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Network vulnerability analysis based on the overall and inequity impacts of the distribution of the added travel time to the network users

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Vulnerability has been a major concern in the performance evaluation of transportation networks. In the concept of vulnerability, the key step is to identify the critical link, which are the ones susceptible to severe operational degradation caused by any type of failure. Several studies have been devoted to this issue by introducing indicators that illustrate the network's operational degradation's overall impact. However, the impact of the interrupted network on users can be further evaluated using the inequity perspective. Here, we present a method to assess network vulnerability to operational degradation based on spatial inequity impacts. The importance of a link is determined by calculating the Gini-coefficient of the distribution of added travel time to the users when the link is disabled. Furthermore, the overall impact of link failure is calculated based on the total extra travel time. The final link's importance is determined by the non-dominated sorting method based on the Pareto optimality concept considering both overall and inequity objectives. Measures quantifying overall and inequity impacts of link failure allow planners to determine how this influences the disadvantaged distribution and help them make decisions associated with maintenance plans that consider link failure's equity impact.

***Keywords**: network vulnerability, equity analysis, critical infrastructure, urban road network, pareto optimality, non-dominated sorting.*

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1. Introduction

The concept of vulnerability is an important measure to evaluate the performance of transportation networks. Although there are many different definitions for this concept, all of them have a common base: How does a transportation network encounter events which may jeopardize network performance (Berdica, 2002; de Oliveira et al., 2016; Mattsson & Jenelius, 2015). These events can be human-made or natural disasters (Humphrey, 2008; Koetse & Rietveld, 2009) and lead to the endangerment of many transportation network infrastructures such as bridges and tunnels. System failure, disaster situations, or even traffic congestion can cause degraded network performance, and this can have significant social and economic impacts (Taylor & Susilawati, 2012). In one well-known definition, the vulnerability in the road transportation system defines as "a susceptibility to incidents that can result in considerable reductions in road network serviceability" (Berdica, 2002). Accordingly, vulnerability assessment to maintain transportation network performance is an important transportation planning tool in the budget management and priority determination to act against transportation network performance reduction under the acceptable level. Indeed, it is essential for transportation planners to determine sections of the network which, if broken or damaged, would have a considerable negative impact on the performance of the network (Burgholzer et al., 2013).

Typically, vulnerability analysis tries to measure the overall network performance before and after a link failure while ignoring the inequity of the disadvantage distribution of network disruption among network users. Many research studies in the network vulnerability analysis have determined the criticality of the links in a network based on the link failure's overall impact, and to achieve this, several indicators have been provided (Balijepalli & Oppong, 2014; Berdica & Mattsson, 2007; de Oliveira et al., 2016; Jenelius et al., 2006; Nagurney & Qiang, 2007; Scott et al., 2006). While these studies give a general view of link closeness's effect on the network performance, the impacts of a link failure on travelers with different spatial distribution are limited in scope.

The transportation network's principal function is to provide accessibility for all members of society to different urban opportunities and services (Tahmasbi et al., 2019). Unequitable accessibility in an area is not consistent with the principles of society's sustainable development (Gudmundsson & Höjer, 1996). Therefore, when the consequences of the disruption in the network are evaluated in the context of vulnerability, the failure of a link should not severely affect some specific travelers and drastically degrade some regions' accessibility. Indeed, a network vulnerability assessment aligned with sustainable development goals should consider the inequity issue and the disadvantages imposed on the network users. The significance of this issue becomes more explicit when seen from the perspective of planners and policymakers. They are responsible for deriving decisions on where to direct funds to reinforce the network to potential threats (Gilbert et al., 2003; Taylor, 2017).

This study investigates transportation network vulnerability and critical links determination and assesses their impact on social justice, particularly horizontal equity. Little attention has been given to how changes in the transportation network, due to a link failure, affect different travelers in different locations. In one study, Jenelius (2010) shed light on considering user equity in link importance measure. He considered the link closure's equity aspect by calculating the coefficient of variation of the user's increased travel time during the closure. However, since the added travel time to the users, due to distribution in the network, might not follow the normal distribution, the use of the coefficient of variation is associated with caution (as it has been previously discussed that the coefficient of variation would not be an appropriate index to measure inequality if the data did not follow a normal distribution (De Maio, 2007)). Moreover, an arbitrary weighting has been applied to consider both efficiency and inequity impacts, which does not provide a straightforward method for policymakers to prioritize the network links. After that, Mattsson and Jenelius (2015) provided an overview of research about vulnerability and mentioned the lack of attention to the equity aspect.

This paper presents a new method to consider both the overall and equity impacts of the network's link failure. The importance of a link is determined by calculating the Gini coefficient index when the link is disabled, and network vulnerability is investigated. Also, the overall impact of a link failure is computed, and the final importance of the link is determined using the concept of Pareto optimality and the non-dominated sorting method based on two calculated objectives. The proposed method is examined on Isfahan transportation network data, specifically on bridges.

The remainder of this paper consists of six sections. In section 2, relevant studies about transportation network vulnerability and various proposed indicators are discussed. In section 3, the concept of spatial equity and its importance in transportation planning is expressed. In section 4, the proposed methodology is discussed, and network vulnerability is surveyed by calculating the Gini index to evaluate each link's importance from the perspective of spatial justice. The data used, its implementation, and results are expressed in sections 5 and 6, respectively. Finally, in section 7, the study's conclusion is presented.

2. Methods used in vulnerability analysis and related indices

To evaluate transportation network vulnerability, it is necessary to identify more important links to maintain network performance than the rest (Rupi et al., 2015; Sullivan et al., 2010). In fact, the failure of critical infrastructures has the most serious impact on the whole transportation network, and for authorities and policymakers, considering the critical infrastructures in general, especially the transport network infrastructures, is now an important concern (Murray & Grubesic, 2007).

By using an appropriate method to evaluate vulnerability, it is possible to improve transportation infrastructure maintenance planning, optimize budget assignment, and reduce socio-economic costs caused by events. In the past two decades, various indicators were provided to evaluate transportation network vulnerability. The classification of these indicators depends on the point of view that the vulnerability issue is seen. In one classification, Murray et al. (2008) proposed four basic typologies of network vulnerability approaches scenario-specific, strategy specific, simulation, and mathematical modeling. Scenario-specific approaches evaluate the impact of specific disruption scenarios (or a small set of them). In these approaches, based on the given information, the network's performance before and after the failure of a specific link is measured. Subsequently, the potential impact of the failure is assessed. Strategy-specific approaches evaluate the impact of losing a sequence of network links. In this approach, after ranking the network links or nodes according to their importance, the links are successively removed, and the network performance is assessed at each stage. Simulation assessment evaluates network vulnerability without any prior assumptions. In fact, many possible scenarios, such as the impact of the loss of one or two pairs of network links, are evaluated. Finally, several approaches had been developed through mathematical modeling to facilitate identifying the most vital network infrastructures.

From the evaluation procedure aspect, Mattsson and Jenelius (2015) separated vulnerability analyses into two topological and systematic traditions. Accordingly, in topological vulnerability analysis, a real transport network is represented in the form of an abstract network (graph), i.e., an ordered pair comprising a set of nodes (or vertices) and a set of links (or edges) that are connected to each other by shortest paths (Crucitti et al., 2006). The second tradition is system-based vulnerability analysis, where the transport system is analyzed through a simulation model based on the interaction between supply and demand. In the system-based approach, the analysis is mostly based on inelastic demand assumption, i.e., a link failure is as long enough to reach a new User Equilibrium (UE) traffic pattern but not so long to change demand distribution and mode choice significantly. In the system-based approach, many studies have concentrated on indicators obtained from traffic assignment simulation results and can be effective in analyzing the vulnerability of transportation networks. Balijepalli and Oppong (2014), in a general classification, divided indicators into two distance-based and cost-based categories. According to their definition, distance-based indicators are relevant to sparse regional networks, so drivers may need to take longer detours to reach their destinations if a link is blocked.

Several cost-based methods have been proposed for network vulnerability assessment and identification of the critical links. Cost-based indicators are based on the route with minimum cost for each origin-destination pair. Researchers developed various cost-based vulnerability assessment indicators such as generalized cost measure (Taylor et al., 2006), network efficiency/performance measure (Nagurney & Qiang, 2007), importance measure (Jenelius et al., 2006), Network Robustness Index (de Oliveira et al., 2016; Scott et al., 2006), and Network Vulnerability Index (Balijepalli & Oppong, 2014). Taylor et al. (2006) described a vulnerability analysis methodology for transport networks to determine the most critical links and examined this methodology in Australian National Transport Network. They introduced an index to calculate the change in generalized cost incurred when a link is removed from the network. This measure is dependent on the demand and the minimum cost of the route between each origin-destination pair. For assessing the criticality of a specific link, the amount of change in generalized cost when the whole network is in performance is calculated. This link's importance is determined by comparing the value of the generalized cost changes with index values calculated for other links. Similar to this index (Dehghani et al., 2014) used Vehicle Miles Traveled (VMT) index as the total distance travelled by all vehicles in the network. This index calculates by the summation of the product of link length and link volume over all the links of the network.

