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Optimal roadworks schedule in multi-agent transportation models

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Abstract

In this paper we consider the problem of the optimal roadworks scheduling in the road traffic network optimization. The main goal of this research is to find the optimal order of roadworks and minimize their negative impact on the transportation network throughput.

In order to model the dynamics of a transportation system, we propose a large-scale, multi-agent vehicle routing modelling framework for traffic simulation and impact of roadworks on traffic throughput.

We present a quick and effective heuristic algorithm for streamlining the road repairs. Finally, the numeric experiment in a real-world setting is conducted. The obtained results indicate the flexibility of the proposed algorithm, which can be effectively applied by decision makers who need to carry out roadworks with limited time and resources.

1 Introduction

Road transport has become one of the most important branches of the global economy due to a rapid growth in the number of private vehicles in the world over the last few decades. Although constant development of the transport industry yields a lot of benefits for modern societies, it also creates new challenges in the fields of traffic networks designing and urban planning because of the increasing level of the traffic congestion. Goodwin (2004) defined congestion as the impedance vehicles impose on each other, due to the speed-flow relationship, in conditions where the use of a transport system approaches its capacity. The economic costs of the increased traffic have turned out to be one of the biggest concerns of current traffic management systems. According to Inrix (2018), American commuters spent averagely 97 hours in traffic jams in 2018, which costed the country around $87 billion ($1348 per driver). Such a large material impact inclined scientists to address this issue in their researches. Dewees (1979) estimated the cost of highway traffic jams as an externality in terms of economic efficiency and to some degree equity due to differences in congestion imposed by different travel modes. Victoria Transport Policy Institute (2016) published a framework incorporating congestion costs borne by individuals in a travel time and vehicle operating costs. In the paper the formula to measure the congestion impact has been introduced based on roadway volume to capacity ratios (V/C). On the other hand, Choi et al. (2013) focused in their research on the negative social aspects of road bottlenecks and indicated that traveling in the crowded streets reduces well-being of the commuters.

One of the biggest challenges for overloaded road systems is maintaining its efficiency during traffic disruptions caused by random accidents or planned in advance roadworks. Hyari and Kandil (2010) distinguished following impacts of work zone activities: safety, mobility, economic, environmental, user as well as contractor's maintenance costs. Social and economic implications were also examined by Jamous and Balijepalli (2017) where the influence of roadworks on travel time reliability has been investigated separately for private cars and public buses. All of these factors need to be taken into consideration by cities governments as optimization algorithms can be successfully implemented in a decision support process. The example of such application was shown by Vallati et al. (2019). The Automated Planning approach using existing domain-independent planning engines has been proposed and validated as a decision support tool in roadworks planning.

Considering various impacts of roadworks, transportation systems can be modeled at a particular level of detail, depending on the purpose of analysis. Traffic networks can be designed on a microscopic (focused on individual traffic participants such as a single vehicle), mesoscopic (local clusters of vehicles and group decision making), or macroscopic (efficiency optimization of the entire traffic system) levels, (Sanderson et al. 2012). The consequences of scheduling long-term work zone activities from the perspective of traffic agencies and jurisdictions by applying a bi-level genetic algorithm (GA) has been examined by Hyari and Kandil (2010). On the more microscopic level, the route-changing behavior of road users and a scheduling model have been analyzed using Ant Colony Optimization (ACO) heuristic algorithm leading to the identification of a near-optimal roadworks schedule, (LEE and TSENG 2011).

The problem of roadworks scheduling (which is essentially a problem of finding an appropriate sequence of edges which will be temporarily removed from the graph representing a route network) falls into a broad family of discrete, combinatorial optimization problems. Unfortunately, in the case of road networks, experimenting with real life systems is costly and inefficient. Such issues have often been addressed through various simulation methods. Different models of dedicated simulators have been presented by Barcelo et al. (2005). A proper simulation might be employed as a surrogate of the real-world data and used as a tool for the scheduling of the roadworks. Simulating the traffic flow in the entire city is a costly process and finding an optimal schedule by brute force might take a significant amount of computational power which reduces its usability as a decision–supporting tool, (Görmer et al. 2011). Furthermore, computational models based on historical data are not able to properly utilize the learning mechanism and changing expectations of drivers. As a consequence, it is important to employ an efficient strategy of dealing with the optimization process. In such situation agent-based models are perfect solutions. A good example of such approach in the literature was described by Guo et al. (2019) where a multi-agent simulation was used for estimating route selections of travelers and solving road maintenance scheduling problem as Network Design Problem (NDP). The bi-level optimization model applied local branching algorithm and NetLogo simulation respectively in the upper and lower models. The proposed algorithm was validated with the simulation of a small network comprised of 11 links and 8 intersections and the numerical results provided good feasibility and a valid reference for decision makers.

Agent-based simulations can be effectively applied for macroscopic as well as microscopic modeling. On a macroscopic level illustrations of such optimization in traffic management systems can be found in works by Carpenter and Mehandjiev (2010) and Hernández et al. (2002). However, in our analysis we focus purely on a microscopic simulation of individual traffic participants and their route selections in the face of existing roadworks. Applications of multi-agent programming for microscopic simulations have been well-documented in the literature as artificial agents naturally represent single traffic participants, (Bazzan and Klügl 2013), (Adler and Blue 2002).

In this paper we propose a new scheduling tool dedicated for roadworks optimization with the potential applications in traffic management. Most of the available frameworks consider a problem of road repairs as a traffic flow optimization assignment (from the point of view of road users) and not as a task of finding the best roadworks schedule minimizing traffic disruptions. One of the applicable frameworks, presented by Calvert et al. (2010), focuses on the consequences in travel time due to changes in traffic demand and roadways capacity in the face of the road repairs. The main practical function of that model is providing pre-trip travel information. Roadworks planning for various workzone setups to evaluate the best approach can be recognized as a natural scope extension of the framework. Furthermore, the algorithm proposed by Calvert et al. (2010) is partially based on historical data. Our solution introduces an agent-based road traffic simulator to better reflect current real-world conditions. Subsequently we propose an efficient algorithm for solving large scale problems of scheduling road maintenance works. Proper combination of a traffic simulator and a dedicated heuristic method allows us to present a novel approach to deal with these types of problems.

The major contribution of our paper can be summarized as follows. Firstly, we establish a new, multi-agent, large-scale, vehicle routing modeling framework for simulation of transportation system suited for modeling of the roadworks schedule. In contrast to the approach of Guo et al. (2019), we test and validate the model and its scalability in a real-world scenario. Secondly, we propose a fast and easy to implement heuristic optimization algorithm of finding the optimal sequence of roadworks. Vallati et al. (2019) in their paper minimized the total duration of roadworks by implementing the proper schedule of maintenance works without investigating its effects on traffic participants. The goal of the proposed algorithm is to plan road repairs in order to minimize traffic disruptions at any time during the roadworks. Our approach is dedicated for solving scheduling optimization tasks and is easily scalable to successfully deal with real-world large roadworks problems. Lastly, we validate the proposed approach showing it usability for practical scenarios.

The remainder of the paper is organized as follows, In Section 2 we define the discrete optimization problem. In section 3 we describe the optimization approach and the analysis framework. Subsequently, we present the experiment design and results of our simulations. Finally, we present the conclusions and future directions of this work in Section 4.

2 Problem Statement

The goal of this section is to formulate the mathematical description of a transportation network and roadworks optimization model. Let us start by presenting the list of symbols used throughout the text in Table 1.

