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## Departure time choice and bottleneck congestion with automated vehicles: Role of on-board activities

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It is widely expected that automated vehicles (AVs) will revolutionise travel experience by better facilitating various on-board activities. While these activities could make travel more pleasant, as is often supposed, they could also affect daily schedules, the related travel choices, and finally, the aggregate travel patterns – possible influences that are still insufficiently studied. For example, a morning commuter deciding to perform some home or work activities during travel, instead of at home or work, could also reconsider his departure time to work. More such travellers together could reshape traffic congestion. This paper models exactly this scenario. It formulates new scheduling preferences, which account for home and/or work activities during morning commute, and uses these (1) to analyse the optimal departure times when there is no congestion, and (2) to obtain the equilibrium congestion patterns in a bottleneck setting. If there is no congestion, it is predicted that AV users would depart earlier (later), if the on-board environment supports their home (work) activities. If there is congestion, AV users that perform home (work) activities during travel skew the congestion to earlier (later) times, and AV users that perform both activities increase both early and late congestion. Engaging in any activity during travel worsens congestion, at least when assuming that AVs do not increase bottleneck capacity. If future AVs would be specialised to support only home, only work, or both home and work activities, and would do so to a similar extent, then ‘Work AVs’ would increase the congestion the least.

*Keywords:* automated vehicles, bottleneck model, departure time choice, on-board activities, scheduling preferences, traffic congestion.

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### 1. Introduction

Among the core expected benefits of automated vehicles<sup>2</sup> (AVs) is their promise to let their users perform new non-driving activities, or engage more efficiently in current non-driving activities, while being on the way. The literature (see Soteropoulos et al., 2019, for a recent review of

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<sup>2</sup> This paper considers primarily the so-called level 5 or fully automated vehicles, according to SAE International (2016) standards.

modelling studies) commonly anticipates that this would make travel more pleasant, thus reducing the 'penalty' associated with travel time. This, the reasoning goes, may lead to acceptance of longer travel times, thereby increasing traffic congestion, which may be (partly) offset by shorter headways and increased throughput expected from AVs. The possible net congestion effects of AVs have been extensively modelled and discussed (e.g., van den Berg & Verhoef, 2016; Wadud et al., 2016; Auld et al., 2017; Milakis et al., 2017; Simoni et al., 2019).

However, a thought experiment can demonstrate that *the substance of on-board activities* may directly influence the timing preferences for a trip, and in so doing affect congestion patterns in ways that would not be predicted using travel time penalty. For example, an AV user may consider shifting or extending the pre- or post-travel activities into the trip. In the context of the morning commute, an individual may choose to perform in the AV 'home activities', such as getting ready, preparing and eating breakfast, getting a little more sleep, or 'work activities', such as replying to emails, planning the day, adjusting meeting schedule. This shift might reduce the aversion to longer travel and encourage AV users to depart at peak times. At the same time, it might result in a desire to depart from the origin earlier, while shifting origin-type activities to the trip, or to depart and arrive at the destination later, while shifting destination-type activities to the trip. That on-board activities may have varied influence on the preferred departure times, may also be expected knowing that various on-board activities differently influence the value of travel time or travel satisfaction (as found by Ettema & Verschuren, 2007; Susilo et al., 2012; Rasouli & Timmermans, 2014; Frei et al., 2015; Correia et al., 2019), as well as pre- and post-trip activities and daily time-use (Banerjee & Kanafani, 2008; Pawlak et al., 2015, 2017; Das et al., 2017; Krueger et al., 2019; Pudāne et al., 2018, 2019; Kim et al., 2020). Lastly, work or leisure during travel matters for the value of travel time savings, according to the time-use theory and the widely-used Hensher's equation (Hensher, 1977; Batley, 2015), and this link was recently examined by Pudāne and Correia (2020) in the AV context.

Yet, most current models that aim to predict mobility and congestion patterns in the AV era do not consider that various on-board activities may differently affect departure times (e.g., Correia & van Arem, 2016; Lamotte et al., 2017; Simoni et al., 2019; F. Zhang et al., 2020). The possibility to model multiple scenarios there is largely lost whenever the effects of a multitude of possible activities are condensed into a single travel time penalty (such as value of travel time). This treatment implies that the travel behaviour effects of various on-board activities are the same and indistinguishable from increased comfort of travel (such as more comfortable seats).

This study proposes a more flexible modelling approach, which lets to investigate, first, how various on-board activities may influence departure times of AV users and, second, how they may affect traffic congestion. It starts by formulating new scheduling preferences that let the analyst specify how suitable the travel environment is for home and work activities. After, it uses the new scheduling preferences to analyse the optimal departure times for users of different AVs. Finally, it obtains equilibrium congestion patterns in a minimalistic bottleneck setting, where a number of travellers with the same scheduling preferences move from a single origin to a single destination on a single route.

Thereby, this study contributes to two streams of literature: first, to the study of the potential travel behaviour impacts of on-board activities (in AVs or other modes), especially those reaching beyond the effects on the value of travel time, and second, to the rich tradition of using the bottleneck model to analyse the impact of behaviour changes on congestion. With regard to the former, this study relates to the work of Pawlak et al. (2015, 2017), which uses a scenario with two out-of-vehicle activities connected by a trip, during which two in-vehicle activities may be performed. Pawlak et al. analysed a multidimensional choice in this setting: choice of activity types, departure times, duration and switching times between on-board activities, mode, route and use of ICT. Relatedly, Rasouli and Timmermans (2014) explored the impact of on-board activities on activities directly preceding or following the travel episode: interactions in what they named 'activity envelope'. More broadly, this work contributes to the literature that studies daily time-use effects of on-board activities (Banerjee & Kanafani, 2008; Pudāne et al., 2018; Kim et al., 2020).

With regard to the latter, this study provides an on-board time-use module to the classical bottleneck framework (conceived by Vickrey, 1969, and Arnott et al., 1990, 1993). The bottleneck model has been instrumental in investigating various factors influencing congestion (see the reviews by de Palma & Fosgerau, 2011, Small, 2015, and Li et al., 2020), and notably, it often allows to obtain analytic as opposed to simulated results. Related to the present work, time-use aspects were included in the bottleneck model by Gubins and Verhoef (2011), who studied the effects of teleworking on congestion, and by Xiaoning Zhang et al. (2005) and Li et al. (2014), who integrated bottleneck-based departure time choice in a whole-day activity pattern.

Moving forward, Li et al. (2020), in their review of the bottleneck model development over the past half a century, emphasise the need to include different properties of new transport technologies, such as automated vehicles, in the bottleneck model. This work contributes to this goal, along with other recent studies that have modelled congestion patterns when AVs and conventional vehicles use different roads (Lamotte et al., 2017), the congestion impacts of AVs being able to park themselves (Liu, 2018; Xiang Zhang et al., 2019; Tian et al., 2019), and the congestion patterns in the long run, when travellers can choose between conventional vehicles and AVs (F. Zhang et al., 2020). In particular, this work furthers the study of van den Berg and Verhoef (2016), who investigated the effects of AVs on congestion in a bottleneck, while assuming that any on-board activities contribute to a decreasing travel penalty. They concluded that AV users would concentrate in the middle of the peak congestion. The same conclusion was reached also by Fosgerau (2019) and F. Zhang et al. (2020). Finally, this study aligns with other ongoing work that looks into departure time effects of on-board activities in AVs: Yu et al. (2019) and Abegaz and Fosgerau (personal communication, September 2019). Yu et al. (2019) analyse congestion patterns given  $\alpha - \beta - \gamma$  preferences<sup>3</sup>, where an on-board activity simultaneously substitutes home and work activities to various degrees. They also derive market and AV-provision effects. Abegaz and Fosgerau model on-board activities as a separate class of mobile activities in a general scheduling preference framework and derive changes in value of time and reliability. The present work contributes to this ongoing research in two main directions. First, it shows how optimal departure times depend on the activities performed during travel – home-, work- or both home and work activities –, even if there is no congestion. Second, it provides a derivation of congestion patterns given a different (compared to Yu et al.) set-up within the  $\alpha - \beta - \gamma$  preference framework, which also enables to capture a situation where traveller switches from performing home to work activities during travel.

The remainder of the paper is structured as follows. Section 2 introduces the scheduling preferences that capture the possibility to shift home or work activities to the trip. It also introduces three types of AVs that are used further in the paper. Section 3 analyses the departure times for a single traveller or multiple travellers that do not create congestion. Section 4 analyses congestion changes with AVs in bottleneck setting. Section 5 compares the current approach with travel time penalty method, discusses the assumption of  $\alpha - \beta - \gamma$  scheduling preferences and other aspects of the model set-up, assesses the validity and applicability of the developed model to other transport modes, and recommends directions for further research. Section 6 concludes and discusses the implications of this study for AV-related transport policy.

## 2. Model set-up

### 2.1 Scheduling preferences considering on-board activities

A general form of scheduling functions (based on Vickrey, 1973) assumes that marginal home and work utilities  $h[x]$  and  $w[x]$  are positive and monotonously decreasing and increasing functions, respectively, of the clock-time  $x$  in a morning time interval  $[0, \Omega]$ . Conventionally, it is assumed that the individual cannot participate in any home or work activities during travel and therefore

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<sup>3</sup> The  $\alpha - \beta - \gamma$  preferences, also called the step model, are the most commonly used scheduling preferences and are further explained in section 2.1.

does not gain any home or work utility at that time. In the context of AVs however, I assume that individuals may continue with their home activities during travel or start to perform their work activities while on the way to work, but a specified share of the utility of these activities would be lost, reflecting some inconvenience of performing these activities in the vehicle. This loss is expressed using multiplicative efficiency factors  $e_h, e_w \in [0,1]$  for home and work activities, respectively.<sup>4</sup>

Figure 1 illustrates the model set-up. It shows the marginal utilities of home and work activities (y-axis), which depend on time (x-axis) in a morning time interval. As can be seen from the distance between the solid and dashed lines, this figure illustrates a situation where home activities are better facilitated on board than work activities:  $e_h > e_w$ . Shaded areas represent the total utility gained from activities at home, at work and during travel.

