,P}^w$ is the travel cost for passenger vehicles on path k; $c_{k,P}^w = \sum_{a \in \boldsymbol{L}_P} t_{a,P} \cdot \delta_{a,k}^w$, and $c_{k,P}^w = \sum_{a \in \boldsymbol{\Gamma}_P} t_{a,P} \cdot \delta_{a,k}^w = 1$ if path k uses link a and 0 otherwise. θ is the dispersion parameter. u is the degree of nesting, $0 < u \leq 1$; $\alpha_{m,k}^{w}$ is the inclusion coefficient, incorporated to overcome the route overlapping problem. It is computed as follows:

$$\alpha_{m,k}^{w} = \left(\frac{L_m}{L_k^w}\right)^{\gamma} \delta_{m,k}^{w} \tag{8}$$

where L_m and L_k^w are the lengths of link m and path k of OD pair w, respectively, and $\delta_{m,k}^{w} = 1$ if path k uses link m and 0 otherwise.

The detailed explanation of the CNL model can be found in Prashker and Bekhor (1999). They also formulate the traffic assignment problem for CNL model. However, this problem is built upon the flow of paths belonging to different nests, making it computationally very expensive and hard to expand to formulate the network equilibrium flow in the context of multiple types of vehicles. To address this problem, this study uses the VI-based problem for CNL traffic assignment proposed by Wang et al. (2019). To characterize the CNL equilibrium state of the mixed traffic flow, let $C_{k,P}^{w}$ and $C_{k,z}^{w}, \forall z \in Z$ be the generalized travel costs for passenger vehicles and trucks on path k. They are defined as follows:

$$C_{k,P}^{w} = c_{k,P}^{w} - \frac{u}{\theta} ln\left[\sum_{m\in\boldsymbol{\Gamma}_P} \left(\alpha_{m,k}^{w}\right)^{1/u} \left(\sum_{l\in\boldsymbol{R}_P^w} \left[\alpha_{m,l}^{w} \exp\left(-\theta c_{l,P}^{w}\right)\right]^{1/u}\right)^{u-1}\right] + \frac{u}{\theta} ln\left(\frac{f_{k,P}^{w}}{q_P^w}\right) \tag{9a}$$

$$C_{k,z}^{w} = c_{k,z}^{w} - \frac{u}{\theta} ln\left[\sum_{m\in\boldsymbol{\Gamma}_T} \left(\alpha_{m,k}^{w}\right)^{\frac{1}{u}} \left(\sum_{l\in\boldsymbol{R}_z^w} \left[\alpha_{m,l}^{w} \exp\left(-\theta c_{l,z}^{w}\right)\right]^{\frac{1}{u}}\right)^{u-1}\right] + \frac{u}{\theta} ln\left(\frac{f_{k,z}^{w}}{q_z^w}\right), \forall z \in \boldsymbol{Z} \tag{9b}$$

where $c_{k,z}^{w}$ is the travel cost for type z trucks on path k, $c_{k,z}^{w} = \sum_{a\in\boldsymbol{\Gamma}_T} t_{a,T} \cdot \delta_{a,k}^{w} = 1$ if path k uses link a and 0 otherwise; θ is the dispersion parameter.

According to Wang et al. (2019), $\mathbf{f}^* = [\mathbf{f}_P^* \quad \mathbf{f}_{LDT}^* \quad \mathbf{f}_{MDT}^* \quad \mathbf{f}_{HDT}^*]$ is at the CLN equilibrium state of the mixed traffic flow if the generalized travel cost of all paths for the corresponding OD pair is the same for each vehicle type, i.e., $\mathbf{f}^*$ satisfies

$$C_{k,P}^{w}(\mathbf{f}^*) = C_P^{w*}, \text{ for } \forall w \in \boldsymbol{W}_P, \forall k \in \boldsymbol{R}_P^w \tag{10a}$$

$$C_{k,z}^{w}(\mathbf{f}^*) = C_z^{w*}, \text{ for } \forall w \in \boldsymbol{W}_z, \forall k \in \boldsymbol{R}_z^w, z \in \boldsymbol{Z} \tag{10b}$$

where C_P^{w*} and $C_z^{w*}, \forall z \in \boldsymbol{Z}$ are constant values.

Based on the above discussion, it can be shown that $\mathbf{f}^*$ is at the CNL-CNL-CNL-CNL equilibrium state of the mixed traffic flow if and only if it satisfies the following VI problem.

$$\sum_{w\in W_P} \sum_{k\in R_P^w} C_{k,P}^{w}(\mathbf{f}^*)\left(f_{k,P}^{w} - f_{k,P}^{w*}\right) + \sum_{z\in Z} \sum_{w\in W_z} \sum_{k\in R_z^w} C_{k,z}^{w}(\mathbf{f}^*)\left(f_{k,z}^{w} - f_{k,z}^{w*}\right) \geq 0 \tag{11}$$

where $[\mathbf{f}_P^T, \mathbf{f}_z^T, \forall z \in Z] \in \boldsymbol{\Omega}_{\mathbf{f}} = \{[\mathbf{f}_P^T, \mathbf{f}_z^T, , \forall z \in \boldsymbol{Z}] | \Lambda_P \mathbf{f}_P = \mathbf{q}_P;\ \Lambda_z \mathbf{f}_z = \mathbf{q}_z; \mathbf{f}_z \geq 0; \mathbf{f}_z \geq 0\}$.