Table 1. List of symbols used in the paper

Variable	Explanation
$G = (V, E)$	graph representing transportation network consisting of a set of vertices (intersections) V and a set of edges E (road sections)
K	population of agents
k	a traveling agent, $k \in K$
N	total number of agents ($N = \|K\|$)
v	a vertex, $v \in V$
e	an edge $e \in E$
s	a path in a graph $s = (v_1, \dots, v_j)$; $(v_i, v_{i+1}) \in E$
s_{AB}	a route (path) between vertices A and B
d_e	the length of an edge (road section) e
t_e	time of traversing section (edge) e
n_{eu}	number of agents at route / edge e at the time u
N_e	capacity of route / edge e
$\mathcal{V}_e$	commuter's velocity at edge e
$\mathcal{V}_{max}$	maximum commuter's velocity at an edge e
$\mathcal{V}_{min}$	minimum commuter's velocity at an edge e
$\bar{t}$	total average time spent commuting by a single agent
$\tilde{E}$	a set of edges planned for repairs
$\tilde{e}$	a single edge under repair
$\tilde{E}_u$	a set of edges planned for repairs at the time u
Z	total number of roads to be repaired at the time u
$\tilde{G}_u$	a transportation network under repairs at the time u
$\tilde{t}$	vector of all available time slots for road repairs
$\mathbf{r}$	permutation of the set $\tilde{E}$
x_{ru}	the edge number r is being repaired in the period u, $x_{ru} \in \{0, 1\}$
p	probability of acceptance of degradation of the solution quality
I	maximum number of iterations

2.1 *Transportation Network Model*

Let a transportation network be presented as a strongly connected, weighted, directed graph $G = (V, E)$, where V denotes a set of vertices and E stands for a set of edges. Furthermore, with edge $e \in E$ three types of weights are assigned:

- d_e — the length of an edge (road segment) e
- v_{min} — the minimum commuter's velocity
- v_{max} — the maximum commuter's velocity

For simplicity, we will assume further in the text that the road system has a homogeneous quality of routes and hence the maximum and the minimum commuter velocities are equal for edge in the graph. We will denote this minimum and maximum vehicle speeds by v_{min} and v_{max} respectively.

Let the set K be a population of agents who are commuting using the transportation network. An agent follows a route s consisting of n vertices defined as:

$$s = (v_1, \dots, v_n) \quad \forall e_i \equiv (v_i, v_{i+1}) \in E \tag{1}$$

When we consider a set of possible agents' routes between points A and B we denote such a set as S_{AB}:

$$S_{AB} = \{s: v_1 = A \ \wedge \ v_n = B\} \tag{2}$$

In the model we assume that for every agent k ($k \in K$) starting point A_k and destination point B_k have been predefined. The agent considers routes $s_{AB} \in S_{AB}$. Drivers are assumed to be rational and prefer the fastest possible routes. However, the travel time between A and B, namely $t(s_{AB})$, depends on decisions of other traffic participants as well as speed limits on specific road sections, thus not always the shortest route. As time spent in the traffic is the only cost of traveling in our model, a rational agent, from all possible routes between points A and B, will always choose the fastest one. We will denote the fastest route between nodes A and B as s^*_{AB}, however, since agents are always traveling the fastest route, we can omit the star (*) and simply write s_{AB}.

We can denote the edge i of the route s_{AB} as $e_i(s_{AB})$ and define it as:

$$e_i(s_{AB}) = (v_i, v_{i+1}) \in s_{AB}. \tag{3}$$

Commuter's velocity v_e (for a given edge e) is not constant because it depends on the number of vehicles traveling that particular edge. We calculate the travel speed conditioned on the road load with the classic equation defined by Gerlough and Huber (1976) that has become one of most popular approaches across many research models - eg. see (Rampf et al. 2023), (Lee et al. 1998), (Vasirani and Ossowski 2014). The velocity has been defined as follows:

$$v_e = (v_{max} - v_{min}) \times (1 - \frac{n_e}{N_e}) + v_{min} \tag{4}$$

where n_e is the number of drivers driving a particular edge and N_e is a total theoretical capacity of that edge. Note that this equation ensures a minimum speed of v_{min} where full route capacity is reached and on the other hand when only a single vehicle drives the route the speed is slightly lesser than v_{min}.

Each edge of the graph has fixed length d_e representing the distance required to travel. The actual time t_e required to travel an edge e depends on the traveling speed that is:

$$t_e = d_e / v_e \tag{5}$$

.

The value of n_e/N_e can be interpreted as relative traffic load of the edge e. Hence, we calculate the travel time as:

$$t_e = \frac{d_e}{v_{max} - (n_e/N_e)(v_{max} - v_{min})} \tag{6}$$

It can be clearly seen that the travel time inversely depends on the the relative traffic load.

Total agent's time required to travel the route s_{AB}, $t(s_{AB})$ is calculated as follows:

$$t(s_{AB}) = \sum_{e_i \in \{(v_i, v_{i+1});\ (v_i, v_{i+1}) \in s_{AB}\}} t_{e_i}. \tag{7}$$

Considering that the time of travel (see Equation 6) depends on the decisions of other agents, we assume the following sequential approach to allocate all agents: at the beginning the first agent selects the fastest route assuming that no one is traveling in the route system. The second agent includes in theirs decisions the traffic "generated" by the first one and each subsequent agent considers in their decision making the traffic induced by all previous agents by calculating the velocity value v for edge e using the Equation 4 (and hence the travel timer presented in Equation 6). The proposed approach matches the situation where each agent uses a mobile phone app to plan theirs trip and looks at an up-to-date traffic at the moment of time when the trip starts. Therefore, we assume that the proposed simplification allows our model to capture the reality with a reasonable level of details.

For a given transportation network $G = (V, E)$, set of agents K, their sources and destinations A_k, B_k, we define the total average time spent commuting as:

$$\bar{t} = \frac{\sum_{k \in K} t(s_{AB})}{N} \tag{8}$$

The total travel time $\bar{t}$ is an aggregate of all sequentially optimal decisions assuming random activation sequence for agents traveling within the graph. Since the time $\bar{t}$ depends on the structure of G we will sometimes denote it by $\bar{t}(G)$.

2.2 Roadworks Optimization Problem Formulation

Having a transportation network $G = (V, E)$ we have defined the travel times for all agents. Now let us consider a situation where there is a sequence of edges $\tilde{E} = (\tilde{e}_1,\ldots,\tilde{e}_z)$ that needs to be renovated. For simplicity we assume that it takes one unit of time to renovate a single edge (it would be possible to achieve different renovation times in the proposed model by placing the same edge to the multiset $\tilde{E}$ more than once provided that the ongoing road repairs will be completed without any interruptions.

Let $T = \{1,\ldots,\tilde{t}\}$ be the set of available time slots for repairs and $r = 1,\ldots,z$ is the identifier of edge $\tilde{e}_r$ to be repaired. We define a decision variable $\mathbf{x} = [x_{ru}]$, $x_{ru} \in \{0, 1\}$, where $x_{ru} = 1$ means that the edge number r will be repaired in the period u, $u \in T$. We assume that the edge must be repaired exactly once over the time period T and hence:

$$\sum_{u \in T} x_{ru} = 1 \qquad \forall r \in 1,\ldots,z. \tag{9}$$

Hence, x is a repair plan for all edges scheduled for the repair $\tilde{E}$. We will denote the repair plan for a given time period $u \in T$ as $\tilde{E}_u$:

$$\tilde{E}_u = \{\tilde{e}_r: \quad r \in 1,\ldots,z \quad \wedge \quad x_{ru} = 1\}. \tag{10}$$

The transportation system $\tilde{G}_u$ under repair at the period $u \in T$ is defined as follows:

$$\tilde{G}_u = (V, E \setminus \tilde{E}_u) \tag{11}$$

that is, $\tilde{G}_u$ is simply the graph G with edges $\tilde{E}_u$ removed at the period $u \in T$ (unavailable in the period u due to roadworks).