Nagurney and Qiang (2007) proposed a network performance measure that is used to evaluate the efficiency of a transportation network and the importance of its links. It was assumed that there exists a positive demand for all pairs of origin and destination nodes. The calculation of their index is based on the user equilibrium assignment model. According to their definition, after calculation of the network transportation efficiency measure, the importance of a network component is measured by relative network efficiency drop after that component is removed from the network. Importance Measure is another index based on the User Equilibrium (UE) assignment model presented by (Jenelius et al., 2006). In their research, it has been assumed that all drivers behave according to the UE principle, so they have to use a route that minimizes their travel cost (time). They also assumed that an event causes a link or a group of links to be completely disrupted or closed, so travelers must choose another route. For calculating Importance Measure, the change in travel cost for each origin-destination pair when one link has failed is calculated and weighted based on demand between origins and destinations pairs. The summation of these changes is considered as the importance of that link. Network Robustness Index (NRI) is proposed by Scott et al. (2006) and is defined as the change in travel time associated with rerouting all traffic in the system. This measure assumes that the disruption will cause complete closure of the link and drivers to follow user equilibrium in route choice. The NRI is based on a comparison of the total amount of time or trip cost of the network in situations with and without the link under analysis (de Oliveira et al., 2016). Also, Balijepalli and Oppong (2014) introduced the Network Vulnerability Index (NVI) that considers the serviceability and importance of each link on the network. They tried to make a measure that considers both partially and fully damaged roads in network vulnerability assessment.

In this paper, the most important methods and corresponding indicators, based on the system-based approach, for assessing the network vulnerability and identifying the importance of the links are briefly reviewed. A summary of the reviewed indices is presented in Table 1, including the formulation of indices and explanations and parameters of each one. These indicators mathematically describe the consequences of a link failure of the network. By applying these indicators, the overall impact of the occurrence of a disruption event in the network is computed. The common aspect of these methods is evaluating the system's performance through a cost measurement analysis where the most important links are those whose failure imposes the most cost to the system. While assessment of the performance of the transportation networks associated with disruptions in the network by measuring the cost imposed on the system is reasonable and important, it does not cover all aspects and goals associated with the role of the transportation network. Beyond the overall impact of the disturbance in the network, the inequity aspect of the disturbance in the network is also important.

Table 1. Summary of system-based vulnerability assessment indices

Authors	Index Name	Formulation	Explanations
(Taylor et al., 2006)	The change in generalized cost	$V^e = \sum_i \sum_j q_{ij} \Delta c_{ij}^e$	Δc_{ij}^e is the difference between the minimum cost of the route between *i* and *j* when the link *e* is intact and when it's removed q_{ij} is the demand between *i* and j
(Nagurney & Qiang, 2007)	Relative network efficiency	$\varepsilon = \dfrac{\sum_i \sum_j {q_{ij}}/{c_{ij}}}{N_k}$ $I(e) = \dfrac{\Delta\varepsilon}{\varepsilon}$	c_{ij} is the cost on the shortest used path(s) with positive flow between each OD pair N_k is the number of all OD pairs that is equal to $k(k-1)$
(Jenelius et al., 2006)	Importance measure	$Importance(e) = \dfrac{\sum_i \sum_j w_{ij} \Delta c_{ij}^e}{\sum \sum w_{ij}}$	w_{ij} reflects the significance of *i* and *j* OD pair in relation to the other pair
(Scott et al., 2006) and (de Oliveira et al., 2016)	Network Robustness Index	$NRI_e = \sum t_i' \times v_i' - \sum t_i \times v_i$	t_i (t_i') is the time or cost of the link *i* in the undisturbed network (in the situation without link *e*) v_i (v_i') is the traffic volume of link *i* in the undisturbed network (in the situation without link *e*)
(Balijepalli & Oppong, 2014)	Network Vulnerability Index	$NVI = \sum_n [(\frac{v_i^{before}}{r_i^{before}})\ t_i^{before}] - \sum_n [(\frac{v_i^{after}}{r_i^{after}})\ t_i^{after}]$	r_e is the serviceability of link *e*, which is the total available capacity of the link divided by standard hourly link capacity

From the perspective of equity, limited research explored the network vulnerability by evaluating the inequity aspects of a link failure and its consequences on different individuals. In one single study, Jenelius (2010) presented a method to incorporate user equity aspects into a road importance measure. He combined two components: the total increase in travel time and disparity in distribution among individual users. The coefficient of variation was used as an equity measure; then, the weighted efficiency importance and equity importance of a link after normalization are combined into a single importance index.

3. The concept of equity in transportation

The realization of justice has always been one of the most important human objectives. Over the past decades, as an increasing proportion of the population lives in cities, equity and social justice in transportation issues are one of the most critical challenges of governments and authorities (Mercier, 2009). Equity in transportation refers to the equitable distribution of transportation impacts (benefits and costs induced by transportation) between travelers (Litman, 1996). Equity is also an important planning goal and a prerequisite for sustainable development, which balances economic, social, and environmental objectives (Litman & Burwell, 2006).

The objective of evaluating equity in urban and transportation studies is to avoid the policies that would be uneven across society (Tahmasbi & Haghshenas, 2019). Indeed, minimizing the system users' inequity should be taken into account in developing strategies for allocating resources. Applying social equity concepts to public administration allows authorities to assume that the effects of good management, efficiency, and the economy would be evenly and fairly distributed among the citizens. Currently, equity took its place along with efficiency and economy as the third pillar of public administration (Frederickson, 2010). Due to different points of view on equity, the objective of equity is often vague (Martens et al., 2012). In this regard, several practical approaches are provided to this concept, including horizontal equity, vertical with regards to income and social classes, vertical with regards to needs and abilities, and intergenerational equity (Litman & Brenman, 2012). Horizontal equity focuses on the equal treatment of people in equal circumstances (Bertolaccini & Lownes, 2013), while vertical equity is denoted as the unequal treatment of unequals (Crampton, 2010).

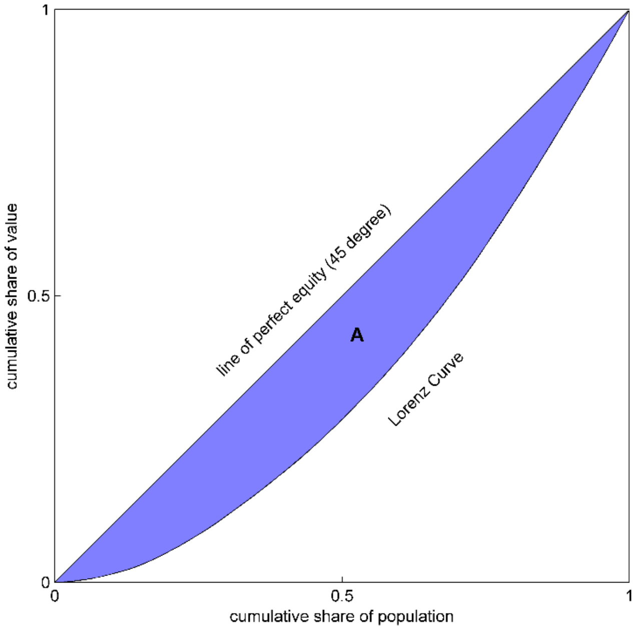


Figure 1. Lorenz curve and Gini index. The Gini index equals twice the area between Lorenz curve and the 45-degree line. The 45-degree line means perfect equality where the Gini index equals to zero.

To analyze equity, various measures are developed and applied like the Gini coefficient (Gini, 1921), variance, the coefficient of variation (Allison, 1978), and the Theil entropy index (Shorrocks, 1980). Even though these indices are adopted first to assess inequity in economic data, they can be adopted for any distribution in other fields (Ramjerdi, 2006). Gini coefficient has proven the most popular measure of inequity which, besides the economic aspect, has been widely used in many transportation planning applications (Bertolaccini & Lownes, 2013; Currie et al., 2009; Levinson, 2002; Li & DaCosta, 2013; Litman, 1996; Litman & Brenman, 2012; Lucas et al., 2016; Mackett et al., 2008; Preston, 2009; Tahmasbi et al., 2019). This index has a value between zero and one, where zero represents the most equitable condition (each person has an equal share), and one indicates wide inequality. Lorenz curve has an important role in calculating this index. The Lorenz Curve plots the cumulative proportion of the population against the cumulative proportion of the value of interest, in which the purpose is to assess its inequality. In this regard, a 45-degree line indicates perfect equality, and by any variation inequality in the data, the curve has deviated from this line (Figure 1). A point on the Lorenz curve represents *X%* of the population receives *Y%* of the target value (Bertolaccini & Lownes, 2013). The Gini coefficient reflects the ratio of the area of the gap between the Lorenz curve and the line of complete equality (a 45-degree line in the same space) over the total area under the line of complete equality (Shirmohammadli et al., 2016). The Gini coefficient can be approximated as (Brown, 1994):

$$G = 1 - \sum_{i}^{n} (Y_{i+1} + Y_i)(X_{i+1} - X_i) \tag{1}$$

where Y_i is the cumulative proportion of the variable of interest over i areas and X is the cumulative proportion of the population.