Problem (11) is the MTA model that characterizes the route choices of both the passenger vehicle user and truck user based on the CNL model. This traffic assignment model overcomes the perfect assumption of travel information in the UE problem and the route overlapping issue of the logit-based SUE problem. Therefore, it can better characterize the equilibrium network flow.

Clearly, if set $u = 1$, then $C_{k,P}^{w} = c_{k,P}^{w} + \frac{1}{\theta} ln\left(\frac{f_{k,P}^{w}}{q_{P}^{w}}\right)$, $C_{k,z}^{w} = c_{k,z}^{w} + \frac{1}{\theta} ln\left(\frac{f_{k,z}^{w}}{q_{z}^{w}}\right)$, $\forall z \in \boldsymbol{Z}$, then problem (11) will become logit-based MTA.

2.3 *Solution algorithms for the MTA model (10)*

The MTA model (11) contains four class of vehicles, which is hard to solve for an equilibrium solution. To tackle this problem, the route-swapping model proposed in Wang et al. (2019) will be used to solve the MTA model. One advantage of this algorithm is that the algorithm use an analytical model to computes the descent direction at each iteration, which helps to circumvent the traditional projection method that is very computational expensive. In each iteration, the route swapping-based algorithm computes the descent direction based on an analytical model, which helps to circumvent the traditional projection method that is very computational expensive. Let $\mathbf{f} = [\mathbf{q}_{P} \quad \mathbf{q}_{LDT} \quad \mathbf{q}_{MDT} \quad \mathbf{q}_{HDT}]$ be the vector of the flow of all routes in the augmented network, and $\mathbf{f}_n$ be the value of $\mathbf{f}$ at iteration n. At iteration $n+1$, the flows of all routes in the network are updated using the following models

$$\mathbf{f}_{n+1} = \mathbf{f}_n + \beta_n \boldsymbol{\Phi}(\mathbf{f}_n) = \begin{bmatrix} \mathbf{f}_{n,P} \\ \mathbf{f}_{n,LDT} \\ \mathbf{f}_{n,MDT} \\ \mathbf{f}_{n,HDT} \end{bmatrix} + \beta_n \begin{bmatrix} \boldsymbol{\Phi}_{P}(\mathbf{f}_n) \\ \boldsymbol{\Phi}_{LDT}(\mathbf{f}_n) \\ \boldsymbol{\Phi}_{MDT}(\mathbf{f}_n) \\ \boldsymbol{\Phi}_{HDT}(\mathbf{f}_n) \end{bmatrix} \tag{12}$$

where β_n is the step, which can be computed using the greedy method proposed in Wang et al. (2021). $\boldsymbol{\Phi}_P(\mathbf{f}_n) = \left(\Phi_{k,P}^{w}(\mathbf{f}_n), \forall k \in \boldsymbol{R}_P^{w}, w \in \boldsymbol{W}_P\right)$ and $\boldsymbol{\Phi}_z(\mathbf{f}_n) = \left(\Phi_{k,z}^{w}(\mathbf{f}_n), \forall k \in \boldsymbol{R}_z^{w}, w \in \boldsymbol{W}_z, z \in \boldsymbol{Z}\right)$ are updated by:

$$\Phi_{k,P}^{w}(\mathbf{f}_n) = \sum_{g \in \boldsymbol{R}_P^{w}} \left[f_{g,P}^{w}(n) \left(C_{g,P}^{w}(\mathbf{f}_n) - C_{k,P}^{w}(\mathbf{f}_n) \right)_+ - f_{k,P}^{w}(n) \left(C_{k,P}^{w}(\mathbf{f}_n) - C_{g,P}^{w}(\mathbf{f}_n) \right)_+ \right] \tag{13a}$$

$$\Phi_{k,z}^{w}(\mathbf{f}_n) = \sum_{g \in \boldsymbol{R}_z^{w}} \left[f_{g,z}^{w}(n) \left(C_{g,z}^{w}(\mathbf{f}_n) - C_{k,z}^{w}(\mathbf{f}_n) \right)_+ - f_{k,z}^{w}(n) \left(C_{k,z}^{w}(\mathbf{f}_n) - C_{g,z}^{w}(\mathbf{f}_n) \right)_+ \right] \tag{13b}$$

Based on above discussions, the steps for solving the MTA model (11) can be found in Figure 4.