In the paper we are considering the two goal functions for the optimization problem:

$$f(\mathbf{x}) = \max_{u \in T} \bar{t}\left(\tilde{G}_u\right) \rightarrow \ min \tag{12}$$

and

$$g(\mathbf{x}) = \sum_{u \in T} \bar{t}\left(\tilde{G}_u\right) \rightarrow \ min \tag{13}$$

both models are subject to the following condition:

$$\tilde{G}_u \qquad \text{is strongly connected} \tag{14}$$

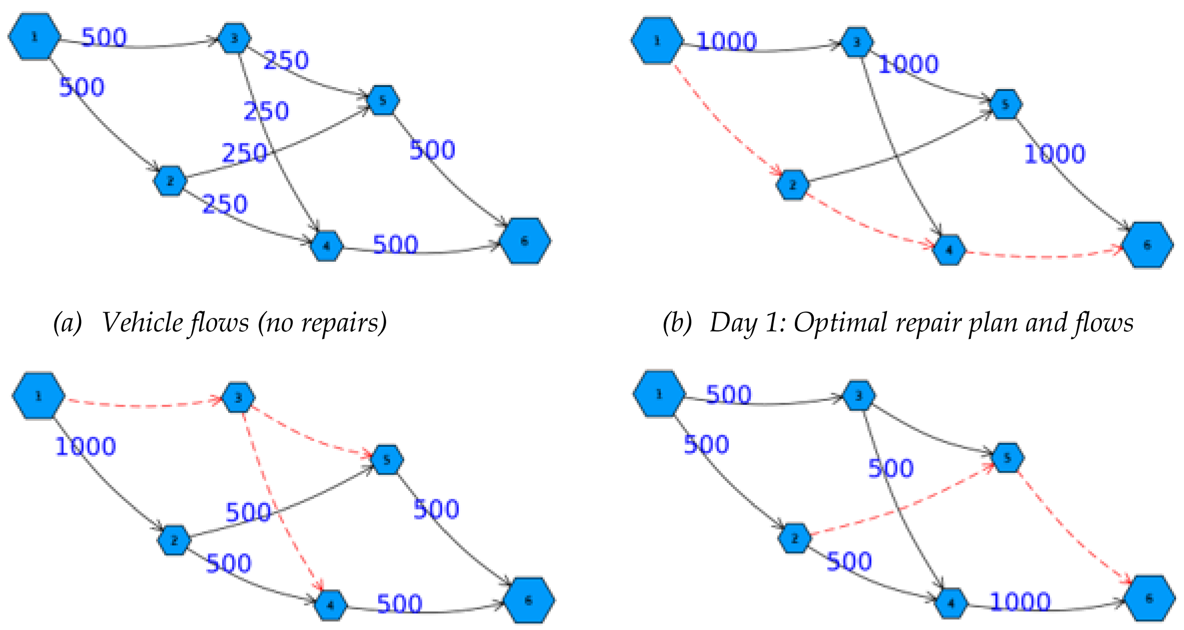
The condition in Equation 14 is required so it is still possible to travel around the city even during roadworks. There is no point in the optimization of the roadworks schedule in cases where a closure of one road would cause a total stop of all traffic due to lack of any possible detours. The optimization goal is to find such a repair plan $\mathbf{x} = [x_{ru}]$ that minimizes the maximum time spent by commuters in the traffic in any period $u \in T$. This naturally means that we try to find such allocation of repairs that are least disruptive for the transportation system at any given time. Therefore, we can assume that the predefined path sets for all agents are still connected in the graph $\tilde{G}_u$ as otherwise the optimization result would approach infinity which certainly could not be the optimal solution of the problem.

Note that the specifications of both models whose goal functions are defined in Equations 12 and 13 are invariant to permutations of periods $T = \{1, \ldots, \tilde{t}\}$. We assume that the only goal of the decision maker is to reduce nuisance of the roadworks over the entire considered period. This assumption significantly reduces the search space of our problem. In a real-life scenario this gives a freedom to the decision maker to layout the plan with regard to secondary criteria (such as seasonal traffic changes) - this process is however beyond scope of the model considered in the paper. The starting date of each work zone project will significantly impact network performance. In the extreme scenario, with roadworks commencing at the same time, the capacity of the entire road network could be reduced to a very low extent.

In the subsequent subsections we will solve how the proposed model can be solved in small scale using the MINLP (Mixed Integer Non-Linear Programming) approach and how it can be addressed in large scale with a heuristic approach.

2.3 *Toy Example and MINLP Formulation*

In order to illustrate the problem, let us consider a simple example of a transportation network presented in Figure 1a. The directed graph representing the road network consists of 6 vertices (representing intersections) and 8 edges (representing road lanes). Note that the edges (3, 4) and (2, 5) do no intersect (e.g. there could be a road overpass). In this example we assume that 1000 vehicles are traveling from the vertex 1 to the vertex 6, the travel times are calculated according to the Equation 6. In numerical computations we assume $v_{min} = 10$ and $v_{max} = 100$. Figure 1a shows vehicle flows when no repairs are being carried out (that is flows that minimize the total travel times in the network). The goal is to find the optimal schedule of roadworks in that minimizes the time spent in the traffic.



(a) Vehicle flows (no repairs)
(b) Day 1: Optimal repair plan and flows
(c) Day 2: Optimal repair plan and flows
(d) Day 3: Optimal repair plan and flows

Figure 1. Vehicle flows between nodes 1 and 6 for on different days before the optimal repair plan is started

For this example, we can formulate the MINLP (Mixed Integer Non-Linear Programming) model for the problem. Let us define the binary decision variable x_{ru}, where r is the identifier of the edge to be repaired u is the period of time. Since we need to model dependencies between nodes in the path we will denote $e = (i, j)$, to represent the edge between nodes i and j. Further, we will use the notations $(i, j) \in E$ and $e \in E$ interchangeably. In this formulation for simplicity, we assume that the cars are starting from only two locations. Let A be a start node and B be a destination node. Hence, an edge starting at the node A and ending at a vertex i can be denoted as (A, i). By μ we denote the total flow from node A to node B, which is constant for all days and all cars.

$$\sum_{i:(A,i)\in E}(n_{(A,i),u}) = \mu \qquad \forall u \in T \qquad (15)$$

$$\sum_{i:(i,B)\in E}(n_{(i,B),u}) = \mu \qquad \forall u \in T \qquad (16)$$

$$\sum_{j:(j,i)\in E}(n_{(j,i),u}) = \sum_{j:(i,j)\in E}(n_{(i,j),u}) \qquad \forall i \in V \setminus \{A,B\},\ \forall u \in T \qquad (17)$$

$$n_{e,u} \le N(e) \qquad \forall e \in E, \forall u \in T \qquad (18)$$

$$\sum_{u\in T}(x_{e,u}) = 1 \qquad \forall e \in \tilde{E} \qquad (19)$$

$$\sum_{u\in T} x_{e,u} = 0 \qquad \forall e \in E \setminus \tilde{E} \qquad (20)$$

$$n_{e,u} \le (1 - x_{e,u})\mu \qquad \forall e \in E, \forall u \in T \qquad (21)$$

The constraint in equation (15) ensures that the total flow from the starting node A is equal to μ at each day. The constraint in equation (16) ensures that the total flow to the ending node B is equal to μ at each day. The constraint in equation (17) ensures that the traffic flow is conserved at each node. The constraint in equations (19) and (20) ensure that each edge designated for the repair is repaired exactly once. The constraint in equation (21) ensures that the flow is not allowed on the edge that is being repaired at the time period u.

Note that the proposed model formulation will not allow the transportation network on any day to be disconnected in such a way that the travel between A and B would have been impossible.

With the above constraints we can formulate the objective function as follows:

$$f(\mathbf{x}) = \max_{u\in T}\left(\sum_{e\in E}\frac{d(e)\cdot n_{e,u}}{v_{max}-\frac{n_{e,u}}{N(e)}(v_{max}-v_{min})}\right) \to min \qquad (22)$$

This function minimizes the total time on the worst day. Note that the goal function 2.3 can be easily used in the actual MINLP solver by replacing it by using a standard approach of replacing it by a set of inequalities $\sum_{e\in E}\frac{d(e)\cdot n_{e,u}}{v_{max}-n_{e,u}(v_{max}-v_{min})/N(e)} \le \theta$ for $\forall u \in T$ combined with the objective function $\theta \to min$.

If we want rather to minimize the total travel time of all vehicles in the network, we can use the following objective function:

$$g(\mathbf{x}) = \sum_{e\in E, u\in T}\frac{d(e)\cdot n_{e,u}}{v_{max}-\frac{n_{e,u}}{N(e)}(v_{max}-v_{min})} \to min \qquad (23)$$

We have implemented the MINLP model with the objective function $f(\mathbf{x})$ in Equation (22) utilizing the Julia programming language and solve it with the Juniper.jl (Kröger et al. 2018) solver combined with the Ipopt solver (Wächter and Biegler 2006) run via the Ipopt.jl Julia library.