4. Methodology

In this paper, the network vulnerability is assessed by quantifying the importance of links within the network. Our focus is to measure the inequity of the disadvantage distribution because of the failure of a link among network users. Furthermore, a link failure's overall effect is calculated based on the total extra travel time added to the system. A comparison of the results will finally be presented. Our analysis is based on some assumptions about network and user behavior, which are the same as those used by previous vulnerability assessment studies (Balijepalli & Oppong, 2014; Jenelius & Mattsson, 2010; Mattsson & Jenelius, 2015; Murray-Tuite & Mahmassani, 2004; Scott et al., 2006). First, we assume that the interruption will cause a complete failure of the link. Another assumption is that drivers behave in user equilibrium, i.e., they choose a route for their trips that minimizes their travel cost. The interruption duration is long enough such that a new user equilibrium in the network is established. On the demand side, we assume that the link's failure does not significantly affect the travel demand (during the link's failure, the travel demand is considered constant). Also, we assume that each network link (bridges, here in our case study) has an equal failure probability, and the inherent vulnerability (fragility) of each link is beyond the scope of this research.

Spatial equity is evaluated in this paper, and we use the Gini coefficient to measure the inequity of the increase in user travel times. First, under the normal situation where all network links are up and running, for each *ij* origin-destination pair, the travel time, t_{ij}, is calculated based on the user equilibrium traffic assignment criteria. Then, under the scenario that the link *e* is removed from the network, the same OD demand matrix is assigned to the new network using the user equilibrium method. The new travel time, t^e_{ij}, is then calculated for all origin-destination pairs. $\Delta t^e_{ij} = t^e_{ij} - t_{ij}$ is the change in travel time for origin-destination pair *ij* when the link *e* is failed. Δt^e_{ij} represents how much extra time travel from zone *i* to zone *j* (q_{ij}) is incurred if the link *e* is failed from the network. The amount of travel time that is increased due to the failure of the link *e* can be calculated as follows:

$$T^e = \sum_i \sum_j q_{ij} \Delta t^e_{ij} \tag{2}$$

T^e represents the overall effects of the closure of the link *e*. In order to assess the inequity effects of change in travel time, the distribution of Δt^e_{ij} among travelers is considered. Different individuals are affected differently by removing a link from the network. Under the user equilibrium assumption, the failure of the link *e* may arise several possibilities for change in user travel time. Some users are not affected, and their travel time remains constant. Some are forced to change their route due to the failure of link *e* that was already on their route. Another group of travelers does not directly affect (because link *e* was not on their route), but they take a longer time to travel because of traffic added to their route. In fact, the failure of a link from the network has some direct effects, which cause some travelers to switch their routes. Additionally, indirect impacts may occur because of extra traffic added to other links. In this research, the Gini coefficient is used to produce an estimate of the inequity in the distribution of Δt^e_{ij} over the travelers between origin-destination pairs. For this purpose, consider the demand between origin *i* to destination *j* as q_k and $\tau^e_k=\Delta t^e_{ij}$ (the sub-index *k* is used to represent *i* and *j* OD pair) according to equation (1), the Gini coefficient to assess the inequity among the travelers can be calculated as:

$$G^e = 1 - \sum_{1} (Q_{k+1} + Q_k)(T^e_{k+1} - T^e_k) \tag{3}$$

where Q_k and T^e_k are the cumulative proportion of q_k and τ^e_k, respectively.

While the above procedure considers the equity impact of the occurrence of an interruption in the network, it still needs to determine whether the policymakers can use these findings to develop appropriate plans for reinforcing the network. Policymakers and planners need decision-support tools to assess the consequences of network degradation and identify network elements that are in need of support (Taylor, 2017). Indeed, the overall impact of a link failure represents the system's efficiency in case of the regular occurrence of a disruptive event. In organizing investment in urban infrastructures, relevant consideration should be both efficiency and equity (Bröcker et al., 2010; Monzón et al., 2013). Therefore, it is required to present a method to combine both efficiency and inequity impact of the occurrence of an interruption in the network.

After quantifying the overall and inequity impacts of degradation in the network, it is necessary to present a method to consider these two objectives simultaneously. Actually, the trade-off between overall and inequity impacts for assessing network vulnerability and determining the network's critical links is a multi-objective problem. The goal is to find a sorting method on which decision-makers can agree to determine the most important links. The concept of Pareto dominance allows a comparison of these two objectives and sort the results without adding additional preference information. This aids decision-makers in selecting from the decision space (Emmerich & Deutz, 2018). The sorting procedure is called non-dominated sorting, which compares the value of objectives and divides a solution set into a number of disjoint ranks (subsets) (Tian et al., 2017). There are several multi-objective non-dominated sorting algorithms that we refer to (Deb et al., 2002; Zarei & Rasti-Barzoki, 2019; Zhang et al., 2014; Zitzler et al., 2001).

In multi-objective problems and non-dominated sorting, the aim is to find (or approximate) a set of solutions that can improve one of the objectives without deteriorating the other objectives. This solution is called Pareto-dominated solutions. The Pareto optimal non-dominated sorting steps can be found in (Emmerich & Deutz, 2018) and (Bao et al., 2017). The result of applying this method on a discrete solution space generates the non-dominated fronts in sorted order. After this sorting, the same rank set solutions are considered equally important (Tian et al., 2017). A brief explanation of the non-dominated sorting is as follow:

Consider *f(x)= (f₁(x), f₂(x), …, f$_M$(x))* a set of *M* objective functions that we want to minimize (a similar approach can be developed if the objective is maximization) and *x* is the solution in the decision space. A solution ***X*** is said to Pareto dominate the other solution ***Y*** if both the following conditions are met (Tian et al., 2017):

$$\begin{cases} \forall i \in 1,2,\dots,M : & f_i(X) \le f_i(Y) \\ \forall j \in 1,2,\dots,M : & f_i(X) < f_i(Y) \end{cases} \tag{4}$$

Without the loss of generality, consider the solutions in the population P can be categorized into K divided subsets or ranks $P=\{F_1, F_2, \ldots, F_K\}$. To sort the population in these K subsets based on the non-dominated concept, the following steps are usually performed:

1. Initialize the index i to 1.
2. Assign all non-dominated solutions in population P to the subset (front) i.
3. Remove the solutions assigned to the subset i from the population P and set $i=i+1$.
4. If P is empty, then stop; otherwise, go to Step 2 and continue.

The algorithm first finds all non-dominated candidates from all studied scenarios and assigns them to the 1st Pareto front, which consists of the most important links, and then considers the remaining links and finds all non-dominated candidates from them and assigns them to the 2nd Pareto front. This process is repeated until all studied links have been assigned to a Pareto front set. This study uses this method to rank evaluated scenarios that we study in network vulnerability analysis and determine the most critical links. For this purpose, the value of overall and inequity impact indices is normalized between zero and one (max-min normalization); then, the non-dominated sorting algorithm is applied to sort the studied links in different importance categories. By applying this algorithm in the studied cases, we can achieve the final important ranking. Briefly, the procedure for computing the two inequity impacts and overall effects to assess the importance of the network link is shown in Figure 2.