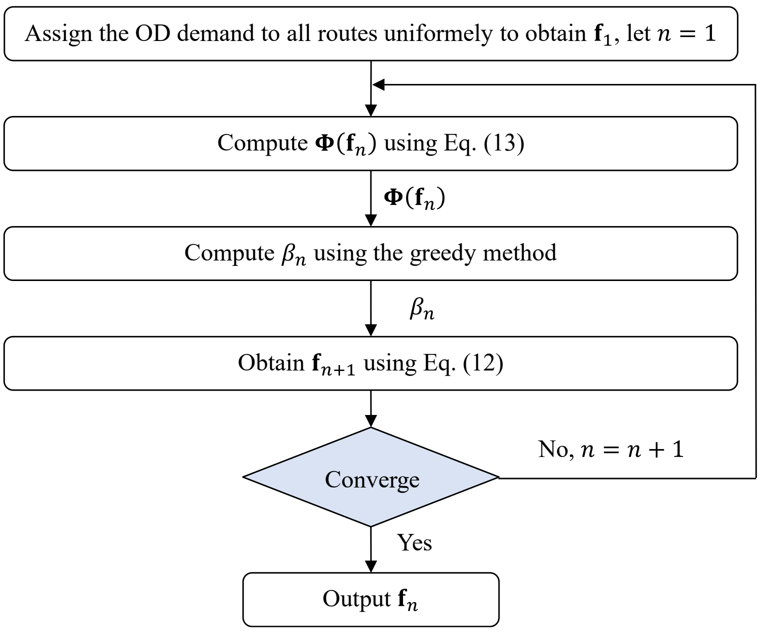


Figure 4. Steps for solving the MTA model (11)

3 Optimizing the freeway toll rates for different types of trucks under truck ban policy in CBD

3.1 Bi-level problem for optimizing freeway toll rates for different types of trucks

As mentioned in the introduction section, most cities have now applied the truck ban policy to reduce the negative impacts of freight transportation in the CBD. However, the main impact of this policy is that trucks cannot access the freeways in the CBD area. To avoid such negative impacts, new tolling strategies must be designed to encourage the trucks to use alternative routes so that the TTC will not significantly increase.

Note that only freeways can charge vehicles in China. Let $\boldsymbol{\Gamma}_T$ be the set of freeways for which the trucks' tolling rate can be controlled and optimized to reduce the negative impacts on freight transportation. $\boldsymbol{\Gamma}_T$ can be easily determined by choosing the alternative freeways for the forbidden freeways (e.g., parallel to the freeways not allowed to be accessed). Let $\boldsymbol{\tau} = [\tau_{a,z}, \forall z \in \boldsymbol{Z}, \forall a \in \boldsymbol{\Gamma}_T]$ be the vector of all the trucks' toll rates of the links in the set $\boldsymbol{\Gamma}_T$. It should be noted that the toll rates for different types of trucks are different. Let $\mathbf{f}^*(\boldsymbol{\tau})$ be the equilibrium path flow solution of the MTA model (11) with toll rates $\boldsymbol{\tau}$. Denote $\varphi_T(\mathbf{f}^*(\boldsymbol{\tau}), \boldsymbol{\tau})$ be the TTC of different types of trucks under toll rates $\boldsymbol{\tau}$ following the truck ban policy (i.e., some freeways are forbidden to access by trucks). Denote $\varphi_T(\mathbf{f}^*(\boldsymbol{\tau}_0), \boldsymbol{\tau}_0)$ be the TTC of different types of trucks under initial toll rates $\boldsymbol{\tau}_0$ before the truck ban policy. $\varphi_T(\mathbf{f}^*(\boldsymbol{\tau}), \boldsymbol{\tau})$ and $\varphi_T(\mathbf{f}^*(\boldsymbol{\tau}_0), \boldsymbol{\tau}_0)$ is formulated as

$$\varphi_T(\mathbf{f}^*(\boldsymbol{\tau}), \boldsymbol{\tau}) = \sum_{z \in Z} \sum_{w \in \boldsymbol{W}_z} \sum_{k \in \boldsymbol{R}_z^w} C_{k,z}^w(\mathbf{f}^*(\boldsymbol{\tau}), \boldsymbol{\tau})\left(f_{k,z}^{w*}\right) \tag{14a}$$

$$\varphi_T(\mathbf{f}^*(\boldsymbol{\tau}_0), \boldsymbol{\tau}_0) = \sum_{z \in Z} \sum_{w \in \boldsymbol{W}_z} \sum_{k \in \boldsymbol{R}_z^w} C_{k,z}^w(\mathbf{f}^*(\boldsymbol{\tau}_0), \boldsymbol{\tau}_0)\left(f_{k,z}^{w*}\right) \tag{14b}$$

The optimal toll rates by links and truck types can be obtained by solving the following bi-level programming problem, where the upper-level problem is formulated as

$$\min_{\boldsymbol{\tau}} \varphi(\mathbf{f}^*(\boldsymbol{\tau}), \boldsymbol{\tau}) = \sum_{w \in \boldsymbol{W}_P} \sum_{k \in \boldsymbol{R}_P^w} C_{k,P}^w(\mathbf{f}^*(\boldsymbol{\tau}), \boldsymbol{\tau}) \cdot f_{k,P}^{w*} + \sum_{z \in Z} \sum_{w \in \boldsymbol{W}_z} \sum_{k \in \boldsymbol{R}_z^w} C_{k,z}^w(\mathbf{f}^*(\boldsymbol{\tau}), \boldsymbol{\tau})\left(f_{k,z}^{w*}\right) \tag{15a}$$

$$s.t.\ \varphi_T(\mathbf{f}^*(\boldsymbol{\tau}), \boldsymbol{\tau}) \le \mu * \varphi_T(\mathbf{f}^*(\boldsymbol{\tau}_0), \boldsymbol{\tau}_0) \tag{15b}$$

$$\boldsymbol{\tau} \le \boldsymbol{\tau}_0, \forall a \in \boldsymbol{\Gamma}_T \tag{15c}$$

$$\boldsymbol{\tau} \ge \mathbf{0}, \forall a \in \boldsymbol{\Gamma}_T \tag{15d}$$

and $\mathbf{f}^*(\boldsymbol{\tau})$ is the equilibrium solution of the MTA problem (11) with input $\boldsymbol{\tau}$. The model framework for developing the bi-level tolling problem can be found in Figure 5.