For the sample toy problem we assume that all roads needs to be repaired over the time horizon $T = \{1, 2, 3\}$. A sample optimal repair plan for a sample toy problem has been presented in Figures 1b, 1c and 1d.

Unfortunately, for the specified MINLP problem the computational time grows exponentially. The number of possible combinations of repair plans surges significantly along with the size of the considered optimization problem, even if we assume that our model is invariant to permutations of periods $T = \{1, \dots, \tilde{t}\}$. For an example the presented repair road plan for 6 vertices and 8 edges takes 1.8 second to compute, but when we scale up the problem to 8 vertices and 13 edges the time increases to 10.5 seconds and for slightly larger problem of 10 vertices and 19 edges the time is already 45 second.

The computational complexity challenge arises to the fact that the presented model is NP-hard. This can be shown by comparing the presented problem to other problems which are known in the literature to be NP-hard: In case of the objective function $f(\mathbf{x})$ (Equation 2.3), we could simplify the model by assuming a constant marginal increase of travel times due to decision $x_{e,u}$ (that is planning repairs of an edge e on a particular day u). The mathematical formulation of such simplified problem would have been an equivalent to a "identical-machines scheduling problem" with fixed number of machines (days) and the goal of minimizing the maximum completion time (sum of delays on a given day). This type of the problem in the literature is known to be NP-hard — see (Garey et al. 1976). On the other hand, when considering the objective function $g(\mathbf{x})$ (Equation 2.3), the mathematical formulation is an equivalent of the bin packing problem with concave costs of bin utilization (where days are bins with limited capacity and edges are items) which is again known to be NP-hard — see (Li and Chen 2006).

The propose MINLP representation is simplified in many ways. Firstly, only one origin and destination point are considered. Secondly, we assumed that all edges need to be repaired over the time period T (note however that those two assumptions could be lifted by making the MINLP representation more complex). Thirdly, we assumed that all agents make their decision at the same time on such way that they minimize the global objective function — which is not the case in the real transportation systems where each decision maker optimizes their own path looking at the current traffic situation. Finally, it does not allow for more complex traffic flow modeling mechanism to be included in the optimization process.

In the next section we demonstrate a heuristic that can be used mitigate those limitation and makes it possible to identify a repair plan for a problem in a larger scale. The heuristic is also more flexible than a MINLP formulation and can be integrated with a stochastic simulation model that can used to evaluate the quality of a given solution $\mathbf{x} = [x_r]$ on the traffic flows.

2.4 *Heuristic algorithm*

In this section we present a an efficient heuristic for road repair planning. Our goal is to optimize the roadworks plan regardless of the total number of repaired roads or the maximum number of roadworks conducted at the same time. Therefore, we use a heuristic approach in our algorithm to minimize the objective functions $f(\mathbf{x})$ and $g(\mathbf{x})$ defined in equations 12 and 12 respectively.

In order to tailor our heuristic algorithm to the needs of the optimization task, we can consider the following assumptions in the process. First of all, we may ignore the condition that $\tilde{G}_u$ is connected in our solver. The reason is that if it is disconnected the solution will certainly not be optimal. In particular, the road between points A and B does not always exist in the disconnected graph, and in such case, we will assume $\bar{t}_{AB} = \infty$.

Next, we assume in the simulator that the functions $f(\mathbf{x})$ and $g(\mathbf{x})$ are monotonous, i.e. adding a road for maintenance work in some period without changing any other assignments does not decrease average travel time in this period. In some rare cases removing a link in a network can actually make the network become more efficient which in the literature is described as the Braess's Paradox (Braess et al. 2005). In our experiment we included checks to make sure that the functions $f(\mathbf{x})$ and $g(\mathbf{x})$ are, in fact, monotonous. Note that in the text and numerical we assume that the travel times can be estimated by Equation 6, however the proposed heuristic will work with other travel estimates such as those acquired by running discrete-event micro-simulations.

Furthermore, we note that the components of $t(\tilde{G}_u)$ from which the value of the functions $f(\mathbf{x})$ and $g(\mathbf{x})$ are computed are independent. If for some period the set of road segments repaired in a given period does not change, then the value of $t(\tilde{G}_u)$ does not change. Also the value of $t(\tilde{G}_u)$ does not depend on actual location of the period u in time. This means that in order to effectively calculate $f(\mathbf{x})$ and $g(\mathbf{x})$ the most efficient method is to memoize already calculated values of $t(\tilde{G}_u)$ and not rerun simulation for them. We use this property in the design of the solver to significantly improve its speed.

Finally, as the considered simulation is stochastic, we use common random numbers technique and averaging for calculation of$t$ $(\tilde{G}_u)$. Technically we use a set of seeds $s_1, \ldots, s_k$ and calculate the average of k simulation runs producing $t(\tilde{G}_u)$ for the same seeds for each period.

After establishing the assumptions of the model in the previous subsection, we can now focus on building the dedicated algorithm. In order to properly capture the attributes of the simulation environment, our algorithm needs to be fast and flexible, hence the application of the heuristic in such a case is reasonable. Our approach is based on a classic Simulated Annealing (SA) method which is effective even for big complex problems. While our algorithm is heuristic, it uses a multi-start approach, as in this way it is easiest to parallelize it over many machines. An implementation detail is that even if doing the calculations in parallel the memoized values of $t(\tilde{G}_u)$ are shared across different nodes to improve the efficiency of the solution calculation.

The single thread of optimization uses two steps: construction of an initial solution and simulated annealing search in the neighborhood of the initial solution. Both steps, in the heuristic employed, use the fact that we are interested to minimize the maximum average traveling time over the periods.

In the initial construction step we take a random permutation of the set $\tilde{E}$, denote it $\mathbf{r}$. Initially we assume that no roads are under construction in any period. In step $i \in \tilde{E}$ we take road segment $\tilde{e}_{ri}$ and assign it to the period $u \in [\tilde{t}]$ that currently has minimum average travel time, splitting ties at random.

In the simulated annealing search phase we start from the initial solution found in the first step of the procedure. We use a standard approach with a predefined maximum number of iterations of the search I and probability of acceptance of degradation of the solution quality p (which in our model remain constant over time). The crucial part of our procedure is definition of the neighborhood of the current solution which is not trivial. We define it as follows. When generating a candidate solution we take one road segment and randomly assign it to a new period. In order to improve the efficiency of the search we select road segments for moving with probability proportional to an increasing function of $t(\tilde{G}_u)$ (so the road segments in periods that have longer travel times have a higher probability of being chosen). On the other hand the selection of target period for the road segment is random but the probability of choosing the period is proportional to a decreasing function of $t(\tilde{G}_u)$ (so that we are more likely to select periods that have short travel times).

It is possible to find an exact solution effectively in our model only for relatively small problems. For all other cases, due to the fact that we are taking maximum or sum of average travel times and that each period is independent from other periods, the following procedure can be applied. The number of all possible solutions is Stirling number of the second kind $S(z, \tilde{t})$, which grows very fast (a rough approximation is $\tilde{t}^z/\tilde{t}!$); still for small values of z and $\tilde{t}$ they can be enumerated. The computational complexity can be partially, we can reduce that number by calculating only $z!\left(\sum_{i=1}^{z-\tilde{t}+1} 1/\big((z-i)!\,i!\big)\right) < 2^z$ values of $t(\tilde{G}_u)$ and memoizing them. However, for larger z values the cost of evaluation of the objective function is not a bottleneck in this process We can also note that this search procedure can be actually further pruned using bounding if the assumption that $t(\tilde{G}_u)$ is monotonous holds. Namely if we have a valid solution $f(\mathbf{x})$ or $g(\mathbf{x})$ and we have some $\tilde{G}_u$ for which $t(\tilde{G}_u)>f(\mathbf{x})$ then we can prune all supersets of $\tilde{G}_u$ from the search space as such solutions cannot be optimal. In practice in the optimal solution each set $\tilde{G}_u$ contains roughly $z/\tilde{t}$ elements (the allocation of roads to periods is approximately balanced)—and this fact greatly reduces the search space; the number of needed objective function evaluations is reduced to the order of $(e\tilde{t})^{z/\tilde{t}}$.