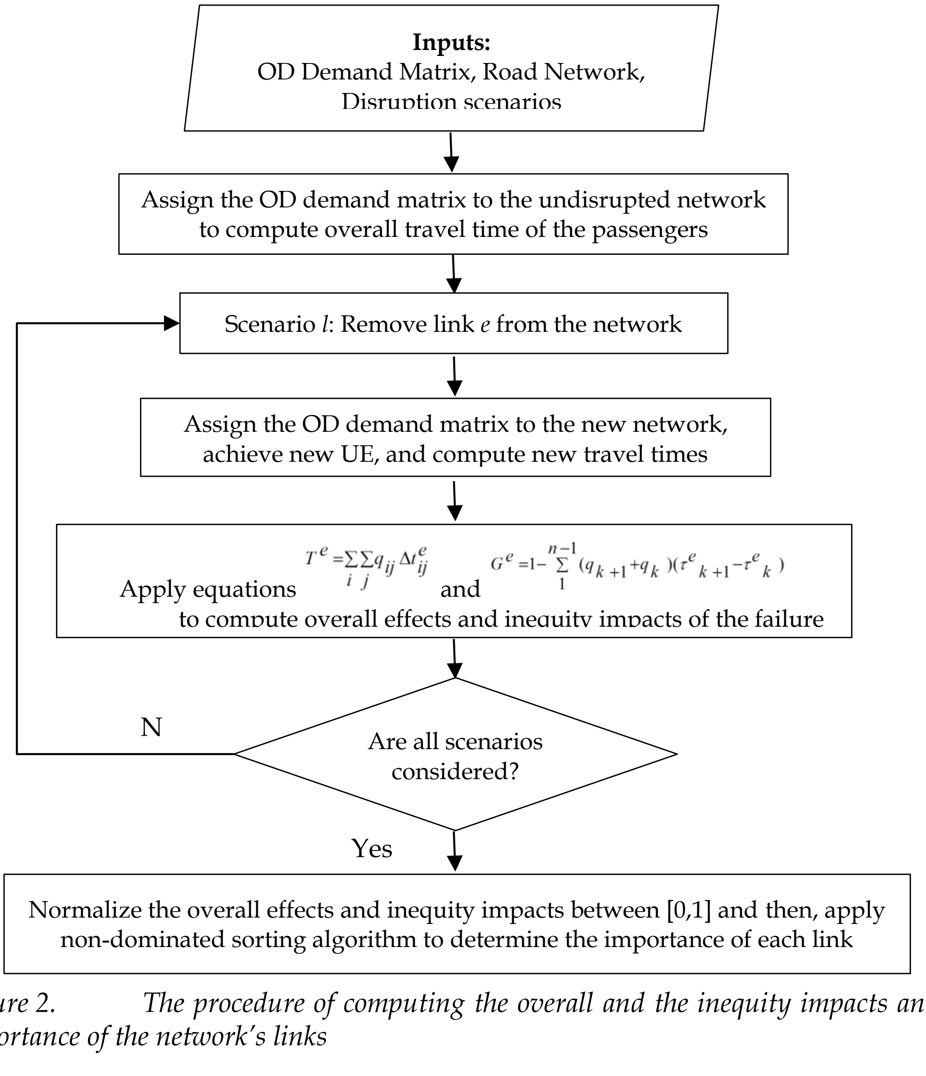


Figure 2. The procedure of computing the overall and the inequity impacts and determining the importance of the network's links

5. Case study and implementation

The methodology developed in section 4 is applied to the Isfahan road network. Isfahan is the third most populated city in Iran. It is a major cultural, commercial, and industrial hub. According to the 2016 national census, the population of Isfahan is near 2,000,000. The city's area is about 600 square kilometers, and the transportation network consisted of 186 Traffic Analysis Zone (TAZ), nearly 10000 links, and 3500 nodes. Bridges are essential links in any road network that are subjected to everyday traffic and vehicle loads. They provide vital access to different parts of the city that must be protected to ensure public safety and serviceability. Furthermore, from a structural perspective, bridges are the most vulnerable elements in transport networks, the damage of which may seriously affect transport mobility (Zanini et al., 2013). In the Isfahan road network, there is no exception to this principle. The presence of the Zayanderud River crossing through the city signifies the importance of bridges. Isfahan road network and its 86 bridges with traffic analysis zones are shown in Figure 3(a).

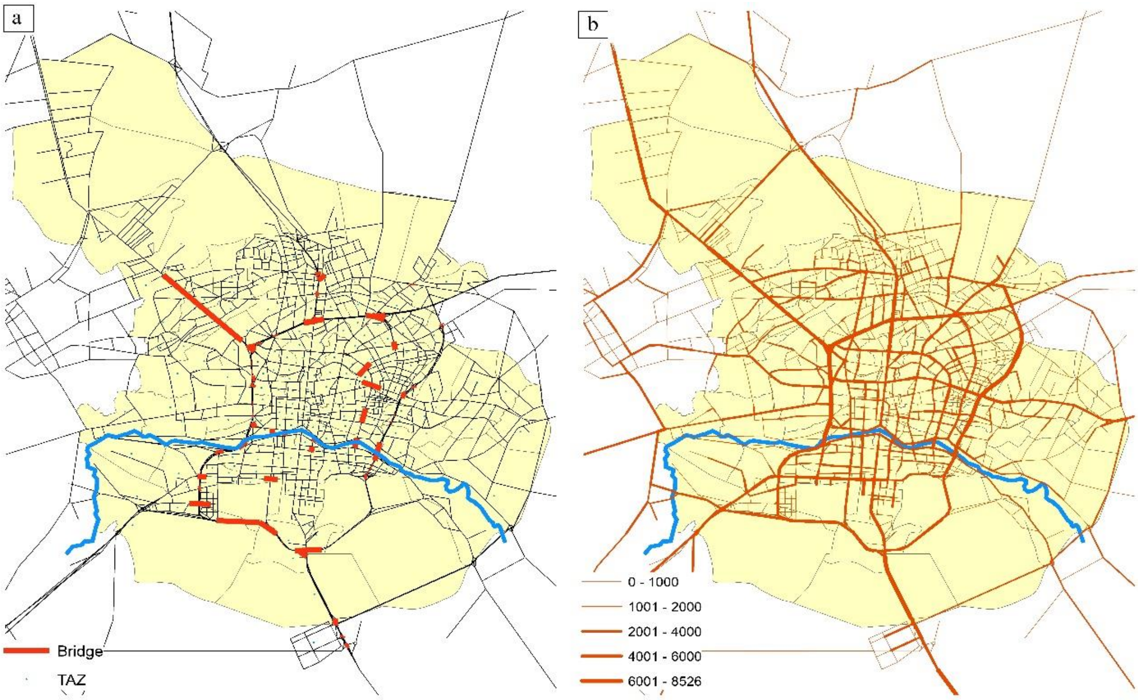


Figure 3. a) Isfahan road network and its bridges with traffic analysis zones. b) Peak hour link flow in the undisrupted road network.

The demand matrix for the trip assignment is the vehicle trip matrix in the morning peak hour. The demand matrix is derived from the people travel survey in the comprehensive transport studies of Isfahan. During peak hour, the majority of trips are for work and school-related. These trips are usually not changed when a disruption in the network occurs. This makes the assumption that inelastic demand is more logical and reasonable. On a normal day, almost 400,000 vehicular trips are made on the Isfahan road network during the morning peak hour. Figure 3(b) shows the user equilibrium flow during the peak hour on each link in the uninterrupted road network.

To implement the proposed method, we need to obtain the travel time between each origin-destination pair. Under the user equilibrium assignment method, in a congested network, the travel time on all used routes for each origin-destination pair is equal (Fisk, 1980). Using this principle, the travel-time cost for each origin-destination pair can be calculated. For this purpose, we used TransCAD 5 and ArcGIS Desktop 10.1. Using TransCAD, we apply the user equilibrium traffic assignment method to assign the OD demand matrix to the road network. The travel time of each link is subsequently calculated. In the next step, using the ArcGIS Desktop network analysis toolbox, the OD travel time matrix is computed based on the fastest path method.

6. Results

6.1 Comparing overall and inequity impact results

The critical links (specifically the bridges) are identified by quantifying the magnitude of inequity using the Gini coefficient. Figure 4 shows the minimum and the maximum Gini coefficient and the corresponding Lorenz curve, which are shown the most and the least important studied links (bridges). The minimum Gini coefficient is 0.5512, which indicates the distribution of change in travel time among the users caused the least inequity. The maximum Gini coefficient is 0.6793. In this case, maximum inequity is occurred among the users due to added travel time to individual trips.

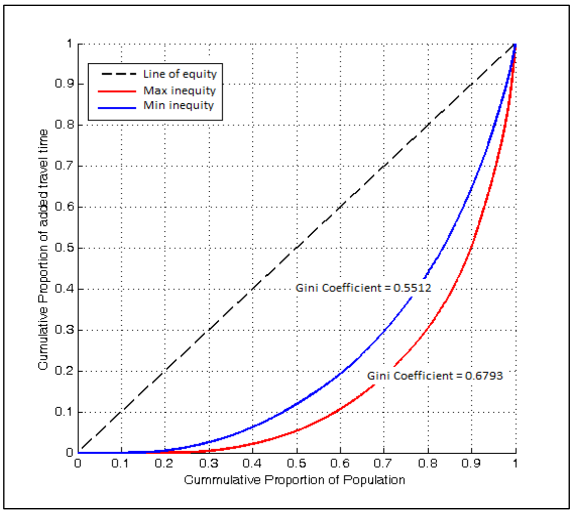


Figure 4. *Maximum and minimum Gini coefficient and the corresponding Lorenz curves*

Figure 5 shows the top 10 most important bridges from two perspectives: overall efficiency effects and inequity. It can be seen in Figure 5 that nine of the top ten critical bridges are the same based on two different ranking strategies but with different rankings. The assessment of inequity impact and the overall effects of link failure have produced different results. Table 2 represents both overall and inequity impacts for the shown links in Figure 5. As seen in Table 2, both average and overall effects have the same trend; however, the Gini index values demonstrate the inequity impact follows a different importance ranking order. As shown in Figure 5 and Table 2, the most important link from the overall impact perspective is in third place in inequity impact ranking. For some links, both overall and inequity impacts give approximately the same ranking; however, for others, a clear difference can be seen in the importance ranking from these two objectives. For example, the third most important bridge in overall impact ranking is the tenth place when it is ranked by inequity impact.