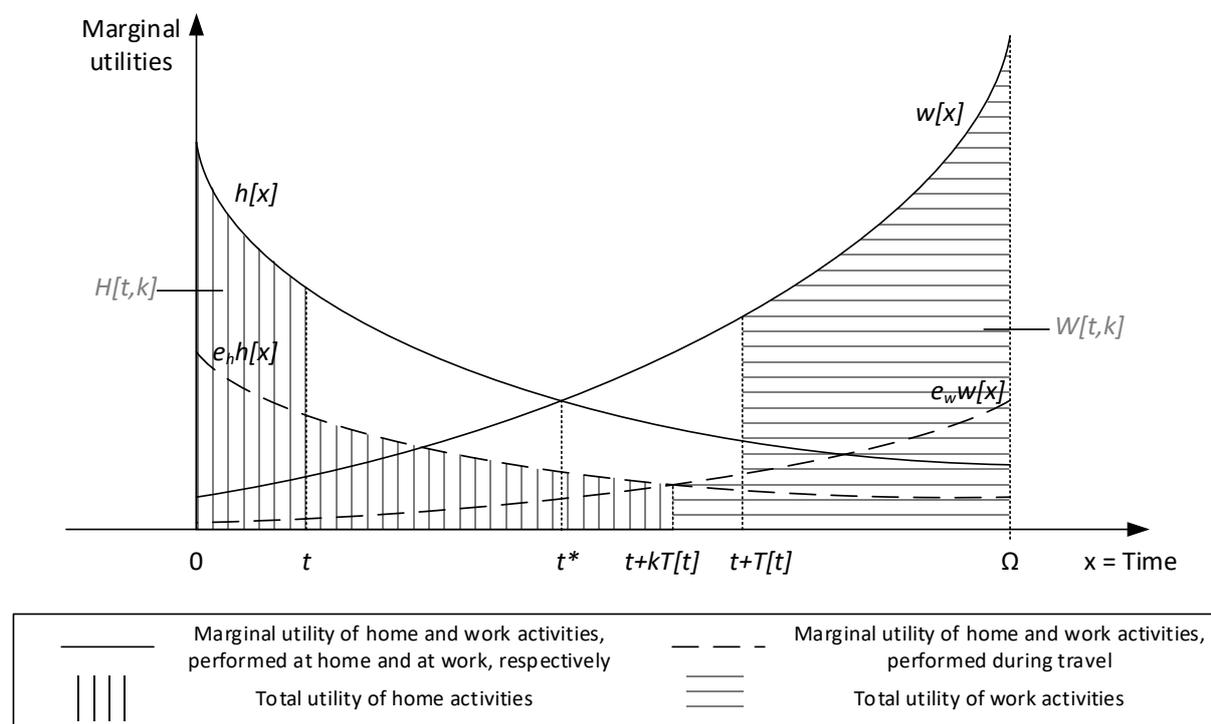


Figure 1. Scheduling preferences including the utility obtained from home and work activities on board: general scheduling preferences

The individual engages in home activity during travel at time  $x$  if  $e_h h[x] > e_w w[x]$  (utility from on-board home activities is higher than utility of on-board work activities) and in work activity if  $e_h h[x] < e_w w[x]$ . Therefore, knowing that  $e_h h[x]$  and  $e_w w[x]$  are monotonously decreasing and increasing with  $x \in [0, \Omega]$ , respectively (due to the above assumptions), the optimal time for on-board home activity (if any) is at the start of the trip, and similarly the optimal time for the on-board work activity (if any) is at the end of the trip.

Furthermore, since marginal home and work utilities  $h[x]$  and  $w[x]$  are assumed to be positive for  $x \in [0, \Omega]$ , the individual would want to continually engage in on-board activities, if they are at least slightly facilitated (i.e., if  $e_h, e_w > 0$ , then utilities  $e_h h[x], e_w w[x] > 0$ ). This setting yields a single optimal switching point between the home and work activities, which can be expressed as a share of the trip duration  $k \in [0,1]$ . Hence, a traveller that departs from home at time  $t$  and arrives

<sup>4</sup> Alternatively, an additive efficiency factor would lead to a situation where higher utility of the activity is associated with a lower utility loss percentage-wise. See Yu et al. (2019) for the congestion derivations given this set-up. The assumption of multiplicative efficiency factors is further discussed in section 5.2.

at work at time  $t + T[t]$  engages in home activity on board during the time interval  $[t, t + kT[t]]$  and in work activity on board during the time interval  $[t + kT[t], t + T[t]]$ . Travel time  $T[t]$  is assumed to depend on the departure time  $t$ , which enables to model the effects of congestion. The boundary cases, where  $k = 0$  or  $k = 1$ , correspond to individual engaging only at work or home activity on board, respectively. If travel took no time at all (the individual would be able to 'teleport' from home to work), then the optimal switch time between home and work would be  $t^*$ .

Total home utility  $H[t, k]$ , total work utility  $W[t, k]$  and total utility  $V[t, k]$  are defined as follows:

$$H[t, k] = \int_0^t h[x]dx + e_h \int_t^{t+kT[t]} h[x]dx, \quad (1)$$

$$W[t, k] = \int_{t+T[t]}^{\Omega} w[x]dx + e_w \int_{t+kT[t]}^{t+T[t]} w[x]dx, \quad (2)$$

$$V[t, k] = H[t, k] + W[t, k]. \quad (3)$$

Every traveller tries to maximise the total utility  $V[t, k]$  by choosing the departure time  $t$  and the switching point between the on-board activities  $k$ . This defines the scheduling preferences that determine the optimal departure times given a broad class of home and work marginal utility functions  $h[x]$  and  $w[x]$ . From here on, I call these 'general scheduling preferences'. While it is possible to use them to analyse the optimal departure times in case of no congestion (section 3), the analysis of equilibrium congestion patterns (section 4) requires that specific forms of  $h[x]$  and  $w[x]$  are used. For this purpose, I select the most widely used scheduling preferences, the  $\alpha - \beta - \gamma$  model<sup>5</sup> (Vickrey 1969; Small, 1982), which can be specified by inserting the following as the home and work utility functions in (1)-(3):

$$h[x] = \alpha, \quad (4)$$

$$w[x] = \begin{cases} \alpha - \beta, & \text{if } x \leq t^* \\ \alpha + \gamma, & \text{if } x > t^*, \end{cases} \quad (5)$$

where  $\alpha, \beta, \gamma$  are positive constants, and  $\alpha$  and  $\beta$  are assumed to have the relationship  $\beta < \alpha$ ;  $t^*$  is the preferred arrival time at work. Parameter  $\alpha$  is the utility of spending time at home;  $\beta$  and  $\gamma$  are the utility differences between home utility and work utility, if work is performed before or after the preferred arrival time, respectively. Figure 2 illustrates the model set-up, using the  $\alpha - \beta - \gamma$  scheduling preferences. The illustrated efficiency factors  $e_h$  and  $e_w$  are such that until the time  $t^*$  it would be optimal for the individual to engage in home activities, but after time  $t^*$  it would be optimal to switch to performing work activities during travel:  $e_h h[x] > e_w w[x]$  for  $x \leq t^*$  and  $e_h h[x] < e_w w[x]$  for  $x > t^*$ .<sup>6</sup> The figure shows a situation where traveller arrives late at work ( $t + T[t] > t^*$ ). As before, the shaded areas represent the total utility  $V[t]$  gained from activities at home, at work and during travel. Note that these utilities no longer contain the switching point  $k$  as a decision variable: if the traveller switches between on-board home and on-board work activities, then he will do so necessarily at time  $t^*$  (see section 2.2 for further explanation).

<sup>5</sup> The advantages of the  $\alpha - \beta - \gamma$  model in the present study are its elegant closed form flow rates (obtained by Arnott et al., 1990) and the conservative predictions for congestion changes with AVs. The drawbacks are the constant home utility assumption, which has been shown to have limited validity, and the restricted subset of  $e_h$  and  $e_w$  values, for which the flow rate computations apply. Section 5.2 further discusses these properties.

<sup>6</sup> Note that the set-up (1)-(3) permits scenarios where the utility of on-board activity is higher than utility of home or work activities just before or after the trip. For example, in the context of  $\alpha - \beta - \gamma$  preferences, home activity during travel may be more valuable than work activity before the preferred arrival time  $t^*$ :  $e_h \alpha > \alpha - \beta$ . In such cases, it is assumed that the individual would still leave the AV once it has arrived, rather than continuing with the home activity in a parked vehicle.

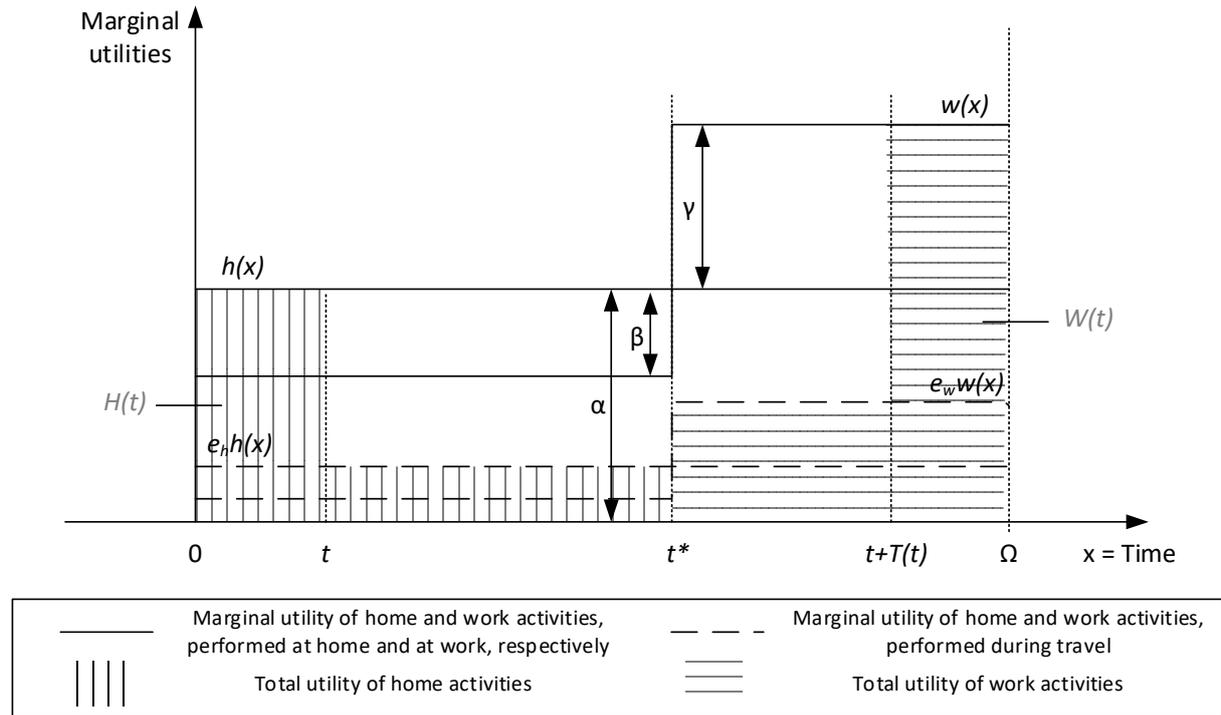


Figure 2. Scheduling preferences including the utility obtained from home and work activities on board:  $\alpha - \beta - \gamma$  scheduling preferences

Finally, note that the  $\alpha - \beta - \gamma$  model does not belong to the class of general scheduling preferences. For the general scheduling preferences, the home and work utility functions are strictly decreasing and increasing, respectively, but in the  $\alpha - \beta - \gamma$  model they are constant and piecewise constant, respectively.

2.2 Three types of automated vehicles

The set-up introduced in section 2.1 allows us to imagine scenarios where AVs are specialised (e.g., via interior design and equipment) to suit the needs of (1) home activities, (2) work activities, or (3) both home and work activities. In the following sections, these AV-types are called ‘Home AV’, ‘Work AV’, and ‘Universal AV’, respectively. However, the precise classification differs between sections 3 and 4. In section 3 with general scheduling preferences (as in Figure 1), the three types are defined using only the efficiency factors:  $e_h > e_w$  characterises the Home AV,  $e_h < e_w$  represents the Work AV, and  $e_h = e_w$  corresponds to the Universal AV.

In section 4, which uses the  $\alpha - \beta - \gamma$  scheduling preferences (as in Figure 2), the definitions involve the parameters of the home and work utility functions ( $\alpha, \beta, \gamma$ ). The resulting definition of Universal AV is such that it would be optimal for the users of this AV to engage in home activities before time  $t^*$  and in work activities after  $t^*$  (as in Figure 2). The Home AV and Work AV facilitate one of the two activities much better than the other, such that, independently of the departure time  $t$ , it is optimal to engage in home activities in Home AV and work activities in Work AV during the entire trip. The parameter combinations that define each AV type in the context of  $\alpha - \beta - \gamma$  preferences are shown in Figure 3. If, for example,  $\alpha e_h$  (the utility of on-board home activity) is smaller than  $(\alpha - \beta)e_w$  (the utility of on-board work activity before  $t^*$ ), then these parameter values correspond to a Work AV.