Input: $\tilde{t}$, $\tilde{E}$, I, p, solution buffer

Output: vector of road segments assignments to periods and number of period; solution fitness

Data: G, reference simulation time with no roadworks

1. **r** := random permutation of the set $\tilde{E}$;
2. assume that no roads are under construction in any period ;
   `/* Initial step */`
3. **for** $i \in \tilde{E}$ **do**
4. take road segment $\tilde{e}_{\mathbf{r}_i}$ and assign it to the period $u \in [\tilde{t}]$ that currently has minimum average
   end
5. calculate solution fitness $f(\mathbf{r})$ and memorize elements of all periods $u \in [\tilde{t}]$ in the solution buffer ;
6. best solution fitness = $f(\mathbf{r})$;
   `/* Simulated Annealing search */`
7. **while** $j < I$ **do**
8. define neighbourhood of the solution $\boldsymbol{r}$: assign one road segment $\tilde{e}_{\mathbf{r}_i}$ to a new period $u \in [\tilde{t}]$ with the probability proportional to an increasing function $f(\mathbf{r})$; period $u \in [\tilde{t}]$ selected with the probability proportional to a decreasing function $f(\mathbf{r})$;
   end
9. generate solution $\mathbf{r}'$ in the neighborhood of **r** ;
10. **if** $f(\mathbf{r}') < f(\mathbf{r})$ or $p < random(0, 1)$ **then**
11. memorize new assignments of roads segments from all periods $u \in [\tilde{t}]$ in the solution buffer ;
12. $\mathbf{r} = \mathbf{r}'$
    end
13. **if** $f(\mathbf{r}) <$ *best solution fitness* **then**
14. best solution = **r** ;
15. best solution fitness = $f(\mathbf{r})$
    end
16. $j = j + 1$; return best solution, best solution fitness

Algorithm 1. Heuristic algorithm for optimal assignment of road segments to roadworks periods

3 Numerical experiments

In this section we present experiment design along the results of numerical experiments.

3.1 Framework design

The simulation framework, built to validate our optimization approach, has been written in Julia programming language (version 1.8.3), (Bezanson et al. 2017). Our framework is comprised of the road traffic agent-based simulator and the roadworks optimization heuristic algorithm which can be easily used to measure the impact of individual road closures on the entire network. The full source code is available with open-source license on Github (https://github.com/OpalskiM/RoadsConstructionOpt).

3.2 Road traffic simulator

The details of the road traffic network have been described in the subsection 2.1. Our agent-based simulator facilitates analyzing road traffic on any transportation system. For testing purposes we downloaded the map of the downtown of New York from the open-source website OpenStreetMap (https://openstreetmap.org). Parsing the OSM (OpenStreetMap) object and transforming it to the form of the graph is possible by using another Julia package – OpenStreetMapX.jl, (Filipowski et al. 2021), (Kaminski et al. 2020).

Each simulation starts with generating a population of artificial agents moving around the city. In our experiment we create 2000 agents with predefined starting points A and destination points B within the map boundaries. Drivers choose their paths sequentially and the order of their actions remains the same in every simulation. The velocity on each road depends on the congestion induced by all other traffic participants. Therefore, for agents' routing calculation we apply commonly used in such situations A-star algorithm which can be seen as an extension of the classic Dijkstra routing algorithm (Kala and Warwick 2015). The final result of our simulation is the total traveling time of all agents. Our base simulation assumes that no roadworks carried out in the transportation network and we will treat it as a reference scenario to compare it further with simulations of roadworks. The presented model specification allows us to easily exhibit travels of all agents, and thereby we can measure traffic loads on every single edge in the graph. The example of this functionality is illustrated in the Figure 2, where we show a number of vehicles moving between two particular nodes (road junctions) in the simulation. Such data can be valuable in the process of identifying the most congested areas in the city.

Figure 2. Example of a traffic load measurement on a particular edge in the reference scenario

3.3 *Roadworks optimization algorithm*

The simulation framework outlined in the previous section has been incorporated in our optimization task of roadworks scheduling. To illustrate how our algorithm works let us consider following real-life problem. A decision maker needs to renovate z roads over a certain period of time and faces a problem of finding the best order of road closures to minimize traffic delays. Such a case can be quickly solved using our algorithm to search the set of all feasible permutations and return the optimal sequence of roadworks.

Table 2. Parameters used for numerical experiments

Parameter	Symbol	Value
Number of agents	N	2000
Number of renovated roads	Z	{5,10,15,20,25,30}
Number of roadworks days	$\tilde{t}$	{1,2,3,4,5}
Maximum number of iterations	I	1000
Probability of acceptance of the solution quality degradation	P	0.001

Following parametrization presented in Table 2, 100 different randomized scenarios have been generated for every combination of two key factors - total number of roadworks (from 5 up to 30) and number of roadworks periods (from 1 to 5). We run so many simulations in order to assess the impact of each of these components on the efficiency of the road transportation system measured by the minimum delay in agents travel times during roadworks. For simplicity we consider that each roadwork takes 1 unit of time, however it is an easily modifiable parameter of the model. To effectively test our algorithm the roads for renovations have been randomly selected in direct proportion to the traffic loads on the edges as closing roads with minimal (or no traffic at all) would not cause any significant disruptions in the network. Figure 3 visualizes all roads that have been chosen for roadworks in our testing scenario.

In the initial step of each simulation we randomly assign all renovated roads to one of the roadworks batches. Next, we optimize our solution by performing up to 1000 iterations, with the fixed probability of moving to the worse solution set to 0.001. After doing all iterations the algorithm returns the optimal order of the roadworks. The result of our optimization is the difference in total traveling time in the roadworks simulation in comparison to the reference scenario with no conducted roadworks. The detailed description of our optimization algorithm has been provided in subsection 2.2.

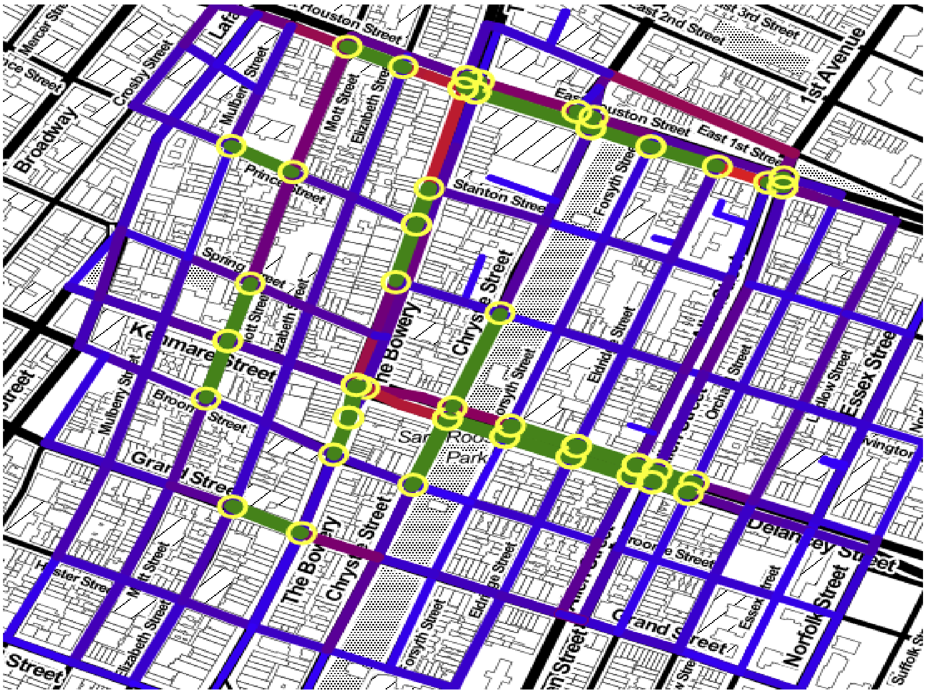


Figure 3. Streets closed for roadworks in the simulation (colored in green)

3.4 Experiment results

In this article we consider two functions: $f(\mathbf{x})$ minimizing the maximum delay on a given day of roadworks, and $g(\mathbf{x})$ that optimizes the total time of delays caused by all repair periods. Depending on the preferences or priorities the decision maker may be interested in both maximum and total delay times. As these are two exemplary functions, the traffic regulator can use a cost function that is their weighted sum in a decision making process. However, for a more complete understanding of how our model works, we consider these two functions separately in the following subsections and at the end we will compare the results of both objective functions.