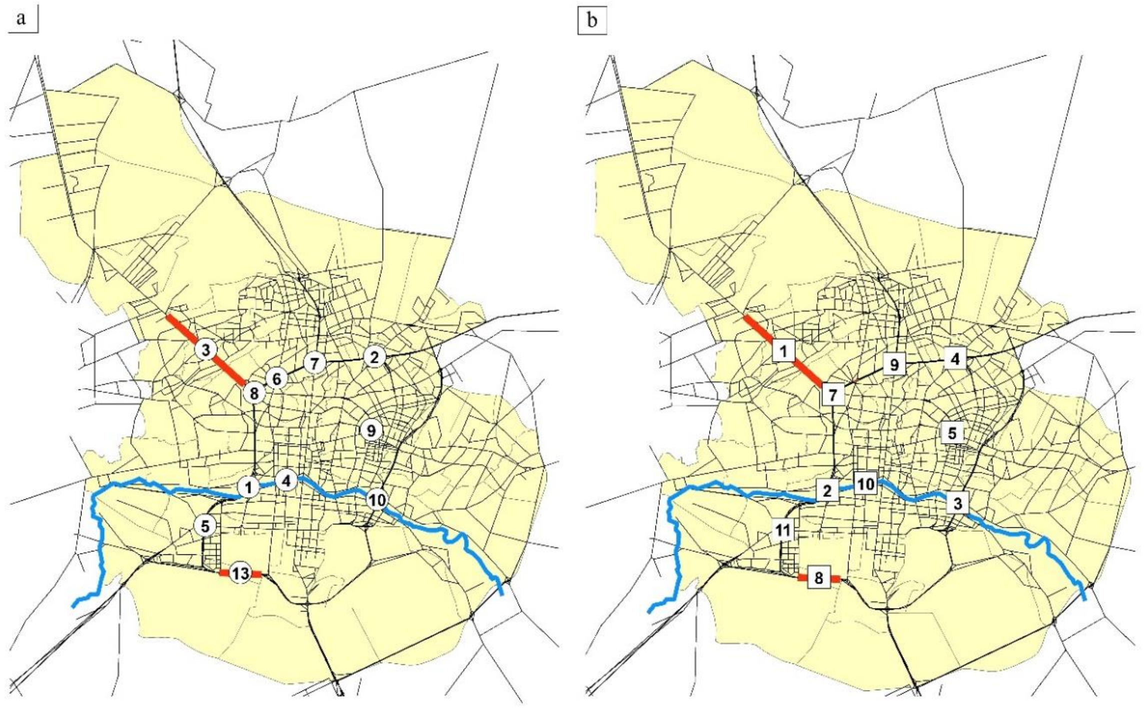


Figure 5. Top 10 importance bridges from the perspective of a) inequity impact b) overall impact

Table 2. The results of the overall and inequity impact analysis for most important links

Name of Bridge	Change in Total Travel time in minutes (Percent of change)	The average change in Travel time in minute/trip (Percent of change)	Gini index	Overall impact rank	Inequity impact rank
Emam	1346306 (6.4 %)	2.74 (5.5 %)	0.657	1	3
Vahid	1330885 (6.3 %)	2.71 (5.4 %)	0.679	2	1
Ghadir	1255382 (6 %)	2.56 (5.1 %)	0.620	3	10
Laleh	1063905 (5.1 %)	2.17 (4.3 %)	0.659	4	2
Ahmad Abad	1063400 (5.1 %)	2.17 (4.3 %)	0.621	5	9
Robat	1053876 (5.0 %)	2.15 (4.3 %)	0.630	6	6
Kharazi	1052996 (5.0 %)	2.14 (4.3 %)	0.623	7	8
Aghareb-Parast	1046487 (5.0 %)	2.13 (4.2 %)	0.609	8	13
Chamran	1026364 (4.9 %)	2.09 (4.2 %)	0.623	9	7
Azar	1018382 (4.8 %)	2.07 (4.1 %)	0.637	10	4
Keshavarzi	898455 (4.3 %)	1.83 (3.7 %)	0.632	11	5

Figure 6 shows the correlation between the overall and inequity impact vulnerability ranking. The x-axis represents the ranking of overall effects, whereas the y-axis is the corresponding ranking of the inequity impact. Even there is a relatively high correlation between the two rankings (Spearman's rank correlation coefficient $\rho=0.7$, which is statistically significant at *99%* confident level (*p-value*<*0.01*)), for some links, different vulnerability indices give different ranking results.

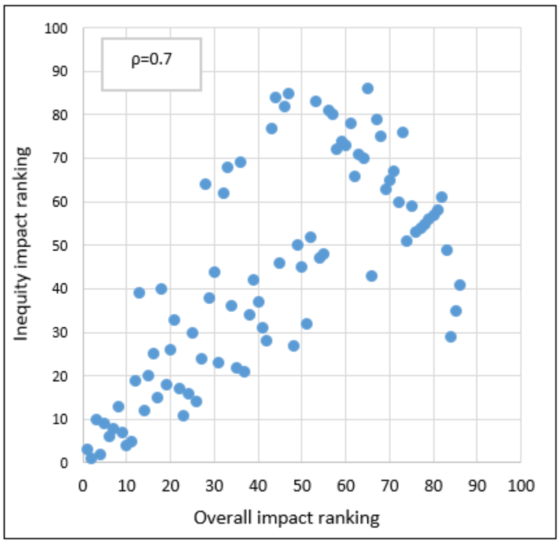


Figure 6. Correlation between overall and inequity impact rankings.

Comparing the histograms of the distribution of added travel time in different link failure scenarios can also be useful to explain the different ranking results. This is achieved by using different indexes to evaluate vulnerability. Figure 7 (a-d) provides histograms of the distribution of added travel time to the network users for four disrupted networks: a) network with the maximum amount of travel time added to the users (the most overall impact), b) network with the minimum amount of travel time added to the users (the least overall impact), c) network with the maximum value of the Gini coefficient of the added travel time distribution (the most inequity impact), and d) network with the minimum value of the Gini coefficient of the added travel time distribution (the least inequity impact). From the overall impact perspective, the Emam (Anushiravan) bridge's failure (the most critical bridge) imposes a total of 1,346,306 minutes extra travel time (see Figure 7). The extra travel time includes a range from under one minute to more than fifty minutes with different frequencies. Failure of the Vahid bridge (second overall impact bridge and the most critical bridge from the inequity aspect) adds 1,330,885 minutes to the user travel time. The Gini coefficient of Δt distribution for these two links is 0.6593 and 0.6793, respectively. Also, considering the link failure's overall impact determines the least important link with 640,447 extra minutes to the network travel time. The Gini coefficient for this bridge equals 0.5733, which is ranked 41 in the inequity impact rankings. Comparing the histograms of these Δt distribution shows that while the failure of one link causes to increase in overall travel time, another one propagates more inequity among the users. On the other hand, the link with the least inequity impact with 0.5512 Gini coefficient causes to add 677,664 minutes extra travel time to the network users, which is ranked 65 in the overall impact ranking. Comparing these four histograms shows how link failure's inequity impact leads to different vulnerability analyses and, therefore, different critical link determination.