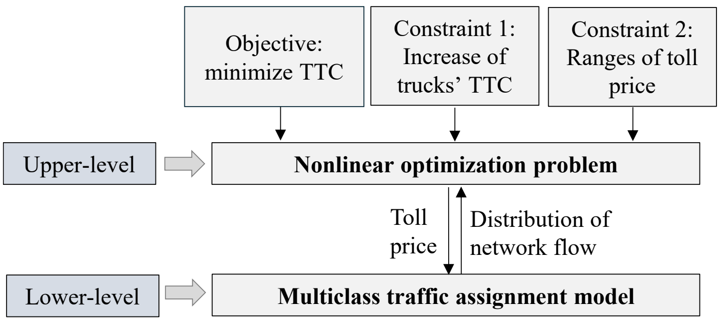


Figure 5. Framework for developing the bi-level tolling problem

In problem (15), the objective is to optimize the toll rates $\boldsymbol{\tau}$ to reduce the total travel cost in the network (including passenger vehicles). $\mu, \mu \ge 1$ is a coefficient. Inequality (15b) seeks to reduce the impacts of the truck ban policy on freight transportation by restricting the increase of the trucks' total travel cost (through reducing alternative freeway toll rates) after the truck ban policy in CBD (e.g., $\mu = 1.05$, means it cannot increase the freight transportation cost by 5%). Inequality (15c) indicates that the new toll rates for different types of trucks after the truck ban policy should be no larger than they were before the policy was released. Inequality (15d) is a nonnegative constraint.

It is worth mentioning that while the proposed bi-level tolling problem is formulated to optimize the toll rates for different types of trucks to mitigate their travel impact after the truck ban on some freeways, it can also be used to optimize the toll rates to balance the traffic load and reduce the total travel cost for a general freeway network. This can be achieved by removing the inequality constraint (15b) (i.e., the truck cost constraint) in problem (15). The proposed solution algorithm presented in the following can still be applied in this context.

3.2 *Solution algorithm for the bi-level problem*

Like the other bi-level problems, the problem (14) is non-convex and hard to solve for an optimal solution. As this problem can be regarded as a mathematical problem with equilibrium constraint (MPEC), we focus on solving the upper-level problem (15a)-(15d) by embedding the lower-level multiclass traffic assignment problem (11) into it (i.e., using. $\mathbf{f}^*(\boldsymbol{\tau})$ to represent the lower-level problem). In the literature, most studies use intelligent algorithms (such as genetic algorithms) to solve the problem (15). However, they are computationally very expensive and cannot ensure convergence. In this study, we will propose a linear search method built upon NRMFD to solve the problem (15). We will show that this algorithm is globally convergent.

Suppose $\boldsymbol{\tau}^n$ is the toll prices for all types of trucks on all tolling links found by this algorithm at iteration n. Then, at iteration $n+1$, a line search method solves the problem (15) based on the following structure:

$$\boldsymbol{\tau}^{n+1} = \boldsymbol{\tau}^n + \alpha^n \boldsymbol{d}^n \tag{16}$$

where $\boldsymbol{\tau}^n$ is the inputs of $\boldsymbol{\tau}$ obtained at iteration n, α^n is the step size and $\boldsymbol{d}^n$ is a feasible descent direction found at $\boldsymbol{\tau}^n$, i.e., there exists $\beta_1^{max} > 0$ such that, for $\forall \beta \in (0, \beta_1^{max})$

$$\text{Condition 1: } \varphi(\mathbf{f}^*(\boldsymbol{\tau}^n + \beta \boldsymbol{d}^n), \boldsymbol{\tau}^n + \beta \boldsymbol{d}^n) < \varphi(\mathbf{f}^*(\boldsymbol{\tau}), \boldsymbol{\tau}) \tag{17a}$$

$$\text{Condition 2: } \varphi_T(\mathbf{f}^*(\boldsymbol{\tau}^n + \beta \boldsymbol{d}^n), \boldsymbol{\tau}^n + \beta \boldsymbol{d}^n) - \mu * \varphi_T(\mathbf{f}^*(\boldsymbol{\tau}_0), \boldsymbol{\tau}_0) \leq 0 \tag{17b}$$

$$\text{Condition 3: } \boldsymbol{\tau}^n + \beta \boldsymbol{d}^n \leq \boldsymbol{\tau}_0 \tag{17c}$$

$$\text{Condition 4: } \boldsymbol{\tau}^n + \beta \boldsymbol{d}^n \geq \mathbf{0} \tag{17d}$$