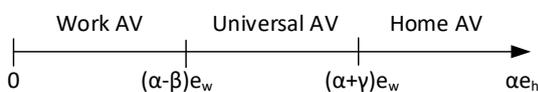


Figure 3. Definition of Home, Universal, Work AVs using  $\alpha - \beta - \gamma$  scheduling preferences

In addition, it could be possible to distinguish a fourth type of AV that only increases the comfort of travel or facilitates such activities on board that do not substitute activities out-of-vehicle (e.g., on-board entertainment). Such an AV could be defined by replacing the home and work functions in the second integrals of equations (1) and (2) with constants (or other time-independent functions). This would define an AV that is modelled by reduced travel time penalty approach. This AV type is discussed as a point of reference in section 5.1.

### 3. Case of no congestion

#### 3.1 Optimal departure times with general scheduling preferences

Having introduced the scheduling preferences, we can analyse the optimal departure time of a single traveller. The derivation would be the same in a hypothetical situation when multiple identical travellers do not create congestion, that is, when the bottleneck capacity exceeds the number of travellers who desire to depart in the given time unit. Formally, this situation can be represented as travel time being independent from the departure time and constant:  $T[t] = T$ . Using the general scheduling preferences, finding the optimal departure time is a 2-variable constrained optimisation problem: choose departure time  $t$  and switching point  $k$  between home- and work-type activities that maximises the total utility  $V[t, k]$  from (3). The optimisation problem is constrained, because switching between activities needs to occur during the trip time ( $0 \leq k \leq 1$ ). These conditions result in the following model:

$$\max V[t, k], \quad (6)$$

subject to:

$$g_1[k] = -k \leq 0, \quad (7)$$

$$g_2[k] = k - 1 \leq 0. \quad (8)$$

Using the definition of  $V[t, k]$  from (3), the Karush–Kuhn–Tucker conditions<sup>7</sup> for this problem are as follows:

$$\frac{\partial}{\partial t} \left( V[t, k] - \sum_{i=1,2} \lambda_i g_i[k] \right) = h[t_0] + (h[t_0 + k_0 T] - h[t_0])e_h - w[t_0 + T] + (w[t_0 + T] - w[t_0 + k_0 T])e_w = 0, \quad (9)$$

$$\frac{\partial}{\partial k} \left( V[t, k] - \sum_{i=1,2} \lambda_i g_i[k] \right) = h[t_0 + k_0 T]e_h T - w[t_0 + k_0 T]e_w T + \lambda_1 - \lambda_2 = 0, \quad (10)$$

$$g_i[k_0] \leq 0 \quad i = 1, 2, \quad (11)$$

$$\lambda_i g_i[k_0] = 0 \quad i = 1, 2, \quad (12)$$

$$\lambda_i \geq 0 \quad i = 1, 2, \quad (13)$$

<sup>7</sup> The Karush-Kahn-Tucker conditions can be applied for this problem, because it fulfils the linear independence constraint qualification (Nocedal & Wright, 2006, p. 320). Since at most one of the constraints  $g_1$  and  $g_2$  is active for any  $k$  value, the independence is trivial.

where the solution is denoted  $(t_0, k_0)$ , and  $\lambda_i, i = 1, 2$  are the Karush-Kuhn-Tucker multipliers. The stationary points  $(t_0; k_0)$  determined by (9)-(13) are the global maximum points of the utility  $V[t, k]$ , because the utility  $V[t, k]$  is concave with respect to  $t$  and  $k$ , as shown next. The second order conditions are

$$\frac{\partial^2}{\partial t^2} V[t, k] = \frac{\partial}{\partial t} (h[t](1 - e_h) + h[t + kT]e_h - w[t + T](1 - e_w) - w[t + kT]e_w) < 0, \quad (14)$$

$$\frac{\partial^2}{\partial k^2} V[t, k] = \frac{\partial}{\partial k} (h[t + kT]e_h T - w[t + kT]e_w T) < 0, \quad (15)$$

and

$$\frac{\partial^2}{\partial t \partial k} V[t, k] = \frac{\partial}{\partial k} (h[t + kT]e_h - w[t + kT]e_w) < 0. \quad (16)$$

The negativity of the second order conditions can be confirmed by recalling that the marginal utilities  $h[x]$  and  $w[x]$  are monotonically decreasing and increasing, respectively. Therefore,  $\partial/\partial x h[x] < 0$  and  $\partial/\partial x w[x] > 0$ . Further, parameters  $t$  and  $k$  enter the marginal utilities  $h[x]$  and  $w[x]$  in (14)-(16) positively, therefore the derivatives of  $h[x]$  and  $w[x]$  with respect to  $t$  and  $k$  maintain their signs. Finally, notice that  $h[x]$  and  $w[x]$  enter the second order conditions with positive and negative signs, respectively. From here follows that all additive terms in (14)-(16) are negative, making all second order derivatives negative. Hence, the utility is concave with respect to  $t$  and  $k$ .

Knowing that (9)-(13) yield the global maximum points, we can analyse the optimal departure times for Home, Universal, and Work AVs. Although these equations do not reveal the optimal points in a closed form, they are nevertheless sufficient to analyse their relationships. To proceed with that, we need to separately consider the non-binding and binding cases of constraints (11).

If (11) are non-binding, then  $\lambda_1 = \lambda_2 = 0$  due to the complementary slackness conditions (12), and the traveller switches from performing home to work activities during the trip. Then (10) can be rewritten as

$$\frac{\partial}{\partial k} \left( V[t, k] - \sum_{i=1,2} \lambda_i g_i[k] \right) = h[t_0 + k_0 T]e_h T - w[t_0 + k_0 T]e_w T = 0. \quad (17)$$

Using this equality, we can simplify the first stationarity condition (9) for the non-binding case. Being an equation with a single unknown, (18) determines the optimal departure time in the non-binding case:

$$\frac{\partial}{\partial t} \left( V[t, k] - \sum_{i=1,2} \lambda_i g_i[k] \right) = h[t_0](1 - e_h) - w[t_0 + T](1 - e_w) = 0. \quad (18)$$

If one of the constraints (11) is binding, then the traveller spends the entire trip performing either home or work activity. Such a situation would arise, when one of the efficiency factors  $e_h$  and  $e_w$  is much higher than the other, as well as when only one of the factors equals zero or one. In the latter case, we can observe that the non-binding condition (17) would not yield a feasible solution if one of  $e_h$  or  $e_w$  equals zero, and the non-binding condition (18) would not yield a feasible solution if one of  $e_h$  or  $e_w$  equals one. The binding cases also necessarily correspond to Home AV ( $k = 1$ ) or Work AV ( $k = 0$ ), except when  $e_h = e_w = 1$  (which would correspond to a Universal AV). We can derive the optimal departure times for the binding cases by inserting the binding  $k$  values in (9):

$$t_1, \text{ when } k_0 = 1: h[t_1](1 - e_h) + h[t_1 + T]e_h = w[t_1 + T], \quad (19)$$

$$t_3, \text{ when } k_0 = 0: h[t_3] = w[t_3]e_w + w[t_3 + T](1 - e_w). \quad (20)$$

Here and further the optimal departure times for Home, Universal, and Work AV users are denoted  $t_1$ ,  $t_2$ , and  $t_3$ , respectively. We can use the obtained conditions (18)-(20) to analyse the relationship between these three departure times. The results are shown in Table 1. The rows in the table differentiate between scenarios where the maximum efficiency factor of 1 is or is not reached. The columns present results for the three AV types. Parameter  $t^*$  is defined such that  $h[t^*] = w[t^*]$ .

**Table 1. Optimal departure times in case of no congestion**

	$t_1$ – Home AV ( $e_h > e_w$ )	$t_2$ – Universal AV ( $e_h = e_w$ )	$t_3$ – Work AV ( $e_h < e_w$ )
$\max(e_h, e_w) < 1$ :	$t_1 > t^* - T$	$t_1 < t_2 < t_3$	$t_3 < t^*$
$\max(e_h, e_w) = 1$ :	$t_1 = t^* - T$	$t_2 \in [t^* - T; t^*]$	$t_3 = t^*$

The relationship between optimal departure times  $t_1 < t_2 < t_3$  in the first line of Table 1 follows from the non-binding solution in equation (18) as well as from binding solutions (19) and (20). In the non-binding case, the definitions of the three AV types:  $e_h > e_w$  for Home AVs,  $e_h = e_w$  for Universal AVs, and  $e_w < e_h$  for Work AVs should be inserted in (18). In the binding case, it can be noticed that both (19) and (20) contain weighted averages on one side of the equality. The following (in-)equalities arise:

$$t_1 \text{ s. t. } h[t_1] > w[t_1 + T], \quad (21)$$

$$t_2 \text{ s. t. } h[t_2] = w[t_2 + T], \quad (22)$$

$$t_3 \text{ s. t. } h[t_3] < w[t_3 + T]. \quad (23)$$

Recalling that home and work marginal utilities are decreasing and increasing, respectively, it follows that  $t_1 < t_2 < t_3$ . Note that equation (22) holds for any  $e_h$  and  $e_w$  values that are smaller than 1. Hence, they include the conventional vehicle, for which  $e_h = e_w = 0$ . This leads to the conclusion that the users of the Home AV would depart earlier and the users of the Work AV would depart later than the conventional vehicle users. The users of the Universal AV would depart at the same time as conventional vehicle users (given general scheduling preferences where  $e_h = e_w < 1$ ).

The earliest and latest optimal departure times  $t_1 = t^* - T$  and  $t_3 = t^*$  for Home and Work AVs (in the second row of Table 1) follow from inserting  $e_h = 1$  and  $e_w = 1$  in the binding cases (19) and (20), respectively. It can be seen that for any efficiency factors lower than 1, these end-points are not reached, leading to the inequalities  $t_1 > t^* - T$  and  $t_3 < t^*$  in the first row of Table 1. Finally, if both home and work activities are perfectly facilitated in the AV ( $e_h = e_w = 1$ ), then the traveller would experience zero disutility in such a Universal AV and would be indifferent between any departure times in the interval  $[t^* - T, t^*]$ , which is determined by conditions (17) and (18) and  $0 \leq k \leq 1$  (constraints (11)).

Hereby, this section has obtained that, in case of general scheduling preferences, travellers whose home activities are better facilitated on board than work activities, would depart earlier than conventional vehicle users. Similarly, travellers whose work activities are better facilitated on board than home activities, would depart later than conventional vehicle users. This result holds even if there is no congestion. The implication of this finding is that a hypothetical traveller

population with identical general scheduling preferences would, upon replacing their conventional vehicles with a mixture of Home, Universal and Work AVs, disperse with respect to their departure times. All departures would however still fit in the interval  $[t^* - T, t^*]$ .

### 3.2 Optimal departure times with $\alpha - \beta - \gamma$ scheduling preferences

It is useful to note that the departure time sequence  $t_1 < t_2 < t_3$  does not hold for the  $\alpha - \beta - \gamma$  scheduling preferences. Due to the discontinuity of  $w[x]$ , we cannot follow the same derivation as in the case of the general scheduling preferences. However, it is intuitive from Figure 2 that the optimal departure time generally equals  $\tilde{t} = t^* - T$ . Formally, it can be shown that only in two cases, the optimal departure time would be  $t^*$  instead of  $\tilde{t}$ : when  $e_w > \gamma/(\beta + \gamma)$  for Work AV and when  $(1 - e_h)/(1 - e_w) > 1 + \gamma/\alpha$  for Universal AV. Further, it can be demonstrated that optimal departure time is necessarily  $\tilde{t}$ , if it is (conservatively) assumed that  $e_w$  does not exceed 0.5 and that  $\beta < \alpha < \gamma$  (as is conventional). The proofs of these results are in Appendix A.