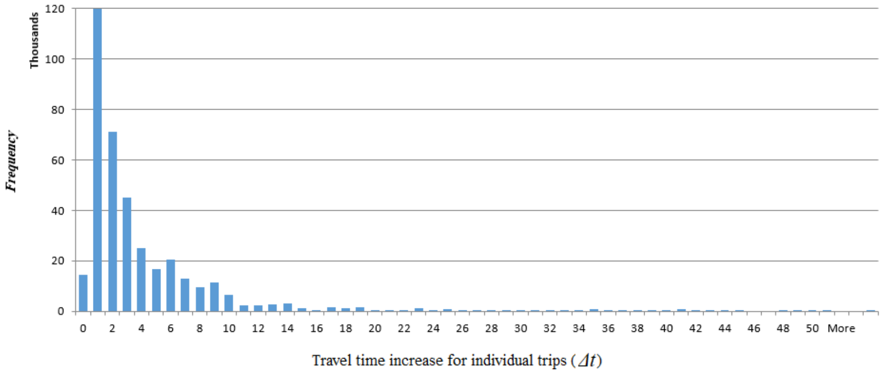
The histograms of the added travel time to the network users in general and, in particular, for the plotted cases in Figure 7 help to get an overall view about the probability distribution of the added travel time to the network users. As seen in Figure 7, the probability distribution does not conform to a normal distribution, which is also proved through the Kolmogorov-Smirnov test. The results of the Kolmogorov-Smirnov test on the added travel time to the network users show that the normal distribution hypothesis is rejected for all cases. This explains why, in comparison with Jenelius (2010), in this research, we did not use the Coefficient of Variation (CV) in order to assess the inequity impact of the link failure. Even the CV is a simple and well-known measure of inequity, it would be an inappropriate and misleading index to measure inequity if the data did not approach a normal distribution.

6.2 Importance ranking based on both overall and inequity impacts

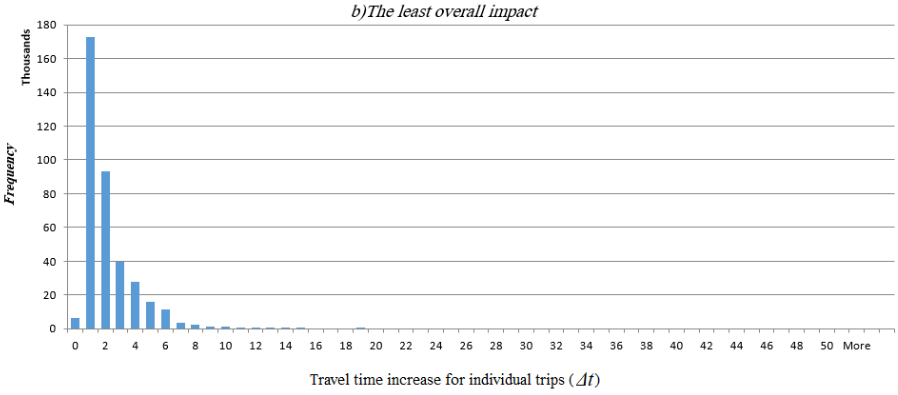
The final step includes presenting a method that considers both the overall and inequity impacts of bridge failure. As mentioned earlier, the decision for ranking the links' importance is a trade-off between the overall and inequity impacts of a disruption in the network's links. In the current case, the so-called decision space consists of 86 links (the bridges of the network), which are being evaluated using two overall and inequity impact objectives. Even there is a high correlation between the two proposed indices, it still needs to present a sorting method to rank all evaluated links and determine which one is the most important one. For example, consider the two most important links in Table 2. For the Emam (Anushiravan) Bridge (the most important bridge from the overall impact perspective), the total added travel time to the network users is 1,346,306 minutes, and for the Vahid Bridge (the second one) is 1,330,855 minutes, which is nearly 15,000 minutes less than the first one. However, the Gini coefficient, as the inequity impact ranking index, for these two links is 0.659 and 0.679, respectively, which shows an increase in the inequity between the network users. This shows that two indices do not have completely the same meaning and behavior.

Applying the non-dominated sorting method helps categorization of the 86 studied bridges in different important sets. Figure 8 show the result of non-dominated sorting applied to the 86 studied links. The x-axis is the normalized value of the overall impact index, and the y-axis is the normalized value of the inequity impact index. As seen in Figure 8, the links are sorted into 19 important sets. For example, the first set Pareto front set consists of two links, which are the most important overall and inequity impact links.

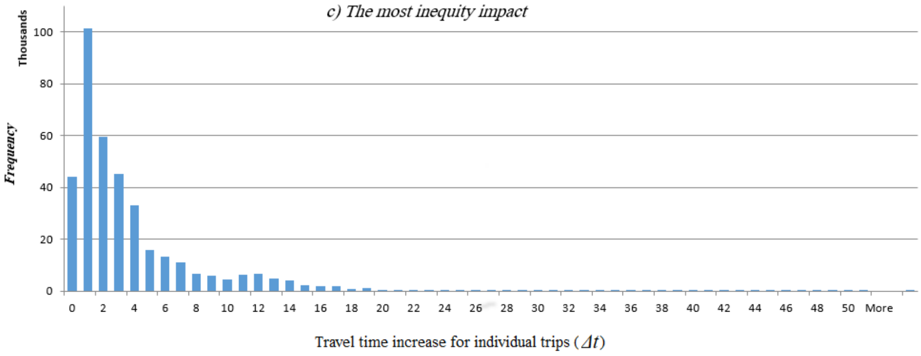
a) The most overall impact



b)The least overall impact



c) The most inequity impact



d) The least inequity impact

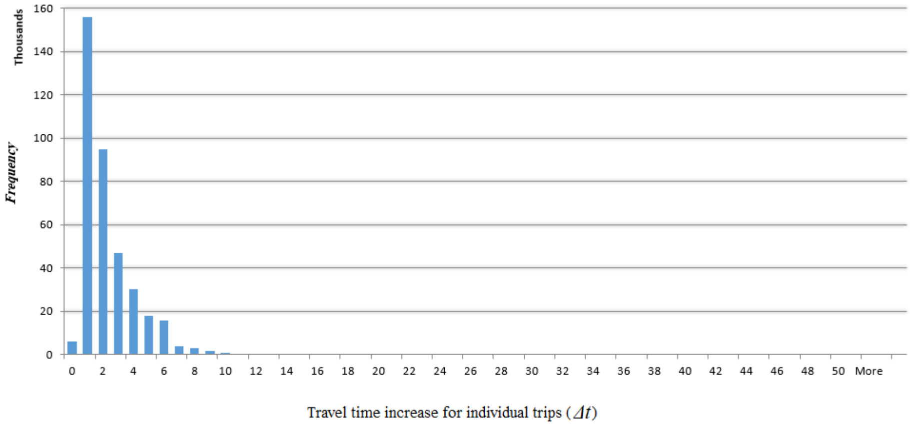


Figure 7. Histograms of the distribution of the added travel time to the network users for four disrupted networks: a) the most overall impact, b) the least overall impact, c) the most inequity impact, and d) the least inequity impact

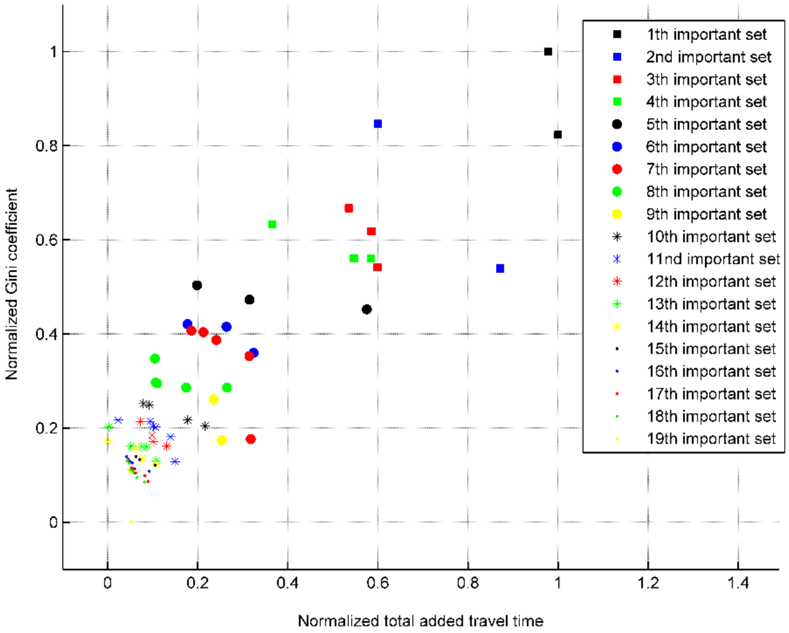


Figure 8. The results of non-dominated sorting

As mentioned earlier, the links (studied bridges) in a critical set rank are viewed equally important, and links in a smaller rank are more important than those in a larger rank. Table 3 represents the top 10 most critical bridges based on the non-dominated sorting method and their related importance rank from overall and the inequity impacts. As seen in Table 3, these top 10 bridges are categorized into five groups, which is consisted of the bridges from the top 11 bridges from the overall impacts view and the top 13 bridges from the inequity impact aspect.