Condition 1 indicates that $\boldsymbol{d}^n$ is a descent direction. Conditions (17b)-(17d) ensures feasibility. Clearly, the most important step to apply this algorithm to solve the problem (15) is to find $\boldsymbol{d}^n$ at $\boldsymbol{\tau}^n$. $\boldsymbol{d}^n$ can be computed using the NRMFD method as follows.

$$\min_{\theta, \boldsymbol{d}^n} \ \gamma + \frac{\sigma}{2} (\boldsymbol{d}^n)^T \cdot \boldsymbol{H} \cdot \boldsymbol{d}^n \tag{18a}$$

$$\nabla_{\boldsymbol{\tau}} \varphi\big(\mathbf{f}^*(\boldsymbol{\tau}^n), \boldsymbol{\tau}^n\big) \cdot \boldsymbol{d}^n \leq \gamma \tag{18b}$$

$$\varphi_T(\mathbf{f}^*(\boldsymbol{\tau}^n), \boldsymbol{\tau}^n) + \nabla_{\boldsymbol{\tau}} \varphi_T(\mathbf{f}^*(\boldsymbol{\tau}^n), \boldsymbol{\tau}^n) \boldsymbol{d}^n - \mu * \varphi_T(\mathbf{f}^*(\boldsymbol{\tau}_0), \boldsymbol{\tau}_0) \leq \gamma \tag{18c}$$

$$\boldsymbol{\tau}^n - \boldsymbol{\tau}_0 + \boldsymbol{d}^n \leq \gamma, \forall a \in \boldsymbol{\Gamma}_T \tag{18d}$$

$$-\boldsymbol{\tau}^n - \boldsymbol{d}^n \leq \gamma, \forall a \in \boldsymbol{\Gamma}_T \tag{18e}$$

where σ is a positive constant; $\boldsymbol{H}$ is a positive definite matrix (PDM). It is typically set as an arbitrary diagonal PDM (Wang et al., 2021); γ is a decision variable; $\nabla_{\boldsymbol{\tau}}\varphi(\mathbf{f}^*(\boldsymbol{\tau}^n), \boldsymbol{\tau}^n)$ and $\nabla_{\boldsymbol{\tau}}\varphi_T(\mathbf{f}^*(\boldsymbol{\tau}^n), \boldsymbol{\tau}^n)$ are formulated as

$$\nabla_{\boldsymbol{\tau}}\varphi\left(\mathbf{f}^*(\boldsymbol{\tau}^n), \boldsymbol{\tau}^n\right) = \nabla_{\mathbf{f}}\varphi\left(\mathbf{f}^*(\boldsymbol{\tau}^n), \boldsymbol{\tau}^n\right) \cdot \nabla_{\boldsymbol{\tau}}\mathbf{f}^*(\boldsymbol{\tau}^n) + \nabla_{\boldsymbol{x}}\varphi(\mathbf{f}^*(\boldsymbol{\tau}^n), \boldsymbol{x})|_{\boldsymbol{x}=\boldsymbol{\tau}^n} \tag{19a}$$

$$\nabla_{\boldsymbol{\tau}}\varphi_T\left(\mathbf{f}^*(\boldsymbol{\tau}^n), \boldsymbol{\tau}^n\right) = \varphi_T\varphi\left(\mathbf{f}^*(\boldsymbol{\tau}^n), \boldsymbol{\tau}^n\right) \cdot \nabla_{\boldsymbol{\tau}}\mathbf{f}^*(\boldsymbol{\tau}^n) + \nabla_{\boldsymbol{x}}\varphi_T(\mathbf{f}^*(\boldsymbol{\tau}^n), \boldsymbol{x})\Big|_{\boldsymbol{x}=\boldsymbol{\tau}^n} \tag{19b}$$

Inequalities (18d) is the first-order approximation of the objective function (15a). It ensures that the computed $\boldsymbol{d}^n$ is descent. Inequalities (18c)-(18e) is a linearized system of the constraints (15b)-(15d). They are set to ensure $\boldsymbol{d}^n$ is feasible.

According to Cawood and Kostreva (1994), problem (18) has a unique solution of $(\gamma^*, \boldsymbol{d}^n)$, in which $\boldsymbol{d}^n$ must be the feasible descent direction for the problem (15).

To solve problem (19), another problem is to compute $\nabla_{\boldsymbol{\tau}}\mathbf{f}^*(\boldsymbol{\tau}^n)$. $\nabla_{\boldsymbol{\tau}}\mathbf{f}^*(\boldsymbol{\tau}^n)$ can be approximated using the perturbation method as follows

$$\nabla_{\tau_i}\mathbf{f}^*(\boldsymbol{\tau}^n) \approx \frac{\mathbf{f}^*(\boldsymbol{\tau}^n + \Delta\tau_i \cdot \mathbf{1}_i) - \mathbf{f}^*(\boldsymbol{\tau}^n)}{\Delta\tau_i} \tag{20}$$

where $\nabla_{\tau_i}\mathbf{f}^*(\boldsymbol{\tau}^n)$ is the gradient of $\mathbf{f}^*(\boldsymbol{\tau}^n)$ with respect to τ_i. $\Delta\tau_i$ is a sufficiently small perturbation value added on τ_i. $\mathbf{1}_i$ is a vector with ith entry being 1 and other entries being 0.