The special case of optimal departure time being  $t^*$  rather than  $\tilde{t}$  is intuitive for large  $e_w$ : excellent facilitation of work during travel should incentivise the traveller to travel when work, rather than home, activities are most valuable, which is the meaning of the preferred work start time  $t^*$ . However, one could argue that this special case also counters the common usage of the  $\alpha - \beta - \gamma$  model: it is usually assumed that being late at work is worse than being early ( $\gamma > \beta$ ). Therefore, the researcher may consider other scheduling preferences in such scenarios. Section 5.2 further discusses the choice of scheduling preferences.

## 4. Case of congestion

In order to analytically study the changes in congestion patterns, we need to assume that travellers have certain shape of departure time preferences. The previous section showed that, while general scheduling preferences lead to changing optimal departure times even if there is no congestion, the  $\alpha - \beta - \gamma$  preferences lead to the same optimal departure time, unless work activity is very well facilitated on board. This makes the  $\alpha - \beta - \gamma$  preferences an interesting case to be studied in the congestion setting: it would provide a conservative prediction for changes in congestion patterns, which can serve as a good starting point. Furthermore,  $\alpha - \beta - \gamma$  preferences have a well-known closed form-solution for the equilibrium flow rate in a bottleneck setting – the number of travellers departing at every time moment, obtained by Arnott et al. (1990) –, which has contributed to their continuing popularity for congestion modelling. For these reasons, I adopt this form of scheduling preferences from now on. However, note that  $\alpha - \beta - \gamma$  preferences in general and in the current application have some limitations; see section 5.2.

The following derivations assume the most minimalistic bottleneck setting, where a number of individuals with the same scheduling preferences travel from a single origin to a single destination on a single route. Free-flow travel time is assumed to be zero, such that the total travel time equals the queueing time at the bottleneck.<sup>8</sup>

### 4.1 Congestion with conventional vehicles

Before proceeding to compute the equilibrium congestion patterns for AVs, it is useful to recap how this is done for conventional vehicles (as per Arnott et al., 1990). As introduced in equations (4)-(5), the  $\alpha - \beta - \gamma$  preferences contain a preferred arrival time  $t^*$ , when the individual starts to value being at work higher than being at home. Because everyone would like to arrive at work at exactly  $t^*$  (assuming homogeneous preferences), congestion arises – travel time is longer for trips that end around  $t^*$ . The departure time that leads to arrival at exactly  $t^*$  and is associated with the

<sup>8</sup> It can be verified that this assumption does not limit the generality of the results: the equilibrium flow rates follow from the travel time changes due to queuing (see the derivations in the next sections 4.1 and 4.2); start and end times of congestion, as well as the on-time departure time would be shifted earlier by a positive free-flow time (due to condition 3 in Appendix B).

longest travel time is denoted  $\tilde{t}$  and called the ‘on-time departure time’. Eventually, it is assumed that the disutility caused by schedule delay and travel time at all departure times is perfectly balanced. This condition corresponds to the Nash equilibrium. In other words, as anyone would consider departing at another time, the gained and lost utility from so doing would cancel each other out.

Figure 4 illustrates a case where a traveller would consider postponing his departure by one time unit. The gained utility from home activity is  $\alpha$ , whereas the lost utility from work activity is  $\alpha - \beta$ , if traveller arrives early, and  $\alpha + \gamma$ , if he arrives late. If we want to obtain the travel utility difference between these two hypothetical departure times, then the utility loss at the destination should be multiplied with the arrival time difference between the two considered departure times (because the travel times may differ at both considered departure times). This arrival time difference time is  $1 + \dot{D}/s$ , where  $\dot{D}$  is the change in queue length at time  $t$ :  $\dot{D} = r[t] - s$ . Here,  $r[t]$  is the number of individuals departing at time  $t$ , and  $s$  is the number of travellers that can pass through the bottleneck (i.e. the bottleneck capacity).

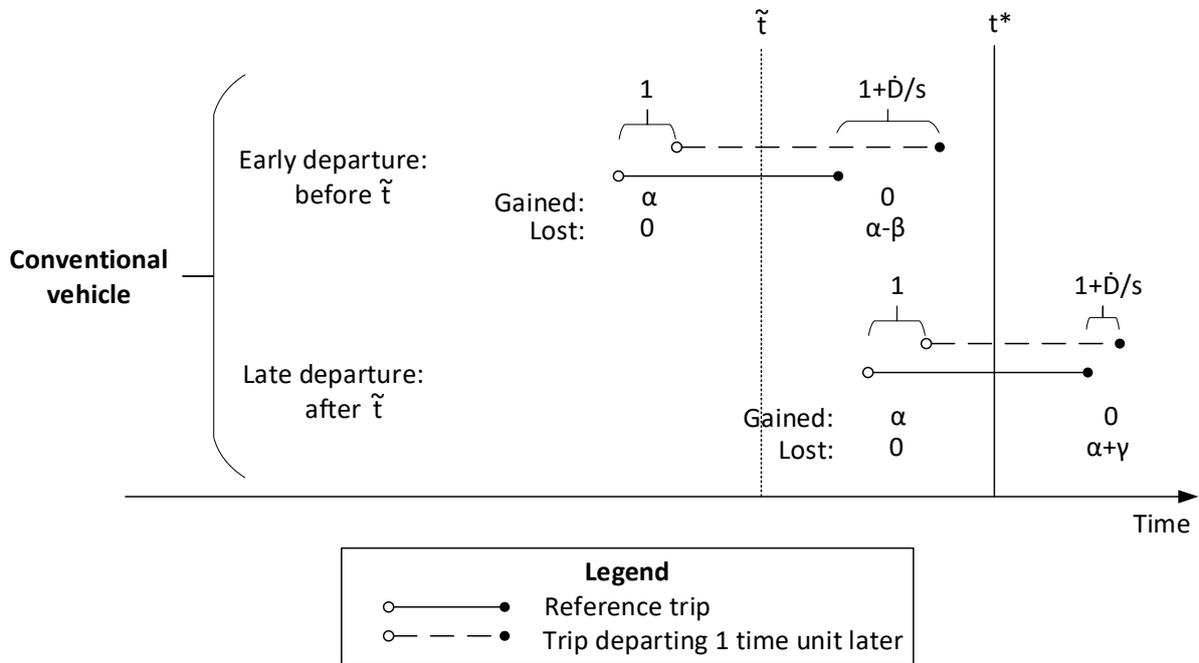


Figure 4. Utility components for computing equilibrium flow rate with conventional vehicles (CV)

By equalling the gained and lost utilities (as illustrated in Figure 4), we can obtain the flow rate  $r[t]$ :

$$r[t] = \begin{cases} \frac{\alpha s}{\alpha - \beta}, & \text{if } t \in [t_q, \tilde{t}) \\ \frac{\alpha s}{\alpha + \gamma}, & \text{if } t \in (\tilde{t}, t_{q'}], \end{cases} \quad (24)$$

where  $t_q$  and  $t_{q'}$  are times at which congestion begins and ends. At these end-points of the congestion, the travel (or queueing) times are zero, but the earliness or lateness (respectively) is at its maximum. Conversely, as explained before, queueing time is longest at the on-time departure time  $\tilde{t}$ . Using an equation system, Arnott et al. (1990) further derived these three times:

$$t_q = t^* - \frac{\gamma N}{\beta + \gamma s}, \quad (25)$$

$$t_{q'} = t^* + \frac{\beta}{\beta + \gamma} \frac{N}{s}, \quad (26)$$

$$\tilde{t} = t^* - \frac{\beta\gamma}{\alpha(\beta + \gamma)} \frac{N}{s}, \quad (27)$$

where  $N$  is the number of travellers. Equations (24)-(27) fully describe the congestion pattern with conventional vehicles.

#### 4.2 Congestion with automated vehicles

Moving on to AVs, it has so far been established that the scheduling preferences of AV users would differ from those of the conventional vehicle users and depend on the activity performed during travel (see section 2). This section analyses the changes in congestion that stem from such scheduling preferences of AV users. Note that this section does not consider the other major source of potentially changed congestion patterns with AVs: their ability to drive closer to each other and thereby increase road capacity, especially in high penetration scenarios (e.g., Wadud et al., 2016). This simplification is made for two reasons. First, omitting the capacity changes allows to isolate the effect of various on-board activities on congestion. Second, the relative magnitude of capacity increase to the changes in scheduling parameters is rather unclear: see van den Berg and Verhoef (2016) for how net congestion patterns (assuming a generic on-board activity) depend strongly on these relative magnitudes.

The most intuitive approach, when studying how changes in scheduling preferences may affect the congestion, is to consider, whether the changes can be expressed as a transformation of the parameters  $\alpha$ ,  $\beta$ ,  $\gamma$ . If such transformation could be found, we could use the results (24)-(27), while only modifying the parameters therein. For the Home AV such a transformation is intuitive. Replacing  $\alpha$  with  $\alpha(1 - e_h)$  leads to the desired result (and replicates the result of van den Berg & Verhoef, 2016). In case of Universal and Work AVs however, it is not immediately clear what transformation of the  $\alpha$ ,  $\beta$ ,  $\gamma$  parameters would capture the AV impact on travel costs (see Figure 2).<sup>9</sup> Therefore, it is necessary to follow the path of Arnott et al. (1990) to obtain the equilibrium flow rates for these AVs. As the on-board activities lead to more complex forms for the equilibrium flow rates, Figure 5 is helpful in the derivations. Similarly to Figure 4 for conventional vehicles, Figure 5 shows all the utility components needed to compute the flow rates for AVs. Compared to the Home AV, it can be seen that the Universal and Work AV results contain an additional line for computing the equilibrium flow rate. This is needed because the utility of time spent in the AV changes depending on the clock time. Before  $t^*$ , the utility during travel is obtained from home activity carried out in a Universal AV or early work activity carried out in a Work AV. After  $t^*$ , the utility is obtained from late work activity carried out in either the Universal or Work AV.

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<sup>9</sup> Note that such transformation can be derived for Home and Work AVs if the utility of stationary and on-board activities is assumed to always differ by a fixed amount – the additive set-up developed in Yu et al. (2019).