3.4.1 Maximum delay

In line with our expectations the temporary closures of busy urban streets considerably affects the road traffic performance. The objective function $f(\mathbf{x})$ described in the previous section has been designed to minimize maximal delay in agents travel time caused by any batch of roadworks. After performing all simulations in our experiment we checked how the number of renovated roads affects the road network efficiency. For every value of number of renovated roads z (from Table 2) the impact on traveling times of agents has been measured depending on the number of $\tilde{t}$ roadworks periods. The results have been presented in Figure 4. Average travel delays have been compared to the reference scenario with no carried out roadworks. Regardless of the number of roadworks days deterioration of the network performance surges with the increase of number of renovated streets. Moreover, the consequences of such repair works are much more acute when roadworks are split into the reduced number of subperiods. In our experiment renovating 30 roads in 5 periods would worsen the total result (in comparison to reference scenario) on average by 9% while splitting work into just 2 periods would increase travel delays by almost 25%.

Figure 5, demonstrates how the traffic network is sensitive in our model to number of roadwork periods in relation to various levels of number of renovated roads. The combined effect of those crucial parameters and their interaction with each other are shown in the heat map. Average delay in agents travel times is dependent on combination of both parameters and the more roadworks is carried out during one period, the more congested become the traffic. The log scale has been used for colors to properly capture the properties of the model. Such analysis can be beneficial for decision makers, who often have limited resources or time, to adaptively balance trade-off between carrying out roadworks over a prolonged time and significant increase of peak traffic congestion within a short time.

Lastly, we performed the analysis how the density of the roadworks may impact the traffic disruptions. We defined roadworks density as the average distance between all renovated streets in each roadworks batch. Our findings have been summarized in Figure 6. Average delay in agents travel time rises with the increase of distance between repaired roads. It means that the more scattered the roadworks are around the city, the more congested as a consequence the traffic network becomes. Furthermore this correlation occurs regardless on the number of roadworks periods. Such results may confirm our intuition - if all roadworks are performed at the same time close to each other, it may be easier for drivers to find just one detour in their travels. The problem arises when restrictions on traffic exist in different parts of the city and it is much more difficult to find a decent workaround. These outcomes may also provide the decision makers with relevant indicators before any new road repairs.

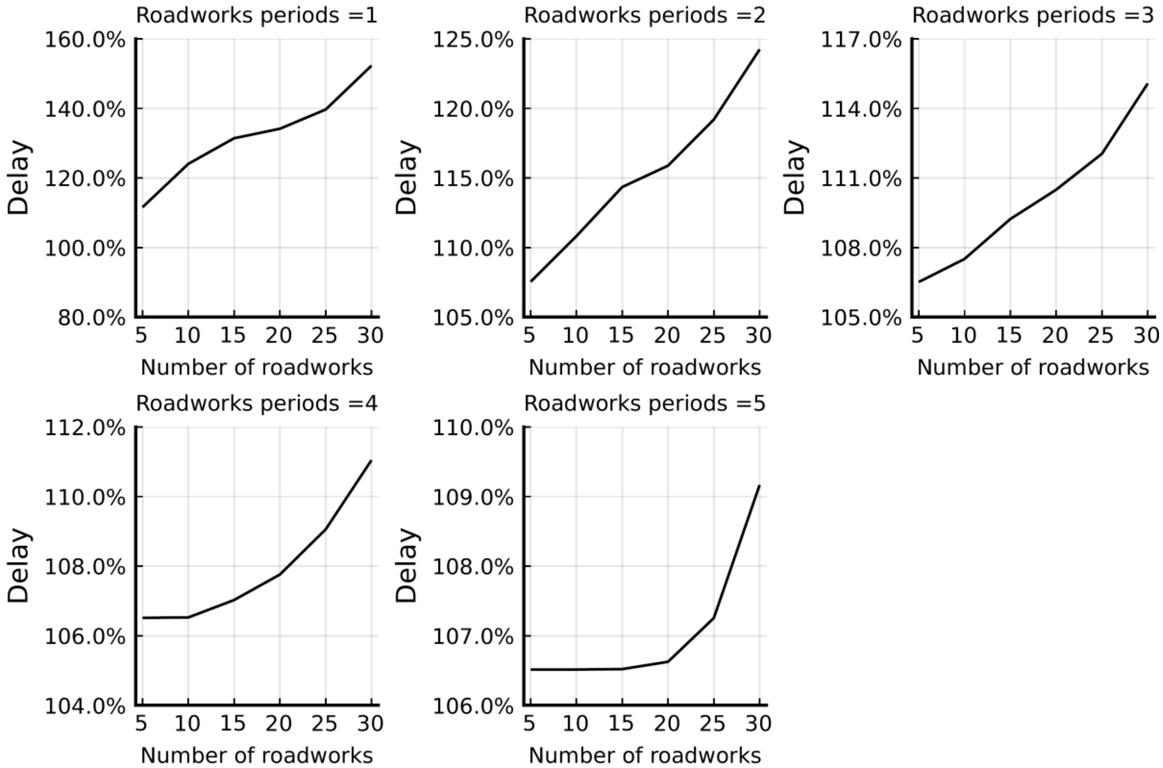


Figure 4. *Average delay in agents travel times depending on the number of roadworks*

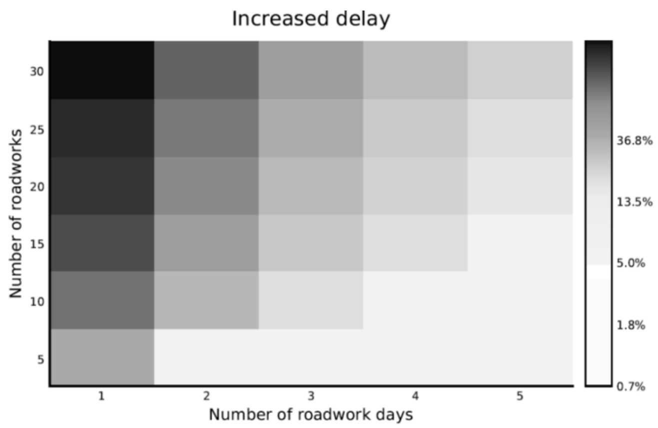


Figure 5. *The average delay in agents travel times depending on the number of road renovations and roadwork periods*