Using the same method discussed in Wang et al. (2021), it can be easily shown that the line search method developed upon the NRMFD is globally convergent. The steps for implementing the solution algorithm for the bi-level tolling problem are shown in Figure 6.

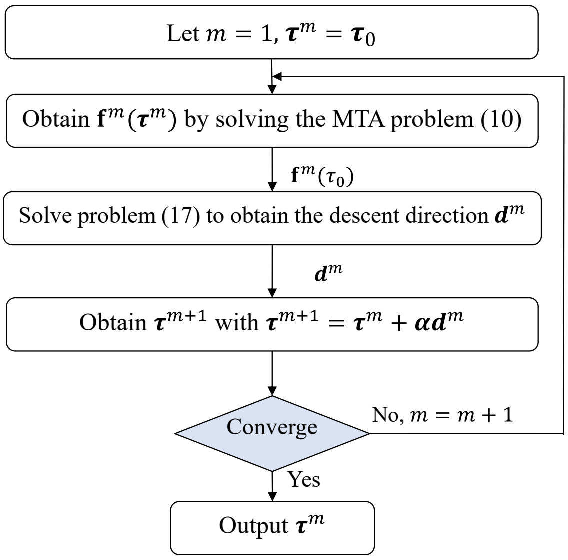
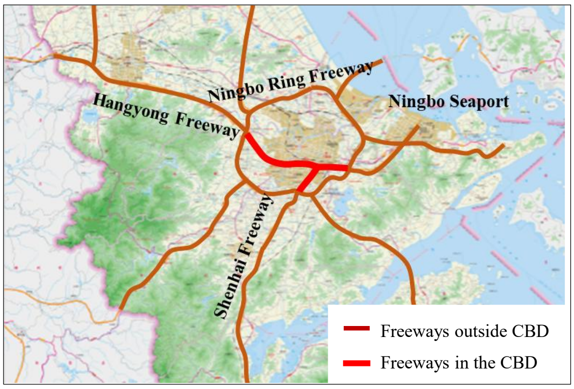


Figure 6. Steps for implementing the solution algorithm for the bi-level tolling problem

4 Numerical example

4.1 Problem description

In this section, the problem (14) will be used to optimize the freeway toll rates for different types of trucks in Ningbo, China. As seen in Figure 7(a), due to rapid urbanization, part of the Hangyong and Shenhai freeway is now within the CBD area (the area within the Ningbo Ring freeway) in Ningbo. The two freeways are also highly congested because both freight transportation demand and passenger transportation demand are very high. To enhance traffic safety, improve traffic efficiency, and avoid the negative impacts (such as noise) on the surrounding residents, the government seeks to apply the truck ban on the two freeways in the CBD (see the red lines in Figure 6(a)).



(a) The freeways in Ningbo

(b) The freeway links whose toll rates are controllable

Figure 7. The truck ban freeways in Ningbo and corresponding freeway links, whose toll rates are controllable

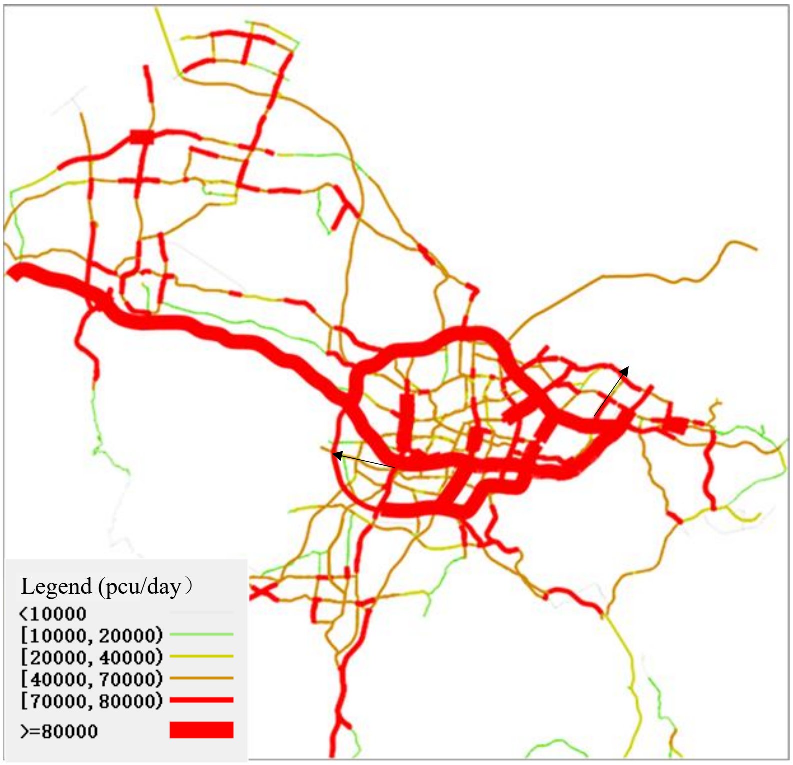


Figure 8. The current distribution of network flows in Ningbo, China

As shown in Figure 7(a), the two freeways are very close to Ningbo Seaport, whose throughput ranks No.1 in the world. Based on Ningbo seaport traffic monitoring system, over 200,000 trucks are shipping cargo to or from the Ningbo Seaport daily using highways. Figure 8 shows the distribution of the link flows detected by over 7000 cameras in Ningbo. As can be seen, the traffic flow on both the Hangyong Freeway and Shenhai Freeway in the CBD is over 80000 pcu/day, and over 70% of them are truck flows that head to the Ningbo Seaport.