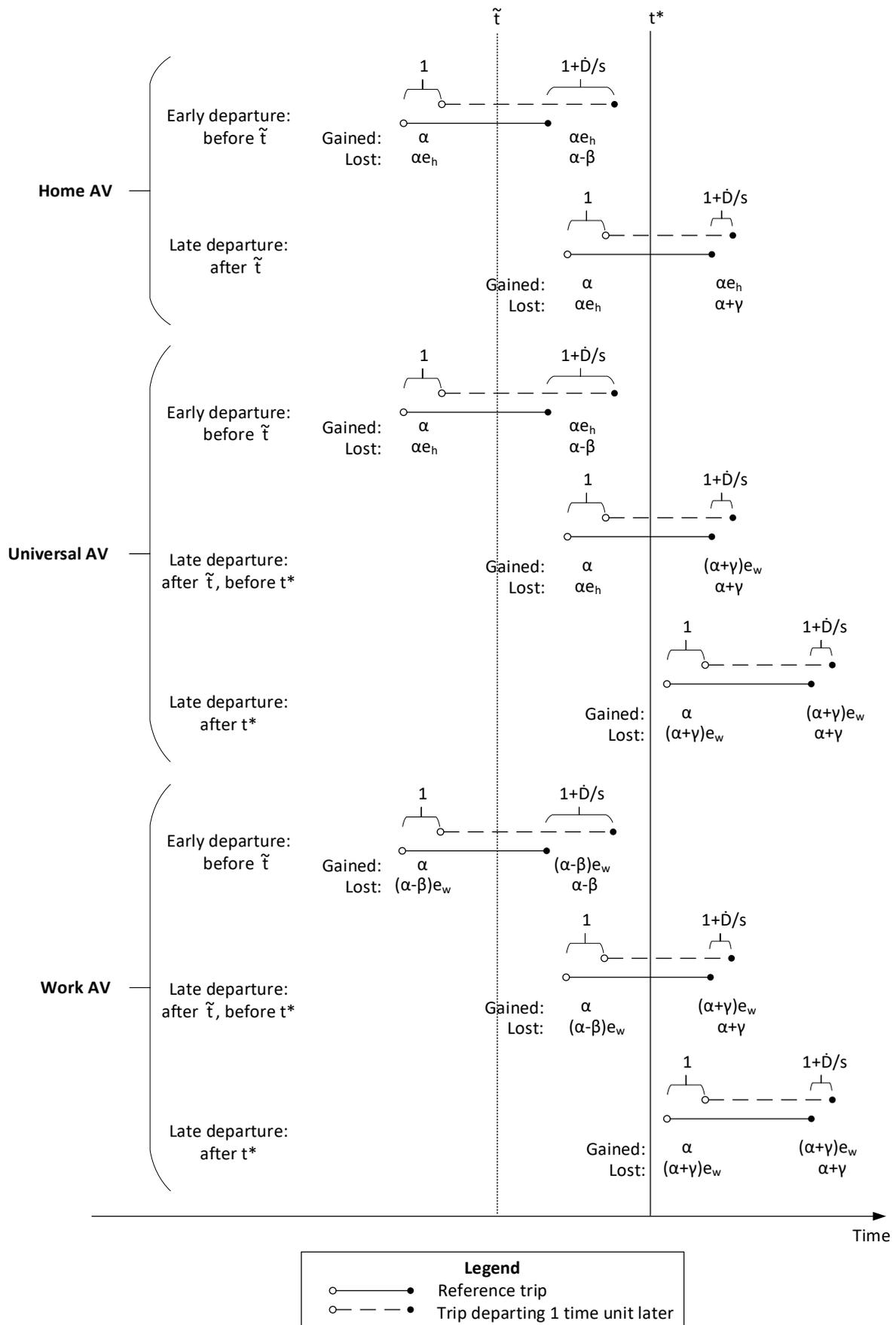


Figure 5. Utility components for computing equilibrium flow rates with AVs

Table 2 shows the results: the parameters needed to fully describe congestion patterns with AVs. The equilibrium flow rates are derived from Figure 5 by balancing the utility components in each line. The congestion start, end and on-time departure times are derived in Appendix B. It is found that the start and end times of congestion are the same for conventional vehicles and all AVs, while the on-time departure time is earlier for all AV-types than for the conventional vehicles, and even earlier, if the AV facilitates home activities. The last row in Table 2 indicates that the results are valid only for the specified relationships between efficiency factors  $e_h$ ,  $e_w$  and the parameters  $\alpha$ ,  $\beta$ , and  $\gamma$ . These conditions follow from the definitions of the three AV types and from a requirement that the flow rates are positive. It can be shown that these conditions are stronger than the sufficient condition for the optimal departure time to be  $\tilde{t}$  in the no-congestion case (i.e.,  $e_w < 0.5$ , as derived in Appendix A). As an example, for common values in the literature  $\alpha = 2$ ,  $\beta = 1$ ,  $\gamma = 4$  (Small, 1982, 2015), the highest possible  $e_h$  that satisfies the conditions in Table 2 would be 0.5, and the highest possible  $e_w$  would be 0.33. The values of Small and efficiency factors of 0.3 (for home and/or work activities, depending on AV type) are used for the following illustrations of the congestion patterns.<sup>10</sup>

**Table 2. Flow rates, congestion start, end times, on-time departure times for homogeneous AV population**

	Home AV	Universal AV	Work AV
Optimal on-board activity before $t^*$	Home	Home	Work
Optimal on-board activity after $t^*$	Home	Work	Work
Equilibrium flow rate $r[t]$	In departure time interval $t \in [t_q, \tilde{t}]$ :		
	$\frac{\alpha(1 - e_h)}{\alpha(1 - e_h) - \beta} s$	$\frac{\alpha(1 - e_h)}{\alpha(1 - e_h) - \beta} s$	$\frac{\alpha - (\alpha - \beta)e_w}{\alpha - (\alpha - \beta)e_w - \beta} s$
	In departure time interval $t \in [\tilde{t}, t^*]$ :		
	$\frac{\alpha(1 - e_h)}{\alpha(1 - e_h) + \gamma} s$	$\frac{\alpha(1 - e_h)}{\alpha - (\alpha + \gamma)e_w + \gamma} s$	$\frac{\alpha - (\alpha - \beta)e_w}{\alpha - (\alpha + \gamma)e_w + \gamma} s$
	In departure time interval $t \in [t^*, t_q']$ :		
	$\frac{\alpha(1 - e_h)}{\alpha(1 - e_h) + \gamma} s$	$\frac{\alpha - (\alpha + \gamma)e_w}{\alpha - (\alpha + \gamma)e_w + \gamma} s$	$\frac{\alpha - (\alpha + \gamma)e_w}{\alpha - (\alpha + \gamma)e_w + \gamma} s$
Congestion start time $t_q$	$t^* - \frac{\gamma}{\beta + \gamma} \frac{N}{s}$	''	''

<sup>10</sup> The efficiency of 0.3 may appear low at first, considering that Wardman and Lyons (2016) summarised several studies that found comparable productivity of work during travel and outside of it: i.e.,  $q \approx 1$  in Hensher (1977) equation. Note, however, that this productivity applies only to the 0-50% of travel time that is used for work in different modes (parameter  $p$  in Hensher's equation). If the remaining travel time would be characterised by lower productivity (hence, the travellers not using it for work), then an average factor of 0.3 does not seem entirely unrealistic. Nonetheless, clearly, much future work is needed to calibrate these parameters, considering also AV and commute trip contexts; see more discussion in sections 5.2 and 5.3.

On-time departure time $\tilde{t}$	$t^* - \frac{\beta\gamma}{\alpha(1-e_h)(\beta+\gamma)} \frac{N}{s}$	$t^* - \frac{\beta\gamma}{\alpha(1-e_h)(\beta+\gamma)} \frac{N}{s}$	$t^* - \frac{\beta\gamma}{(\alpha - (\alpha - \beta)e_w)(\beta + \gamma)} \frac{N}{s}$
Congestion end time $t_{q'}$	$t^* + \frac{\beta}{\beta + \gamma} \frac{N}{s}$	''	''
Conditions	$e_h < \frac{\alpha - \beta}{\alpha} e_w$ $e_w < \frac{\alpha + \gamma}{\alpha} e_h$	$\frac{\alpha - \beta}{\alpha} e_w < e_h < \frac{\alpha - \beta}{\alpha} e_w$ $\frac{\alpha + \gamma}{\alpha} e_h < e_w < \frac{\alpha + \gamma}{\alpha} e_h$	$e_h < \frac{\alpha - \beta}{\alpha} e_w$ $e_w < \frac{\alpha + \gamma}{\alpha} e_h$

The resulting congestion shapes for all AV-types and the base conventional vehicle are illustrated in Figure 6. This and later figures use queuing time as an indicator for the severity of the congestion. The queuing time  $T[t]$  is a function of the departure rate  $r[u]$ :

$$T[t] = \int_0^t \frac{r[u] - s}{s} du. \tag{28}$$

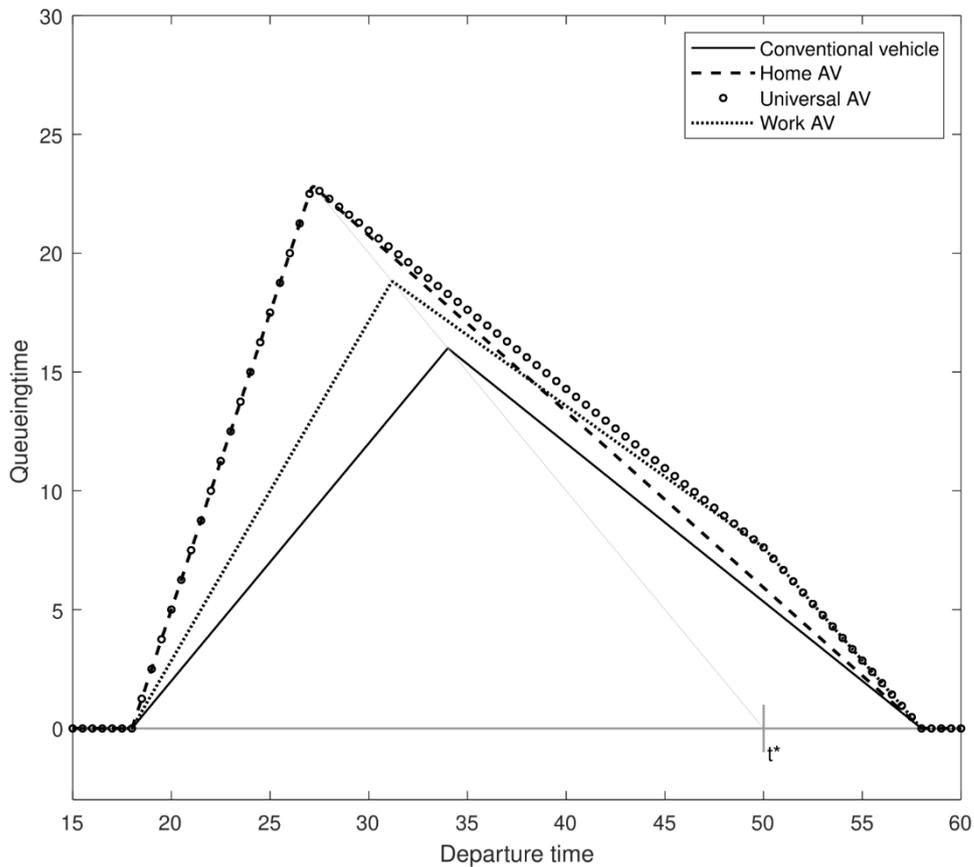


Figure 6. Development of queuing times for conventional vehicles and AVs.  $N = 200$ ,  $s = 5$ ,  $(\alpha, \beta, \gamma) = (2, 1, 4)$ ,  $t^* = 50$ ,  $(e_h; e_w) = (0.3; 0)$  for Home AVs,  $(e_h; e_w) = (0.3; 0.3)$  for Universal AVs,  $(e_h; e_w) = (0; 0.3)$  for Work AVs.