Table 3. The final most critical bridges

Name of Bridge	Non-dominated rank	Overall impact rank	Inequity impact rank
Emam	1	1	3
Vahid	1	2	1
Ghadir	2	3	10
Laleh	2	4	2
Ahmad Abad	3	5	9
Azar	3	10	4
Kharazi	4	7	8
Chamran	4	9	7
Keshavarzi	4	11	5
Aghareb-Parast	5	8	13

The top 10 most important bridges based on the non-dominated sorting procedure for considering both the overall and the inequity impacts indices are shown in Figure 9. Compared to the methods that only concentrate on the overall impact of a link failure, the proposed method's results would lead to deeper and broader insight regarding the consequences of disruptions in the network. In fact, it is not enough to assess the consequences of any disturbance in the transportation network. For example, the overall added travel time to the network users, change in total accessibility level, and total travel cost on the network (as previously studied presented). In order to have a deeper and wider outlook, it is also necessary to consider the inequity aspect. This kind of view of the network vulnerability is more aligned with the sustainable policies where equity is assured.

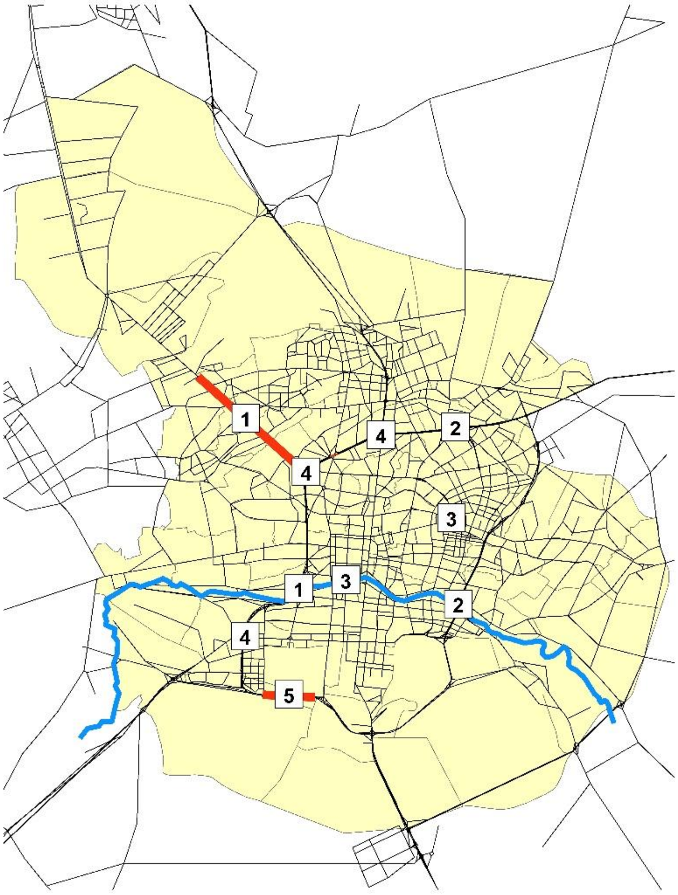


Figure 9. Top 10 most important bridges based on integrated overall and inequity impact

7. Discussion and Conclusion

The aim of this paper is vulnerability assessment of transportation networks with a focus on equity considerations and to develop a methodology to achieve this purpose. The vulnerability of bridges, as the most vulnerable links in surface transportation networks, is evaluated because of their unique role and structural features. The failure of a link has consequences for the network users, which determines the importance of that link. Previous studies mostly concentrated on the total increase of people travel time, while in this paper, we also focused on the distribution of the added travel time to the network users. In this context, equity is a measure of the distribution of the added travel time among users. The Gini coefficient is used to assess how evenly a link's failure changes the user's travel time. Also, the overall impact of a link was determined by calculating the total travel time added to the network in case of that link's failure. The case study results showed that the proposed method is based on inequity impact consideration for vulnerability assessment, which positively correlates with the overall impact method. For some links, the importance ranking based on these two methods is distinctively different. We compared the results of these two approaches by identifying the most vulnerable links. Finally, both overall and inequity impacts of a link failure have been considered to present a final importance rank by applying the non-dominated sorting method.

One of the direct results of determining links' importance in network vulnerability analysis is to determine how and where to direct funds and resources towards the reinforcement of the transportation network vulnerable to potential threats. One factor associated with sustainable development is involving equity and distributive justice in the planning process and resource allocation. Thus, when it comes to answering where to spend funds, not only should the system's overall performance be considered, but also equity issues should be taken into account. This paper's proposed method can be useful for policy recommendations in maintenance and rehabilitation plans, which take the impacts of the link's failure into account. The results indicate how the inequity distribution of added travel time to network users gives different importance rankings of the network links. Furthermore, it is not enough to evaluate the network vulnerability only based on inequity ensued by a link's failure. For example, in the occurrence of a big distribution that severely damages the transportation system (like an earthquake), the inequity index may be lower than other disruptions. Therefore, it is reasonable to consider both the overall and the inequity impact on determining the link's importance, which gives the final importance ranking of the network's links.

Another result that can be useful for transportation authorities is mapping the average delay time of each TAZ for every disturbance event and determining the most vulnerable TAZ where delays are concentrated. This can serve as a guideline to implement appropriate policies to reduce the negative effects of a link failure from the perspective of both the overall and inequity impacts. On the whole, this paper concludes that considering the inequity impact of occurring a disruption in the network offers additional insight into the network vulnerability assessment, which can lead policymakers to make better-informed decisions.

Future work may advance the research in several directions. First, as we mentioned, there is a relatively high correlation between overall and inequity impacts of a link failure. This would lead us to conclude that it is likely related to the network topology. Some possibilities for future researches would be to examine the relation between these two measures and network topology. Second, the method just concentrates on horizontal equity, and the next step would be to consider vertical equity. In fact, the impact of socio-economic attributes could be considered and assessed how a disruption in network negatively affects vulnerable groups like low-incomes, elderlies, or people living with disabilities. For example, it could be assessed how an interruption in the network could adversely affect the accessibility to essential urban facilities like health and medical services. Considering both horizontal and vertical equity gives policymakers and planners a wider view of the socio-economic impact of different failures in the network. Third, other modes of transportation, particularly public transportation, could also be considered. The public transportation system in many cities consists of Bus Rapid Transit (BRT) and bus lines, which often have to use the same infrastructure as cars, but all are not the same. For example, bus lines are usually do not pass through highways, and this factor, alongside different demand patterns of public transportation, would lead to a different ranking of the important links and vulnerability analysis. Forth, this study assessed the impact of link failure from both overall and inequity impacts and to show the applicability of the proposed method. The method is applied to the bridges of the network of the city of Isfahan. Similarly, most transportation infrastructures need to consider each infrastructure's inherent vulnerability (fragility). In fact, in order to assign funds to retrofit critical facilities, what should also be considered is the risk of failure, defined as the probability of failure weighted by the consequences.

References

Allison, P. D. (1978). Measures of inequality. *American sociological review*, 865-880.

Balijepalli, C., & Oppong, O. (2014). Measuring vulnerability of road network considering the extent of serviceability of critical road links in urban areas. *Journal of Transport Geography*, *39*, 145-155.

Bao, C., Xu, L., Goodman, E. D., & Cao, L. (2017). A novel non-dominated sorting algorithm for evolutionary multi-objective optimization. *Journal of Computational Science*, *23*, 31-43.

Berdica, K. (2002). An introduction to road vulnerability: what has been done, is done and should be done. *Transport policy*, *9*(2), 117-127.

Berdica, K., & Mattsson, L.-G. (2007). Vulnerability: a model-based case study of the road network in Stockholm. In *Critical infrastructure* (pp. 81-106). Springer.

Bertolaccini, K., & Lownes, N. E. (2013). Effects of scale and boundary selection in assessing equity of transit supply distribution. *Transportation Research Record*, *2350*(1), 58-64.

Bröcker, J., Korzhenevych, A., & Schürmann, C. (2010). Assessing spatial equity and efficiency impacts of transport infrastructure projects. *Transportation Research Part B: Methodological*, *44*(7), 795-811.

Brown, M. C. (1994). Using Gini-style indices to evaluate the spatial patterns of health practitioners: theoretical considerations and an application based on Alberta data. *Social science & medicine*, *38*(9), 1243-1256.

Burgholzer, W., Bauer, G., Posset, M., & Jammernegg, W. (2013). Analysing the impact of disruptions in intermodal transport networks: A micro simulation-based model. *Decision Support Systems*, *54*(4), 1580-1586.

Crampton, F. E. (2010). The economics and financing of urban schools: Toward a productive, solution-oriented discourse. *Educational Considerations*, *38*(1), 7.

Crucitti, P., Latora, V., & Porta, S. (2006). Centrality measures in spatial networks of urban streets. *Physical Review E*, *73*(3), 036125.