To avoid the negative impacts of the truck ban on the two freeways in CBD, the government would like to give toll discounts for trucks using alternative highways. The exist two main alternative routes based on the Ningbo Ring freeways: one is on the north side, and another one is on the south side. To balance the traffic demand, the two routes will be divided into three links, each of which can be charged with different toll rates for various types of trucks (see Figure 7(b)).

4.2 Data collection

To determine the toll rates systematically to reduce the total travel cost of all vehicles, in addition to the freeway network, the road transportation network of Ningbo is built (see Figure 9) based on OpenStreetMap. It contains 8765 links, 468 traffic zones, and 236,196 OD pairs. The OD demands are computed using mobile signalling data. The initial toll rates for the LDTs, MDTs, and HDTs are 0.8 yuan/km, 1.2 yuan/km, and 1.7 yuan/km (1 yuan equals 0.13 dollars), respectively.

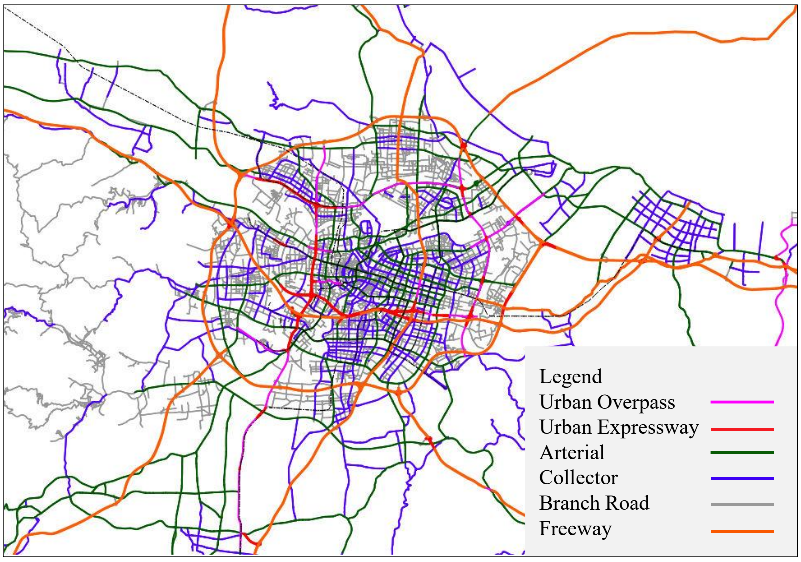


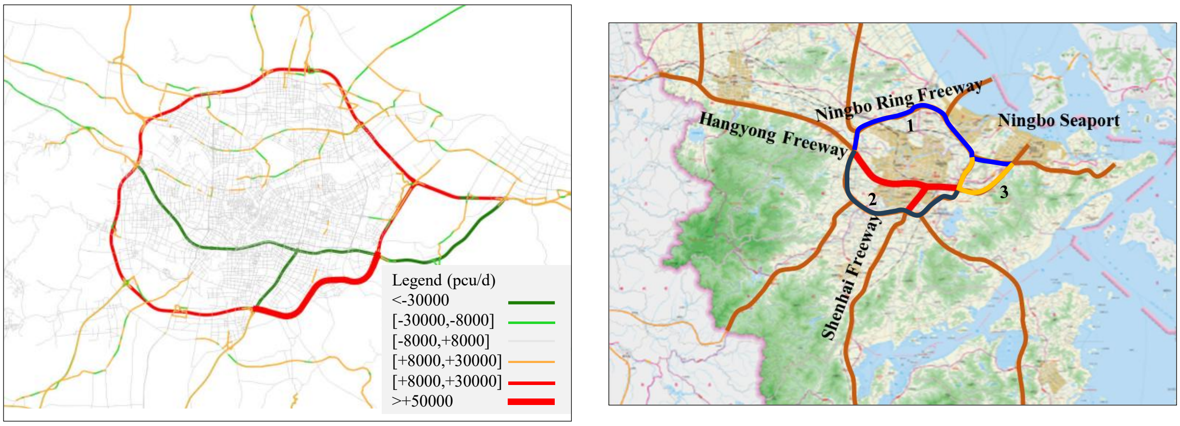
Figure 9. The road network considered in this study for optimizing the freeway toll rates for trucks