Four properties of AV congestion can be observed from Figure 6. First, congestion is more severe with AVs compared to conventional vehicles (at least while not considering any potential capacity effects of AVs). This result is intuitive – performing any activity during travel leads to less negative

experience of travel, and consequently, lower aversion to the most congested and longest travel times. And this result aligns with previous works that do not differentiate between on-board activities (and specifically, with van den Berg & Verhoef, 2016, who use bottleneck models). Second, congestion is more skewed to earlier times for the Home AVs and to later times for Work AVs. This finding aligns with Yu et al. (2019) who found that when an on-board activity closer resembles home- (work-) activity (defined differently than here), then the AV users will travel in the beginning (end) of the peak. Universal AVs (in Figure 6) partially overlap with both Home and Work AV graphs, thereby increasing both early and late congestion. It is noteworthy that these results of skewed congestion follow from the  $\alpha - \beta - \gamma$  preferences, which do not lead to any changes in optimal departure times in the no congestion case (see section 3.2). Intuitively, an even stronger skew in congestion could be expected, if general scheduling preferences were used. Third, Home and Universal AVs lead to longer maximum queueing times than Work AVs. It can be shown that this property holds when  $\alpha e_h^H > (\alpha - \beta)e_w^W$ , where  $e_h^H$  is the efficiency of home activities in the Home or Universal AV, and  $e_w^W$  is the efficiency of work activities in the Work AV. This condition determines that the Home AV is not inferior to Work AV in terms of the on-board activity facilitation. In the converse scenario, the queueing times of Home AVs would be shorter than of Work AVs at all departure times. Fourth, congestion starts and ends at the same time for all vehicles. This leads to the conclusion that, although congestion levels are increasing, the experienced costs of congestion do not change. The proofs of these four properties are in Appendix C.

#### 4.3 Congestion with mixed vehicles

Given that all AV types intensify the congestion, but possibly in different directions, it is useful to see the net congestion effect of having different AVs in the population. Arnott et al. (1994) demonstrated how this can be done using the so-called Travel Equilibrium Frontier (TEF). The idea of the TEF is that the travellers are indifferent between departing at any moment  $t \in [t_q, t_{q'}]$ , given that the queueing times are as depicted in Figure 6 (for their vehicle type). They would therefore not use any departure times when the queueing times are longer – which is whenever a graph of other traveller group lies above theirs. Furthermore, note that decreasing or increasing the number of travellers  $N$  adjusts the graph proportionally ‘down or up’ (the duration of congestion is always  $N/s$ ). Similarly, to obtain the TEF with a specified number of travellers using each vehicle, the graphs need to be ‘scaled down or up’, such that all travellers of each group depart during the time intervals, when their graph lies above other graphs.

Three combinations of vehicles are used to demonstrate the congestion patterns in Figure 7: Home AVs and conventional vehicles (top left of Figure 7), Work AVs and conventional vehicles (top right), and Home and Work AVs (bottom).<sup>11</sup> For every pair, Figure 7 shows scenarios with 25%, 50%, 75% of travellers using each vehicle, as well as the corresponding homogeneous cases from Figure 6. The different shades of grey represent the scenarios with various vehicle shares; lane types (solid, dashed, dotted) represent vehicle types departing at every moment during the congestion. Note that a part of the graph stays unchanged for every mixture – this part overlaps with the congestion graph of the vehicle with lower peak (e.g., the graph of conventional vehicles on the top left of Figure 7). Considering any type of AV, the graphs demonstrate, in line with van den Berg and Verhoef (2016) and F. Zhang et al. (2020), that having a mixture of AVs and conventional vehicles leads to the AVs occupying the central departure time interval, and the conventional vehicles departing as the first and last in the congestion. Furthermore, higher share of AVs in the mixture leads to more severe congestion. This is intuitive: being able to perform any on-board activity reduces the travel time costs and makes AV users less averse to long travel times. However, note once again that higher AV shares do not lead to increased bottleneck capacity in the present model.

<sup>11</sup> The MATLAB code used to create figures can be found in Pudāne (2020), <https://doi.org/10.4121/13247633>.

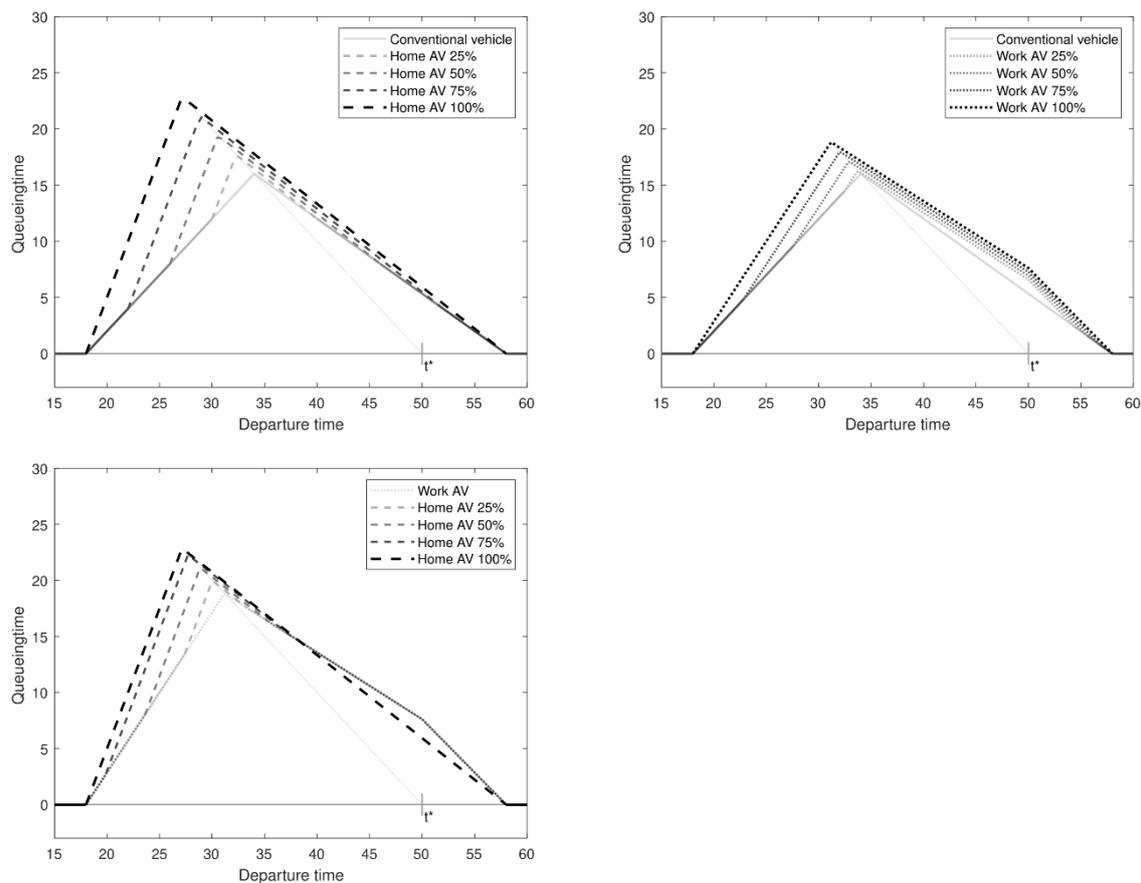


Figure 7. Development of queueing times with mixture of conventional vehicles and Home AVs (top left), conventional vehicles and Work AVs (top right), Home AVs and Work AVs (bottom).  $N = 200$ ,  $s = 5$ ,  $(\alpha, \beta, \gamma) = (2, 1, 4)$ ,  $t^* = 50$ ,  $(e_h; e_w) = (0.3; 0)$  for Home AVs,  $(e_h; e_w) = (0; 0.3)$  for Work AVs.

Considering the congestion effects of different AVs, Work AVs depart later than Home AVs and conventional vehicles (top right and bottom of Figure 7). Given a mixture of Home and Work AVs (bottom of Figure 7), Work AVs reduce congestion, unless Home AVs are inferior to Work AVs in terms of the on-board activity experience (in a sense explained at the end of section 4.2: the third congestion property). However, if Home AVs are inferior, the converse is true and they reduce the congestion; the effect then resembles the combination of Work AVs and conventional vehicles (top right of Figure 7). In general, the higher the efficiency of on-board activities in AVs, the more likely are the AV users to cause severe congestion, and the more likely they are to benefit from sharing a road with travellers whose activities are less well facilitated on board (their graphs are 'scaled down'). See van den Berg and Verhoef (2016) and Yu et al. (2019) for an in-depth discussion of how, in terms of congestion benefits or costs, the AV introduction affects their users and the users of conventional vehicles.

Hereby, this section has demonstrated that, given the  $\alpha - \beta - \gamma$  scheduling preferences and bottleneck setting, travellers whose home activities are better facilitated on board than work activities, would prefer to depart earlier than conventional vehicle users and increase the severity of congestion in its early to middle part. Similarly, travellers whose work activities are better facilitated on board than home activities, would prefer to depart later than conventional vehicle users and increase the congestion mostly in its middle to late part. Given similar levels of activity facilitation on board, the increase in queueing times due to Work AVs is smaller than due to Home AVs. Thereby, Work AVs have a moderating effect on the increasing congestion levels.

## 5. Discussion and suggestions for further research

### 5.1 Comparison with the travel time penalty approach

This work started from a proposition that it is important to differentiate among on-board activities when modelling departure time choice and congestion patterns. It was assumed that, similarly to out-of-vehicle activities, the utility of different on-board activities varies with clock-time. In contrast, the travel time penalty approach assumes that the utility of on-board activities is time-independent. Now we are in a position to ask: has the approach taken in this paper yielded qualitatively different results than the travel time penalty approach would have?

In the case of no congestion and general scheduling preferences, the answer is 'yes'. If the utility of on-board activities did not vary with time, the on-board activities would not influence the departure time preference, and the optimal departure time of conventional vehicles would be maintained. Formally, the second integrals of the total home and work utility functions (1) and (2) would not depend on  $t$ , and hence would disappear when the total utility (3) is differentiated with respect to  $t$ . However, clearly, the conclusion of section 3 was that the optimal departure times depend on the activities performed during travel - or on the use of various AV types that facilitate various activities to different extents.

In the case of congestion and the  $\alpha - \beta - \gamma$  preferences, the answer is 'yes, but with an exception'. Different congestion patterns were obtained for Home-, Work-, and Universal AVs. However, because the  $\alpha - \beta - \gamma$  preferences assume constant home utility, the results of Home AV exactly replicate the travel time penalty approach (as derived in van den Berg & Verhoef, 2016). Since it is furthermore known that a constant home utility is a rough approximation (Tseng & Verhoef, 2008), this correspondence is not desirable. A way to avoid this situation would be to adapt other scheduling preferences where both home and work activity utility varies with clock-time. The following section further discusses this possibility.

### 5.2 Beyond $\alpha - \beta - \gamma$ scheduling preferences and multiplicative efficiency factors

As just mentioned, adopting the  $\alpha - \beta - \gamma$  scheduling preferences has some potential drawbacks. First, it means that the scenario, where the traveller engages in home activities, is constrained to be equivalent to the scenario where AVs generally provide better travel experience (e.g., less stressful, smoother drive), which is embedded in the travel time penalty concept. Second, the assumption of constant home utility has been challenged before (Tseng & Verhoef, 2008). Finally, using the  $\alpha - \beta - \gamma$  preferences allows the researcher to arrive at closed-form departure rates only for low to medium  $e_h$  and  $e_w$  values (see 'Conditions' in Table 2). For these reasons, exploring the effects of other scheduling functions on the congestion changes with AVs, while differentiating between home and work activities performed on board, is a highly recommended direction for further research. The literature offers good alternatives for this endeavour: the so-called slope model (Fosgerau and Engelson, 2011), where the marginal utilities of out-of-vehicle activities are linear functions of time, or exponential scheduling preferences (Hjorth et al., 2015). A closed-form departure rate function for the slope model has recently been derived (Xiao et al., 2017) and would be useful for such study.

It can be expected that replacing the  $\alpha - \beta - \gamma$  model with any type of general scheduling preferences (such as slope model or exponential preferences) would lead to larger congestion differences between conventional vehicles and AVs and among different AVs. Because of this consequence however, the weakness of the  $\alpha - \beta - \gamma$  model is also its strength: the current approach provides conservative results - a lower bound of the possible influence of on-board activities on congestion patterns, which would apply even in contexts with a strong preference for a single work-start time.