Currie, G., Richardson, T., Smyth, P., Vella-Brodrick, D., Hine, J., Lucas, K., Stanley, J., Morris, J., Kinnear, R., & Stanley, J. (2009). Investigating links between transport disadvantage, social exclusion and well-being in Melbourne—Preliminary results. *Transport policy*, *16*(3), 97-105.

De Maio, F. G. (2007). Income inequality measures. *Journal of Epidemiology & Community Health*, *61*(10), 849-852.

de Oliveira, E. L., da Silva Portugal, L., & Junior, W. P. (2016). Indicators of reliability and vulnerability: Similarities and differences in ranking links of a complex road system. *Transportation Research Part A: Policy and Practice*, *88*, 195-208.

Deb, K., Pratap, A., Agarwal, S., & Meyarivan, T. (2002). A fast and elitist multiobjective genetic algorithm: NSGA-II. *IEEE transactions on evolutionary computation*, *6*(2), 182-197.

Dehghani, M. S., Flintsch, G., & McNeil, S. (2014). Impact of road conditions and disruption uncertainties on network vulnerability. *Journal of infrastructure systems*, *20*(3), 04014015.

Emmerich, M. T., & Deutz, A. H. (2018). A tutorial on multiobjective optimization: fundamentals and evolutionary methods. *Natural computing*, *17*(3), 585-609.

Fisk, C. (1980). Some developments in equilibrium traffic assignment. *Transportation Research Part B: Methodological*, *14*(3), 243-255.

Frederickson, H. G. (2010). Social equity and public administration; origins, developments, and applications. *Public Integrity*, *11*.

Gilbert, P. H., Isenberg, J., Baecher, G. B., Papay, L. T., Spielvogel, L. G., Woodard, J. B., & Badolato, E. V. (2003). Infrastructure issues for cities—Countering terrorist threat. *Journal of infrastructure systems*, *9*(1), 44-54.

Gini, C. (1921). Measurement of inequality of incomes. *The Economic Journal*, *31*(121), 124-126.

Gudmundsson, H., & Höjer, M. (1996). Sustainable development principles and their implications for transport. *Ecological economics*, *19*(3), 269-282.

Humphrey, N. (2008). Potential impacts of climate change on US transportation. *TR news*, *256*, 21-24.

Jenelius, E. (2010). User inequity implications of road network vulnerability. *Journal of Transport and Land Use*, *2*(3).

Jenelius, E., & Mattsson, L.-G. (2010). The impact of network density, travel and location patterns on regional road network vulnerability.

Jenelius, E., Petersen, T., & Mattsson, L.-G. (2006). Importance and exposure in road network vulnerability analysis. *Transportation Research Part A: Policy and Practice*, *40*(7), 537-560.

Koetse, M. J., & Rietveld, P. (2009). The impact of climate change and weather on transport: An overview of empirical findings. *Transportation Research Part D: Transport and Environment*, *14*(3), 205-221.

Levinson, D. (2002). Identifying winners and losers in transportation. *Transportation Research Record: Journal of the Transportation Research Board*(1812), 179-185.

Li, Y., & DaCosta, M. N. (2013). Transportation and income inequality in China: 1978–2007. *Transportation Research Part A: Policy and Practice*, *55*, 56-71.

Litman, T. (1996). Evaluating Transportation Equity. *Victoria Transport Policy Institute, Canada*.

Litman, T., & Brenman, M. (2012). *A new social equity agenda for sustainable transportation*. Victoria Transport Policy Institute.

Litman, T., & Burwell, D. (2006). Issues in sustainable transportation. *International Journal of Global Environmental Issues*, *6*(4), 331-347.

Lucas, K., Wee, B., & Maat, K. (2016). A method to evaluate equitable accessibility: combining ethical theories and accessibility-based approaches. *Transportation*, *43*(3), 473-490.

Mackett, R. L., Achuthan, K., & Titheridge, H. (2008). AMELIA: A tool to make transport policies more socially inclusive. *Transport policy*, *15*(6), 372-378.

Martens, K., Golub, A., & Robinson, G. (2012). A justice-theoretic approach to the distribution of transportation benefits: Implications for transportation planning practice in the United States. *Transportation Research Part A: Policy and Practice*, *46*(4), 684-695.

Mattsson, L.-G., & Jenelius, E. (2015). Vulnerability and resilience of transport systems–a discussion of recent research. *Transportation Research Part A: Policy and Practice*, *81*, 16-34.

Mercier, J. (2009). Equity, social justice, and sustainable urban transportation in the twenty-first century. *Administrative theory & praxis*, *31*(2), 145-163.

Monzón, A., Ortega, E., & López, E. (2013). Efficiency and spatial equity impacts of high-speed rail extensions in urban areas. *Cities*, *30*, 18-30.

Murray-Tuite, P., & Mahmassani, H. (2004). Methodology for determining vulnerable links in a transportation network. *Transportation Research Record: Journal of the Transportation Research Board*(1882), 88-96.

Murray, A. T., & Grubesic, T. (2007). *Critical infrastructure: Reliability and vulnerability*. Springer Science & Business Media.

Murray, A. T., Matisziw, T. C., & Grubesic, T. H. (2008). A methodological overview of network vulnerability analysis. *Growth and Change*, *39*(4), 573-592.

Nagurney, A., & Qiang, Q. (2007). A transportation network efficiency measure that captures flows, behavior, and costs with applications to network component importance identification and vulnerability.

Preston, J. (2009). Epilogue: Transport policy and social exclusion—Some reflections. *Transport policy*, *16*(3), 140-142.

Ramjerdi, F. (2006). Equity measures and their performance in transportation. *Transportation Research Record: Journal of the Transportation Research Board*(1983), 67-74.

Rupi, F., Angelini, S., Bernardi, S., Danesi, A., & Rossi, G. (2015). Ranking Links in a Road Transport Network: A Practical Method for the Calculation of Link Importance. *Transportation Research Procedia*, *5*, 221-232.

Scott, D. M., Novak, D. C., Aultman-Hall, L., & Guo, F. (2006). Network robustness index: A new method for identifying critical links and evaluating the performance of transportation networks. *Journal of Transport Geography*, *14*(3), 215-227.

Shirmohammadli, A., Louen, C., & Vallée, D. (2016). Exploring mobility equity in a society undergoing changes in travel behavior: A case study of Aachen, Germany. *Transport Policy*, *46*, 32-39.

Shorrocks, A. F. (1980). The class of additively decomposable inequality measures. *Econometrica: Journal of the Econometric Society*, 613-625.

Sullivan, J., Novak, D., Aultman-Hall, L., & Scott, D. M. (2010). Identifying critical road segments and measuring system-wide robustness in transportation networks with isolating links: A link-based capacity-reduction approach. *Transportation Research Part A: Policy and Practice*, *44*(5), 323-336.

Tahmasbi, B., & Haghshenas, H. (2019). Public transport accessibility measure based on weighted door to door travel time. *Computers, Environment and Urban Systems*, *76*, 163-177.

Tahmasbi, B., Mansourianfar, M. H., Haghshenas, H., & Kim, I. (2019). Multimodal accessibility-based equity assessment of urban public facilities distribution. *Sustainable Cities and Society*, *49*, 101633.

Taylor, M. (2017). *Vulnerability Analysis for Transportation Networks*. Elsevier.

Taylor, M. A., Sekhar, S. V., & D'Este, G. M. (2006). Application of accessibility based methods for vulnerability analysis of strategic road networks. *Networks and Spatial Economics*, *6*(3-4), 267-291.

Taylor, M. A., & Susilawati, S. (2012). Remoteness and accessibility in the vulnerability analysis of regional road networks. *Transportation Research Part A: Policy and Practice*, *46*(5), 761-771.

Tian, Y., Wang, H., Zhang, X., & Jin, Y. (2017). Effectiveness and efficiency of non-dominated sorting for evolutionary multi-and many-objective optimization. *Complex & Intelligent Systems*, *3*(4), 247-263.

Zanini, M. A., Pellegrino, C., Morbin, R., & Modena, C. (2013). Seismic vulnerability of bridges in transport networks subjected to environmental deterioration. *Bulletin of Earthquake Engineering*, *11*(2), 561-579.

Zarei, H., & Rasti-Barzoki, M. (2019). Mathematical programming and three metaheuristic algorithms for a bi-objective supply chain scheduling problem. *Neural Computing and Applications*, *31*(12), 9073-9093.

Zhang, X., Tian, Y., Cheng, R., & Jin, Y. (2014). An efficient approach to nondominated sorting for evolutionary multiobjective optimization. *IEEE transactions on evolutionary computation*, *19*(2), 201-213.

Zitzler, E., Laumanns, M., & Thiele, L. (2001). SPEA2: Improving the strength Pareto evolutionary algorithm. *TIK-report*, *103*.