4.3 Optimal toll prices for different types of trucks on different freeways

Table 1. Convergence of the line search algorithm developed upon NRMFD

Iterations	$\tau_{1,LDT}$	$\tau_{1,MDT}$	$\tau_{1,HDT}$	$\tau_{2,LDT}$	$\tau_{2,MDT}$	$\tau_{2,HDT}$	$\tau_{3,LDT}$	$\tau_{3,MDT}$	$\tau_{3,HDT}$	Total cost ($\times 10^7$)
1	0.800	1.200	1.700	0.800	1.200	1.700	0.800	1.200	1.700	7.7154
2	0.759	1.167	1.675	0.782	1.177	1.678	0.775	1.192	1.662	7.7145
3	0.746	1.138	1.658	0.756	1.17	1.652	0.767	1.177	1.619	7.5054
4	0.73	1.124	1.613	0.726	1.149	1.625	0.735	1.144	1.601	7.3805
5	0.721	1.104	1.594	0.699	1.148	1.613	0.701	1.119	1.556	7.2815
6	0.681	1.079	1.548	0.692	1.116	1.574	0.679	1.104	1.529	7.2054
7	0.643	1.036	1.524	0.674	1.105	1.542	0.659	1.078	1.486	7.1601
8	0.623	1.022	1.505	0.665	1.097	1.538	0.632	1.042	1.463	7.1246
9	0.601	0.976	1.453	0.63	1.091	1.519	0.597	1.029	1.443	7.0765
10	0.589	0.957	1.411	0.6277	1.065	1.505	0.587	1.013	1.419	7.0531
11	0.59	0.958	1.414	0.6281	1.065	1.503	0.589	1.012	1.419	7.0531

To determine the optimal toll prices for different trucks on different freeways following the truck ban policy in CBD. The parameter μ in Eq. (14b) is set as 1.03, which means that the increase in total travel cost of trucks after the truck should be less than 3%. Table 1 shows the evolution of the toll rates for different types of trucks on different freeway links computed by the line search algorithm with NRMD. The algorithm converges fast and only takes 11 iterations to converge. At the optimal tolling state, the TTC of the network reduces by 8.5% compared to the initial state. This is partly because the freeway toll prices for all types of trucks are reduced, and the truck ban on the two freeways can drastically reduce the congestion. Further, as the trucks' toll prices are reduced, the truck ban on the two freeways in CBD does not significantly increase their travel cost. Most trucks would use the two alternative routes on the Ningbo Ring freeway to reach their destination (see Figure 10(a)). Some of them would use the national or provincial highways. This indicates that the new truck toll strategy can effectively encourage the trucks to use the two alternative routes. At the optimal toll state, the TTC for trucks is 1.863×10^7, and an increase of only 1.15% compared to it was before the truck ban in CBD. The truck toll optimization strategy can effectively accommodate the loss of trucks induced by the truck ban policy. This helps to prevent the truck drivers from resisting the policy. Figure 10(b) also shows that after the truck ban was implemented on the two freeways in CBD, more passenger flows would use the two freeways due to improved traffic efficiency.



(a) Difference in truck flow *(b) Difference in passenger vehicle flow*

Figure 10. Difference in truck and passenger vehicle flow between the optimal toll state following the truck ban and the initial toll state before the truck ban

5 Conclusion

This study proposes a bi-level problem to optimize the freeway tolling strategy for different types of trucks following the truck ban policy in CBD. To do this, an MTA model is developed where passenger vehicle and truck travellers are assumed to choose routes based on the CNL model. The traffic cost functions are also designed to model the link costs of different types of vehicles by simultaneously considering the impacts of different types of vehicles on link capacity, the differences in gasoline consumption, and toll prices. The bi-level problem developed upon the MTA model seeks to optimize the freeway toll rates of different types of trucks to reduce the TTC of vehicles in the network. It also places a hard constraint on the increase in trucks' travel costs to reduce the resistance to the truck ban policy. To solve the bi-level problem, a globally convergent solution algorithm is also designed in this study. Application of the proposed model in Ningbo, China, found that the optimal tolling strategy can reduce the TTC of the network by 8.5%. Further, the trucks' travel costs only increased by 1.15% following the truck ban policy because the optimized truck toll rates can effectively encourage them to use alternative freeways with discount toll prices to compensate for their loss.

The proposed method offers several practical applications for traffic planners. First, it can be used to determine differentiated toll rates for various types of trucks, helping to mitigate the adverse effects of truck ban policies on freight mobility. Second, beyond truck ban scenarios, the method applies to situations involving long-term freeway closures—planned or unexpected—such as structural failures or extended maintenance by optimizing freeway toll rates across vehicle types. Third, it can be employed to balance traffic loads within the freeway network by adjusting toll rates for different vehicle categories, thereby alleviating congestion and improving overall traffic flow.

For future research directions, first, this study does not consider the impacts of vehicles' weight on their toll price. In China, the toll price is computed by vehicle type and weight (including the weight of goods). A new multiclass traffic assignment model and corresponding tolling problem will be designed by factoring vehicle weight distribution based on real-world data. Second, this study assumes that all trucks cannot access the CBD after the truck ban. To meet daily demands in the CBD area, some trucks with "green license plates" can access the CBD at certain times (e.g., after 10:00 pm). A time-varying tolling strategy will be designed to address this issue.

Data Access Statement

Data supporting this study are not publicly available due to commercial restrictions. The data will be shared on reasonable request. Please contact the corresponding author at yangbr2024@163.com.

Author and Contributor Statement

Bingjie Yang: Conceptualization and Methodology, Investigation and Writing—Original Draft, Visualization, Methodology, Writing—Review & Editing.

Use of AI

During the preparation of this work, the author did not use AI.

Acknowledgements

I thank my colleagues at Ningbo Development Planning Research Institute for their helpful comments and suggestions.

Conflict Of Interest (COI)

There is no conflict of interest.

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