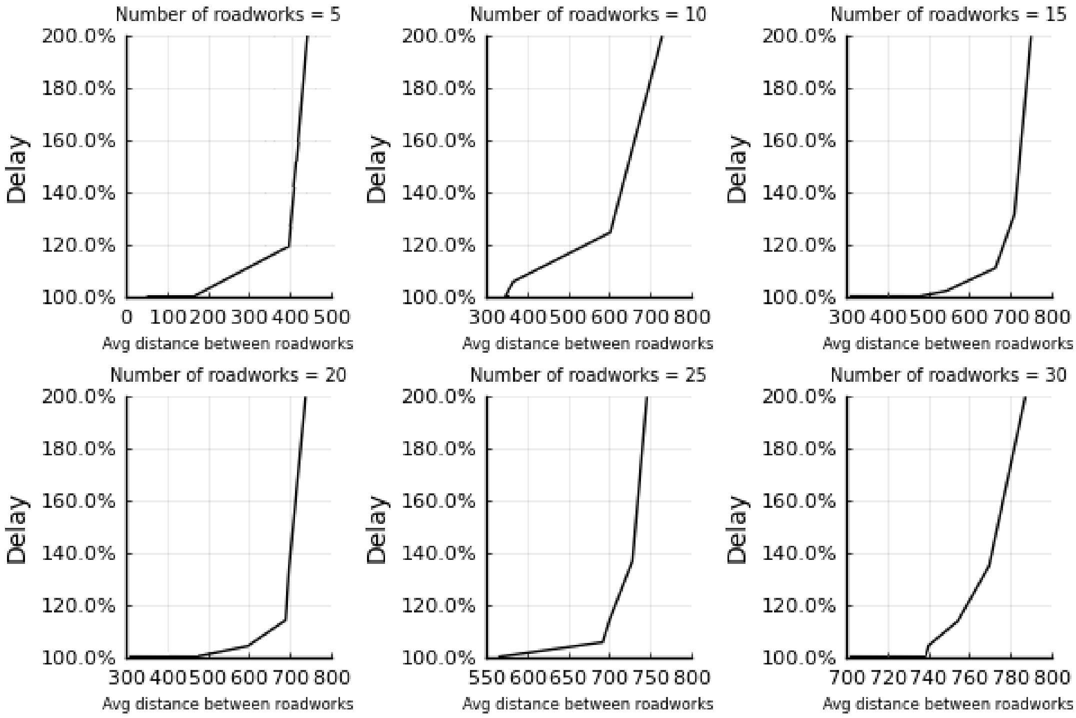


Figure 6. *The average delay in agents travel times depending on the average distance between roadworks*

3.4.2 *Total delay*

The heatmap presented in Figure 8, shows the sensitivity of the function $g(\mathbf{x})$ to the number of roadwork periods in relation to various levels of number of renovated roads. This sensitivity is noticeably lower than for the function $f(\mathbf{x})$ which can provide the decision maker with useful information. If the main priority is to reduce the total delay time, then the regulator should have more flexibility when setting the roadworks schedule and can more easily adjust the repairs timetable to external factors such as weather, weekends etc.

Lastly, in Figure 9 we have presented the impact of the roadworks density on the total commuter's delay. Interestingly, the relationship is not linear for a large number of renovated roads as the total delay is relatively high when the road repairs are a short distance apart. The minimal time of delays occurs with the moderate scattering of roadworks and in the case of large distances between renovations, the objective function begins to take on larger and larger values. It can be explained as follows. The heavy concentration of renovations in one critical location, which has significant implications for the entire city, considerably increases travel times for drivers. As the distance between renovations increases, there is the possibility of detouring the closed areas and traffic disruption decreases. However, the distribution of renovations throughout the entire city significantly adversely affects the traffic performance as all parts of the network become more congested.

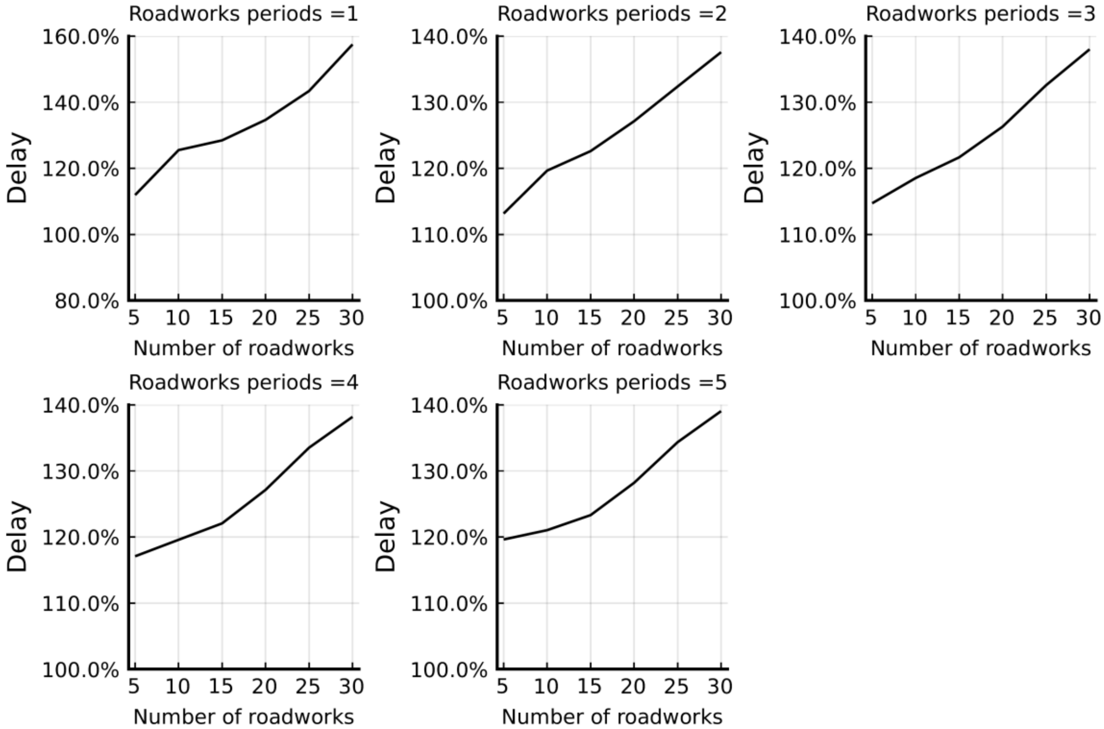


Figure 7. *The total delay in agents travel times depending on the number of roadworks*

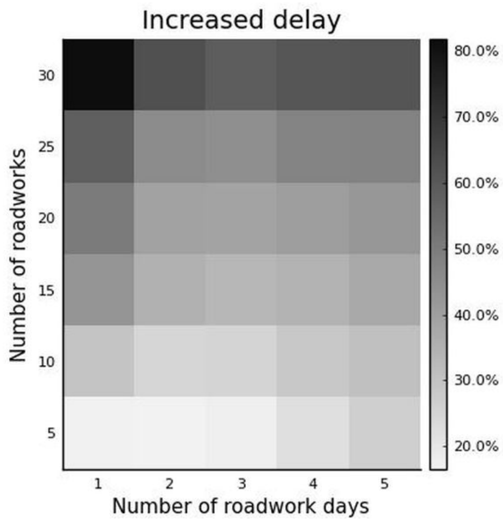


Figure 8. *The total delay in agents travel times depending on the number of road renovations and roadwork periods*

In parallel to the calculations presented in the previous subsection, we have also conducted the experiment for the $g(\mathbf{x})$ function and generated the similar graphs. This objective function has been designed to minimize the total delay in agents' travel times caused by all roadworks batches.

Similarly to the $g(\mathbf{x})$ function the total travel time of drivers increases for every number of roadworks (Figure 7) On the other hand, unlike the optimization of the maximum delay, increasing the number of maintenance days does not significantly improve the result of the $g(\mathbf{x})$ function. This is one of the main differences between these two functions. Even if the renovations are divided into 5 days, delays in drivers' travel times accumulate each day, causing a significant total delay caused by the road repairs.