Another feature of the current model set-up that deserves further discussion is the assumption of multiplicative efficiency factors. This assumption implies that the utility of on-board home and work activities depends on the clock-time in a similar way as the utility of stationary activities.

Considering stationary activities, Fosgerau and Small (2017) discuss how their time-dependent utilities emerge from the benefits of agglomeration in time. That is, it is often important to synchronise the working hours within companies and industries (e.g., teamwork, work with clients), as well as to synchronise them with supporting services (e.g., childcare). Similarly, many leisure or home activities require the members of a household or social circle to be simultaneously available, as well as some leisure activities (e.g., theatre, TV programs, opening hours of bars) are scheduled considering such typical available times. To some extent, this clock-time dependence could be assumed to translate to on-board activities. However, unlike for stationary activities, the utility variations of on-board activities are not influenced by availability of physical spaces (such as opening hours of shops) or presence of other people, unless remote communication with them is sufficient. Hence, future work should further explore whether this utility reduction is, first, proportional, and second, whether it depends only on the utilities of stationary activities. As for the first, other functional forms could be used. For example, Yu et al. (2019) penalise the on-board activities with an additive factor. Other forms could reflect, for example, that some travellers may not be able to work on board after the preferred arrival time, but they may engage in preparatory work tasks during travel. Ultimately, the functional form should be determined empirically. As for the second assumption, it is evident that the utility of on-board activities could depend also on travel-specific conditions that vary within a trip: winding road or congestion may obstruct or ease activities for only some portions of the trip. Furthermore, travellers may choose to switch between home and work activities multiple times during the journey (as was observed by, e.g., Pawlak et al., 2017), especially if both activities are well facilitated on board. Incorporating such dependencies and multiple switches in a bottleneck model would be challenging, and would likely require a simulation approach.

### *5.3 Validity and applicability to public transport and shared automated vehicles*

As with all travel behaviour models, an important aspect is their validation and estimation. While there are not yet sufficient number of AVs on the roads, studies have occasionally turned to public transport to gain insights into possible effects of on-board activities (e.g., Pawlak et al., 2015; Malokin et al., 2019). Hence, a relevant question to the present study is: would the devised models apply and could they be validated using public transport data? Unfortunately, there are several important obstacles to such an application. First, future AVs could be expected to perform significantly better in facilitating on-board activities compared to current public transport. The difference may be even larger when considering on-board activities that substitute out-of-vehicle activities: recall the examples of morning home activities - getting ready, preparing and eating breakfast, getting a little more sleep - or work activities - replying to emails, planning the day, adjusting meeting schedule. Several of these may require privacy, space, silence, continuity (absence of transfers), comfort and facilities that may be available in AVs, but not in public transport. On the flipside, public transport may outperform AVs with regard to proneness to motion sickness. (See Pudāne et al., 2019, for a qualitative discussion of the potential advantages and disadvantages of AVs for on-board activities.) Second, trade-offs involved in departure time choices are fundamentally different for car and public transport users: while car drivers trade off on-time arrival with travel time, public transport users balance on-time arrival with crowding levels and to a lesser extent, travel time and reliability. Third, public transport users face constraints (which the car drivers do not) when choosing departure time: they must choose from a set of scheduled departure times or predicted departure times according to public transport frequency. These characteristics would make the departure time choice model for a public transport user, who is able to engage in on-board activities during travel, fundamentally different from the model presented in this paper. Therefore, other sources of travel behaviour and departure time data could be more useful for estimation and validation of the current models: naturalistic experiments (Harb et al., 2018) or surveys (for example, stated choice experiments), which have been shown to provide trustworthy results in AV contexts (Wadud & Huda, 2019). This is an important direction for further research.

Nevertheless, even before having access to data supporting the current models, it is possible to argue for their face-validity. The current work builds on established microeconomic models of scheduling preferences (Vickrey, 1969, 1973; Small, 1982), which have stood the test of time to predict departure time choice and resulting congestion patterns in a variety of contexts. Furthermore, the analytical results correspond to intuition: the possibility to substitute home or work activities with their on-board counterparts leads to departure time adjustments towards the most desirable time for these activities.

Another often asked and important question is: how would travel experience and behaviour differ between users of privately owned and shared AVs (including both car sharing and ride sharing), and would the same models be valid for these modes? Considering the current departure time choice model, two differences could be anticipated. First, the on-board activities may be facilitated to a different extent in shared AVs. The activities may be impaired by the reduced privacy, storage and personalisation possibilities, which would be available in privately owned AVs. At the same time, the facilitation may be increased, if fleet owners customise the AVs to suit various on-board activity needs. For example, some cars may be equipped with business and conference facilities, while others may be suited for resting and leisure. The net effect of sharing on the efficiency of on-board activities is an interesting question for future research. Second, clients of car and ride sharing may have less flexibility of choosing their departure time as compared to owners of vehicles: they may need to book the car in advance or coordinate with other users. Hence, the departure time choice and congestion models for future AV owners and users of shared AVs may differ somewhat; yet, the present model can provide a good starting point for modelling these scenarios.

#### *5.4 Suggestions for further research*

This work has presented the first steps in a detailed analysis of the impact of different on-board activities on congestion patterns. Nevertheless, and as importantly, it opens up a new field of study into the AV-effect on future mobility – and invites further work to investigate whether the proposed peak-skewing, increasing and moderating effects are also observed in more complex contexts. Previous sections mentioned the need to explore other scheduling preferences and specifications of on-board activity utility (section 5.2), as well as to obtain data to estimate and validate the current models (section 5.3). Following are few other suggestions for further research.

1. A natural extension of the present work would be to simulate the effects of the proposed scheduling preferences in artificial and real city networks, as done by Correia and van Arem (2016), while incorporating heterogeneity in scheduling parameters. An extended simulation would also include other types of choices, such as mode- and route-choices, trip making and destination choice, to balance the effects of departure time changes with other anticipated AV effects, such as induced travel. Potentially increased road capacity in high AV penetration scenarios would also need to be considered.
2. An important extension would be to account for various on-board activities when modelling the full day of a commuter and account for the flexibility of work hours, as is done in the activity-based bottleneck analyses (Xiaoning Zhang et al., 2005; Li et al., 2014; Xiang Zhang et al., 2019) and studies of departure time choice (e.g., Thorhaug et al., 2016). Some flexibility in activity schedules in general and work start times in particular is a prerequisite for the congestion shifting and moderating effects observed in this work.
3. Empirical work should continue in assessing the sources of decreasing travel time disutility in AVs. Note that the peak-mitigating effect would come into play only if on-board activities constitute a significant portion of the AV-benefits. If instead the travellers mainly appreciate the reduced burden and increased comfort when using AVs (as argued by Singleton, 2019) or even experience some disadvantages of converting resting time into busy activity time (Shaw et al., 2019; Pudāne et al., 2019), they would constitute a more homogeneous group, and hence, be more prone to intense congestion.

4. Finally, it would be important to incorporate potential endogeneity effects in the model. Travellers whose work or home activities can be performed on board may self-select to obtain access to certain type of AVs. The approach here could follow F. Zhang et al. (2020), who included a choice between conventional vehicles and (a generic type of) AVs into bottleneck congestion analysis.

## 6. Conclusions and policy implications

The arrival of automated vehicles (AVs) is expected to increase the feasibility and role of on-board activities in people's daily schedules. This paper argued that the current ways of modelling the departure time choice and congestion impacts of the improved on-board activities, based mostly on the idea of a reduced travel time penalty, are not sufficient. While travel time penalty condenses effects of all on-board activities into a single indicator, different activities may in reality have varied impacts on travel behaviour. This intuition was supported in the present paper. A classical microeconomic approach – modelling departure time choices and their congestion impacts using scheduling functions – was extended to consider effects of different on-board activities in AVs. It was obtained that, if travellers are able to perform home activities on board (in Home AVs), they prefer to depart earlier than if they are able to perform work activities (in Work AVs), even if there is no congestion. If there is congestion, results obtained in a minimalistic bottleneck setting indicate that congestion would increase due to on-board activities in AVs, – doing something during travel decreases people's aversion to longer travel times, thereby prioritising on-time arrival and concentrating travellers in the middle of the peak. However, if several AV types are available that facilitate home and/or work activities to a similar extent, then Work AVs increase the congestion levels the least.

The model developed and results obtained in this paper can provide input for one of the key AV-related policy questions: will AVs lead to higher congestion levels and, if yes, how to avoid or mitigate that effect? While congestion can be expected to increase, at least while assuming no increases in road capacity due to AVs, travellers who are able to work during travel seem to mitigate that effect. This offers a valuable tool for policy makers: although some work tasks may be easily transferred to AVs, the mobile work possibilities could be further encouraged by allowing flexible working hours and, perhaps even, making work-equipped AVs available for a broader range of professions. Such measures should be further tested using models that account for possibly diverging effects of different on-board activities (such as the one presented in this paper, but also by Yu et al., 2019), while accounting for the model limitations outlined earlier. If their effects are positive, these measures could help to ensure that the celebrated benefits of AVs – such as allowing individuals to re-allocate their travel time for other activities – are maintained, while their potential downsides are reduced.

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## Appendix A. Proofs of the optimal departure times with $\alpha - \beta - \gamma$ scheduling preferences in case of no congestion

**Proposition A1.** If the optimal departure time without congestion using  $\alpha - \beta - \gamma$  preferences is not  $\tilde{t} = t^* - T$ , then it must be  $t^*$ .

### Proof.

Departure time  $\tilde{t}$  is better than any departure time  $t < \tilde{t}$ . The earlier departure times  $t$  would incur the same costs during travel as departing at  $\tilde{t}$  (being the lost utility due to on-board activity being less efficient than home activity). However, the early departure would also incur costs due to arriving early.

Departure time  $t^*$  is better than any departure time  $t > t^*$ . The later departure times  $t$  would incur the same costs during travel as departing at  $t^*$  (being the lost utility due to on-board activity being less efficient than work activity). However, the later departure would also incur costs due to performing home instead of work activity after  $t^*$ .

Departure times between  $\tilde{t}$  and  $t^*$  have either monotonously increasing or decreasing utility, which depends on whether a travel time unit costs more before or after  $t^*$ , see Figure 2. Therefore, the optimal departure time is either  $\tilde{t}$  or  $t^*$ .

**Proposition A2.** Optimal departure time is  $t^*$  in two cases only: when  $e_w > \gamma/(\beta + \gamma)$  for Work AV or when  $(1 - e_h)/(1 - e_w) > 1 + \gamma/\alpha$ .

### Proof.

The necessary and sufficient condition for  $t^*$  to be the optimal departure time is that unit costs of travel before  $t^*$  is higher than after  $t^*$ .

For Home AVs, the condition equals  $\alpha(1 - e_h) > \alpha(1 - e_h) + \gamma$ , which is never true.

For Universal AVs, the condition leads to  $(1 - e_h)/(1 - e_w) > 1 + \gamma/\alpha$ .

For Work AVs, the condition leads to  $e_w > \gamma/(\beta + \gamma)$ .

**Proposition A3.** Optimal departure time is never  $t^*$  if  $e_w < 0.5$  and  $\beta < \alpha < \gamma$ .

**Proof.**

For Universal AVs, the condition from Proposition 2 requires that  $(1 - e_h)/(1 - e_w) > 1 + \gamma/\alpha$ . Since it is assumed that  $\gamma > \alpha$ , then the strongest form of the condition is  $(1 - e_h)/(1 - e_w) > 2$ . If  $e_w < 0.5$ , then that will never occur, and optimal departure time for Universal AVs will never be  $t^*$ .