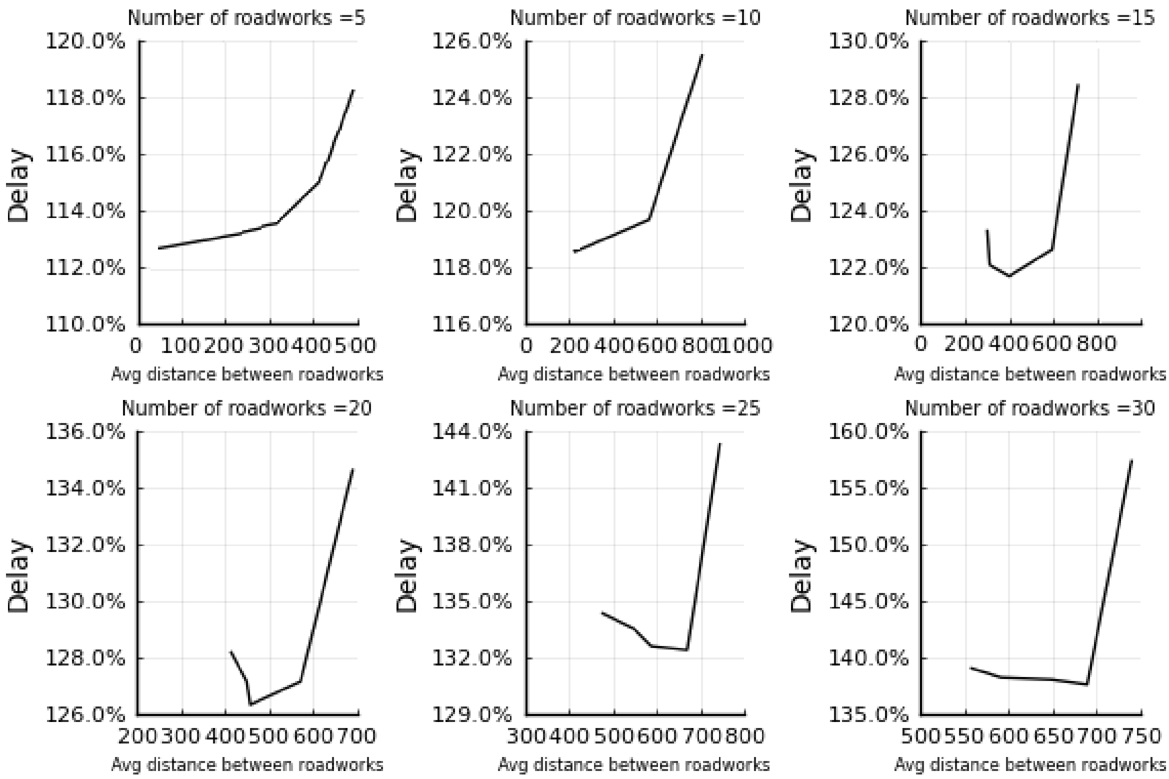


Figure 9. *The total delay in agents travel times depending on the average distance between roadworks*

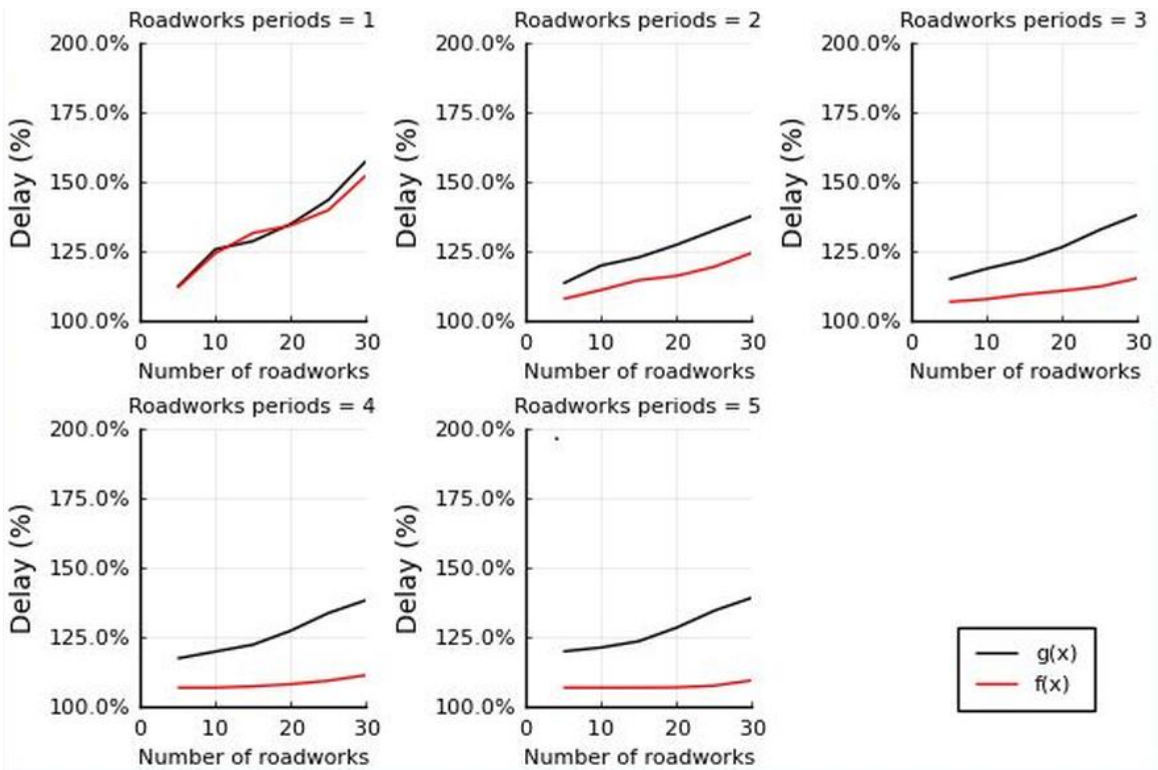


Figure 10. *Comparison of the maximum delay (f(**x**)) and the total delay (g(**x**)) depending on the total number of roadworks and roadworks batches*

3.4.3 Discussion

The regulator can favor either minimization of total travel times (minimize the total societal cost of renovation) or the maximum time (minimize inequalities in the access to the transportation network during the renovation). Depending on preferences, the regulator can make a decision based on a comparison of the two functions that we presented in previous subsections. In the Figure 10 we have shown a comparison of the two functions depending on the number of road renovations and roadworks periods. The more repairs are divided into more renovation periods, the greater the efficiency differences obtained by the two functions. Such a comparison can provide a great deal of insight for traffic regulators. In the case of repairing 30 roads, the decision maker can decide whether he prefers a maximum traffic delay of 10 % on a single day (function $f(\mathbf{x})$) and divide the repairs into 5 periods, or whether the most important factor for him is the total traffic deterioration (function $g(\mathbf{x})$). Then he can decide to carry out the repairs on one day and increase the total single delay by 50%.

4 Concluding Remarks

As the number of private cars in the streets is growing much more quickly than the road system capacity, the issue of road traffic network efficiency became the matter of importance for city managers. One of the ways to increase its performance is minimizing traffic disruptions by optimizing road maintenance works. In this paper we presented a novel optimization algorithm for roadworks scheduling. We proposed a scalable simulation framework dedicated for traffic optimization and a heuristic algorithm, written in Julia programming language, created for finding the best order of road renovations. Numerical tests were employed to validate our model. Results indicated that our approach is well-suited for such problems and enables to easily evaluate the traffic disruptions determined by various parameters such as number and duration of the roadworks or the density of the repairs. Information gathered from using our algorithm can be very valuable for decision makers who face a challenge of effective planning of road renovations. Our tool is flexible and may be easily adjusted to the specific needs of traffic network administrators.

In our analysis we focused on adjusting the repair plan in such a way so as to minimize traffic disruptions whereas existing research such as presented by Calvert et al. (2010) focuses mainly on the optimization of the traffic flow from the point of view of road users. For more complete we have also compared the results with the function minimizing the total delay of all roadwork days. Additionally, we propose a new tool which can be used by decision makers as a support tool in roadworks planning in a similar way as the approach described by Vallati et al. (2019). The algorithm proposed in this paper can be successfully applied to solve similar problems as Network Design Problem (NDP) presented by Guo et al. (2019). The biggest advantage of our approach is the traffic simulator dedicated for multi-agent scenarios and its scalability. Such feature facilitates analyzing and optimizing many scenarios with different assumptions very quickly as well as observing the behavior of single agents in each simulation. Moreover, due to its efficiency, we can easily apply our algorithm to simulate road traffic and schedule roadworks even in the biggest, most complex metropolises.

Future model extensions may include more complex, dynamically changing environment with the different value of time for commuters. Furthermore, the introduction of smart connected cars will inevitably change the way how the transportation networks operate as the intelligent vehicles will be able to acknowledge roadworks in advance and adjust their routing. Implementation of such a technology can lead to the globally optimized system where cars communicate with each other and collectively minimize total time spent in traffic.

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