For Work AVs, the condition from Proposition 2 requires that  $e_w > \gamma/(\beta + \gamma)$ . Since it is assumed that  $\gamma > \beta$ , then the strongest form of the condition is  $e_w > 0.5$ . Hence, if  $e_w < 0.5$ , then optimal departure time for Work AVs is never  $t^*$ .

**Appendix B. Start, end and on-time departure times of AV congestion**

Three conditions determine the start, end and on-time departure times of congestion:

1. Total number of travellers departing equals  $N$ ;
2. Duration of the congestion is  $\frac{N}{s}$ , where  $s$  is the bottleneck capacity;
3. Departing at the on-time departure time leads to the arrival at the preferred arrival time  $t^*$ .

*Derivation for Universal AVs*

Conditions 1 and 2:

$$\frac{\alpha(1 - e_h)}{\alpha(1 - e_h) - \beta} s(\tilde{t} - t_q) + \frac{\alpha(1 - e_h)}{\alpha - (\alpha + \gamma)e_w + \gamma} s(t^* - \tilde{t}) + \frac{\alpha - (\alpha + \gamma)e_w}{\alpha - (\alpha + \gamma)e_w + \gamma} s(t_{q'} - t^*) = N \quad (\text{B1})$$

$$t_{q'} - t_q = \frac{N}{s} \quad (\text{B2})$$

Insert condition 2 into condition 1, and obtain  $t_q$  as a function of  $\tilde{t}$ :

$$t_q = \frac{\frac{\gamma N}{s} - t^*((\alpha + \gamma)e_w - \alpha e_h)}{\alpha - (\alpha + \gamma)e_w + \gamma} - \left( \frac{\alpha(1 - e_h)}{\alpha(1 - e_h) - \beta} - \frac{\alpha(1 - e_h)}{\alpha - (\alpha + \gamma)e_w + \gamma} \right) \tilde{t} \quad (\text{B3})$$

$$\frac{\alpha - (\alpha + \gamma)e_w}{\alpha - (\alpha + \gamma)e_w + \gamma} - \frac{\alpha(1 - e_h)}{\alpha(1 - e_h) - \beta}$$

Condition 3:

$$\tilde{t} = t^* - \frac{D(\tilde{t})}{s} = t^* - \frac{\int_{t_q}^{\tilde{t}} r(u) du - s(\tilde{t} - t_q)}{s} = t^* - \frac{\beta}{\alpha(1 - e_h) - \beta} (\tilde{t} - t_q) \quad (\text{B4})$$

Obtain  $\tilde{t}$  as a function of  $t_q$  from condition 3:

$$\tilde{t} = \frac{(\alpha(1 - e_h) - \beta)t^* + \beta t_q}{\alpha(1 - e_h)} \quad (\text{B5})$$

Insert (B5) into (B3) to obtain  $t_{q'}$ , which, after simplification, coincides with the  $t_q$  for the conventional vehicle case:

$$t_q = t^* - \frac{\gamma}{\beta + \gamma} \frac{N}{s} \quad (\text{B6})$$

Using (B2), the end of congestion  $t_{q'}$  is

$$t_{q'} = t^* + \frac{\gamma}{\beta + \gamma} \frac{N}{s}. \quad (\text{B7})$$

Inserting (B6) into (B5), we can obtain the on-time departure time:

$$\tilde{t} = t^* - \frac{\beta\gamma}{\alpha(1 - e_h)(\beta + \gamma)} \frac{N}{s}. \quad (\text{B8})$$

*Derivation for Work AVs*

Conditions 1 and 2:

$$\frac{\alpha - (\alpha - \beta)e_w}{\alpha - (\alpha - \beta)e_w - \beta} s(\tilde{t} - t_q) + \frac{\alpha - (\alpha - \beta)e_w}{\alpha - (\alpha + \gamma)e_w + \gamma} s(t^* - \tilde{t}) + \frac{\alpha - (\alpha + \gamma)e_w}{\alpha - (\alpha + \gamma)e_w + \gamma} s(t_{q'} - t^*) = N \quad (\text{B9})$$

$$t_{q'} - t_q = \frac{N}{s} \quad (\text{B10})$$

Insert condition 2 into condition 1, and obtain  $t_q$  as a function of  $\tilde{t}$ :

$$t_q = \frac{\frac{\gamma N}{s} - t^*(\beta + \gamma)e_w - \left( \frac{\alpha - (\alpha - \beta)e_w}{\alpha - (\alpha - \beta)e_w - \beta} - \frac{\alpha - (\alpha - \beta)e_w}{\alpha - (\alpha + \gamma)e_w + \gamma} \right) \tilde{t}}{\frac{\alpha - (\alpha + \gamma)e_w}{\alpha - (\alpha + \gamma)e_w + \gamma} - \frac{\alpha - (\alpha - \beta)e_w}{\alpha - (\alpha - \beta)e_w - \beta}}. \quad (\text{B11})$$

Condition 3:

$$\tilde{t} = t^* - \frac{\beta}{\alpha - (\alpha - \beta)e_w - \beta} (\tilde{t} - t_q) \quad (\text{B12})$$

Obtain  $\tilde{t}$  as a function of  $t_q$  from condition 3:

$$\tilde{t} = \frac{(\alpha - (\alpha - \beta)e_w - \beta)t^* + \beta t_q}{\alpha - (\alpha - \beta)e_w}. \quad (\text{B13})$$

Insert (B13) into (B11) to obtain  $t_q$ . Congestion start and end times turn out to be the same for all vehicles. Insert  $t_q$  into (B13) to obtain the on-time departure time for Work AV:

$$\tilde{t} = t^* - \frac{\beta\gamma}{(\alpha - (\alpha - \beta)e_w)(\beta + \gamma)} \frac{N}{s}. \quad (\text{B14})$$

## Appendix C. Proofs of the congestion properties given single AV type and $\alpha - \beta - \gamma$ scheduling preferences

**Proposition C1.** The queuing times are longer with AVs compared to conventional vehicles.

**Proof.**

It is sufficient to show that the inflection points of AV graphs at  $\tilde{t}$  are higher and lie earlier for the AV graphs than the inflection point of the conventional vehicle graph, and that the inflection point at  $t^*$  for Universal and Work AVs also lies above the conventional vehicle graph.

The highest peak at  $\tilde{t}$  is as high as it is far from the preferred arrival time  $t^*$ . This follows from the definition of  $\tilde{t}$  as the departure time that leads to on-time arrival. Knowing this, it can be seen from

Table 2 that  $t^* - \tilde{t}$  increases with  $e_h$  and  $e_w$  for all AV types. Therefore, the inflection point at  $\tilde{t}$  is higher and earlier for the AV graphs than for the conventional vehicle graph, for which  $e_h = e_w = 0$ .

The peak at  $t^*$  for Universal and Work AVs lies above the conventional vehicle graph, because the Work AV graph in segment  $[\tilde{t}, t^*]$  is flatter than the conventional vehicle graph. This is because the departure rate (from Table 2) is higher for Work AV in that interval: it can be verified that  $(\alpha - (\alpha - \beta)e_w)/(\alpha - (\alpha + \gamma)e_w + \gamma) s > \alpha/(\alpha + \gamma)$  is always true. Since Work and Universal AV graphs overlap from  $t^*$  onward, the inflection point of Universal AVs is also necessarily above the conventional vehicle graph.

**Proposition C2.** Congestion is more skewed to earlier times for the Home AVs and to later times for Work AVs. Congestion with Universal AVs is skewed in both directions.

**Proof.**

To prove this property, we need to select an indicator that describes the skew well. I propose the following indicator, which captures the difference between the relative increase of congestion at times  $\tilde{t}$  and  $t^*$ , while taking the congestion with conventional vehicles as a reference point:

$$S^{AV} = \frac{Q_{\tilde{t}}^{AV}}{Q_{\tilde{t}}^{CV}} - \frac{Q_{t^*}^{AV}}{Q_{t^*}^{CV}}, \quad (C1)$$

where  $Q_{\tilde{t}}^{AV}$  and  $Q_{\tilde{t}}^{CV}$  are queuing times at the on-time departure time  $\tilde{t}$  with AV and conventional vehicle (CV), respectively;  $Q_{t^*}^{AV}$  and  $Q_{t^*}^{CV}$  are the corresponding queuing times at  $t^*$ . If  $S^{AV}$  is positive, then the congestion is skewed towards earlier times as compared to the congestion with conventional vehicles; if it is negative, then congestion is skewed to later times.

The skew indicators for the Home AV ( $S^{AV_1}$ ), Universal AV ( $S^{AV_2}$ ) and Work AV ( $S^{AV_3}$ ) are the following:

$$S^{AV_1} = \frac{1}{1 - e_h} - \frac{\alpha + \gamma}{\alpha(1 - e_h) + \gamma} = \frac{\gamma e_h}{(\alpha(1 - e_h) + \gamma)(1 - e_h)} > 0, \quad (C2)$$

$$S^{AV_2} = \frac{1}{1 - e_h} - \frac{1}{1 - e_w}, \quad (C3)$$

$$S^{AV_3} = \frac{\alpha}{\alpha - (\alpha - \beta)e_w} - \frac{1}{1 - e_w} = -\frac{\beta e_w}{(\alpha - (\alpha - \beta)e_w)(1 - e_w)} < 0. \quad (C4)$$

This indicator shows that, indeed, Home AVs skew the congestion to earlier times; Work AVs skew it to later times. The indicator is zero for Universal AVs, if  $e_h = e_w = 0$ , and positive (negative), if  $e_h$  is larger (smaller) than  $e_w$ .

**Proposition C3.** Longer queuing times are reached with Home and Universal AVs compared to Work AVs.

**Proof.**

Having congestion with any vehicle, the longest queuing time occurs at the on-time departure time  $\tilde{t}$ . Following the definition of  $\tilde{t}$ , this queuing time equals  $t^* - \tilde{t}$ . Comparing the distance  $t^* - \tilde{t}$  for Home (or Universal), and Work AVs, it can be obtained that  $t^* - \tilde{t}$  is larger for Home and Universal AVs, whenever  $\alpha e_h^H > (\alpha - \beta)e_w^W$ , where  $e_h^H$  is the efficiency of home activities in the Home and Universal AV, and  $e_w^W$  is the efficiency of work activities in the Work AV. This condition determines that home activities would yield higher utility in Home AV than early work activities (before  $t^*$ ) yield in Work AV. If this condition is not fulfilled, then Home AVs are inferior to Work AVs in terms of the quality of on-board activities, and Home AVs would lead to shorter queuing times than Work AVs (the congestion pattern would be only slightly altered from the conventional vehicle case).

However, if AVs are specialised to support only home, only work, or both home and work activities, and do so to a similar extent (such that none of AVs is inferior to another at all clock-times), then Work AVs would result in a smaller congestion increase than other AV types.

**Proposition C4.** Congestion costs with AVs are the same as with conventional vehicles.

**Proof.**

The start and end times of congestion are the same for conventional vehicles (25) and (26) and AVs (Table 2). At these times, the travel time is zero, and the individual experiences only the costs of being at work too early or too late. Since these costs are not influenced by AVs, the equilibrium costs of all congestion patterns in Figure 6 are the same and equal  $(\beta\gamma/(\beta + \gamma)) * (N/s)$ .

## Appendix D. Code used to create Figures 6 and 7

Code can be found in Pudāne (2020), <https://doi.org/10.4121/13247633>. Code was created in MATLAB R2018b.