A Multi-mode, Multi-class Dynamic Network Model With Queues For Advanced Transportation Information Systems

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In this paper we propose a composite Variational Inequality formulation for modeling multi-mode, multi-class stochastic dynamic user equilibrium problem in recurrent congestion networks with queues. The modes typically refer to different vehicle types such as passenger cars, trucks, and buses sharing the same road space. Each vehicle type has its own characteristics, such as free flow speed, vehicle size. We extend single mode deterministic point model to multimode deterministic point model for modeling the asymmetric interactions among various modes. Meanwhile, each mode of travelers is classified into two classes. Class 1 is equipped travelers following stochastic dynamic user-equilibrium with less uncertainty of travel cost, class 2 is unequipped travelers following stochastic dynamic user-equilibrium with more uncertainty of travel cost. A solution algorithm based on stochastic dynamic network loading for logit-based simultaneous route and departure time choices is adopted. Finally a numerical example is presented in a simple network.

Key Words: ATIS, multi-mode dynamic network model, dynamic traffic assignment, queuing network

1. Introduction

Advanced traveler information systems, as a major part of Intelligent Transportation Systems, are generally believed to be efficient means for improving individual traveler's trip planning, alleviating traffic congestion and enhancing traffic network performance. There are many researches in modeling and evaluating the impacts of ATIS on travelers and transportation systems in order to determine the feasibility, risks and benefits of such technology. Up to now, these studies include path choice surveys (Abdel Aty, et al., 1997; Hato et al., 1999), field deployments (Tsuji et al., 1985), laboratory experiments (Yang, 1993; Reddy, 1995),

computer simulations (Emmerink, 1995; Mahmassani, 1994, 1999) and analytical models (Yang Hai, 1998, 1999; YaFeng Yin, 2003; Hong, 2002a, 2004; William, 2003).

Analytical multi-class traffic models for evaluating the impacts of ATIS on general networks with recurrent congestion are classified into two classes. One class is static models (Harker, 1988; Kanafani, 1991; Van Vuren, 1991; Bennett, 1993; Maher and Hughes, 1995; Yang, 1998, 1999; Hong, 2002a). Static models are a rather simplistic model that might give unrealistic results for determining the benefits of ATIS. Another is dynamic models (Ran et al., 1996, 2002; Al-Deek, 1998; Hong, 1996, 2002b; Lam and Huang, 2004). Compared with static model, dynamic models can reflect the traffic conditions more realistically, but it is at the expense of increases in the complexity of model and computing time. Hong (2002b,2004) made a comparison of differences between dynamic and static models in the appraisal of ATIS, and pointed out that quite reverse results might be achieved using the two different models in similar cases.

Al-Deek et al. (1998) used a composite traffic assignment model which combines a probabilistic traveler behavior model of route diversion and a queuing model to evaluate ATIS impacts under incident conditions. Three types of travelers are considered in the composite traffic assignment model: those who are unequipped with electronic devices, i.e. they do not have ATIS or radio in their vehicles; those who receive delay information from radio only; and those who access ATIS only. Similar studies were carried out by Emmerink (1994, 1995a) in studying the economic impacts of driver information systems by using a dynamic stochastic route choice-modeling framework.

Among the first to use an analytical approach to modeling multi-class DTA were Lo et al. (1996) and Ran et al. (1996). In these papers, travelers were classified into those who follow predetermined routes, those who follow a stochastic dynamic UE assignment, and those who follow a dynamic UE assignment. The three classes of users are integrated into one dynamic traffic assignment (DTA) model, Ran et al. (1996, 2002) give various algorithms for solving the above multi-class dynamic equilibrium problem. Huang and Lam (2004) presented a multi-class dynamic user equilibrium assignment problem formulation in order to assess the impacts of ATIS in general networks with queues. Suppose that users equipped with ATIS will follow the deterministic simultaneous route/departure time equilibrium choice behavior due to complete traffic information, while users unequipped with ATIS will follow the stochastic simultaneous route/departure time equilibrium choice behavior (travel choice behaviors modeling by the nested-logit model) due to incomplete traffic information. A heuristic algorithm based on route/time swapping process was processed for solving the multi-class dynamic user equilibrium problem. However, the multiple transportation modes (car and truck etc) and multi-mode dynamic queuing phenomenon in general networks with ATIS were not considered in the previous analytical static or dynamic multi-class models.

The impacts of multiple transportation modes are generally considered in microcosmic simulation models and are more studied in static traffic assignment models, but they are less studied in dynamic traffic assignment models. Bliemer and Bovy (2000, 2001, 2003) firstly extended single mode dynamic traffic assignment to multi-mode dynamic assignment model, considering different driving characteristic, network usage and route choice behavior. And various solution algorithms were given such as extended time-space network algorithm, nested modified projection method etc.

In this paper, the modes typically refer to different vehicle types such as cars, trucks and buses sharing the same road space. Each vehicle type has its own characteristics, such as free flow speed, vehicle size. We extend single mode deterministic point model to multimode

deterministic point model in modeling the asymmetric interactions among various modes. Then we classify each mode of travelers (truck or car) into two classes; Class 1: each mode of travelers with ATIS who can receive the fairly precise traffic information will follow stochastic dynamic simultaneous route/departure time equilibrium choice behavior with less uncertainty of travel cost. Class 2: each mode of travelers without ATIS who can only capture the imperfect traffic information (perhaps from past experiences) will follow stochastic dynamic simultaneous route/departure time equilibrium choice behavior with more uncertainty of travel cost.

The remainder of this paper is organized as follows. In section 2, a multimode and multi-class dynamic traffic network equilibrium model is proposed on a discrete-time basis. In section 3, the governing multimode and multi-class dynamic traffic network equilibrium conditions are formulated as a finite-dimensional variational inequality problem. In section 4, we propose a diagonalisation algorithm for solving the variational inequality problem. Finally the model and algorithms are tested in a simple network.

2. Discrete-time Network Model

Consider a network $G = (N; A)$, where N is the set of nodes and A is the set of links in the network. Let a denote a link of the network connecting a pair of nodes (i,j) and let p denote a path that is consisted of a series of directed link $(a_1, a_2, \ldots, a_n)$ between origin r and destination s. Let RS denote the set of all OD pairs in the network. P_{rs} denotes the set of routes between OD pair $rs \in$ RS and the entire set of paths in the network by P. Let M denote the set of all modes, examples of modes include passenger cars, trucks and public transportations, etc. The studies horizon is discretized into m intervals of length δ such that $T = m \cdot \delta$. Here, we assume the study horizon is long enough to ensure all traveler can exit from the network after the time T. on the other hand, it is also assumed that the value of δ is small enough so that the discrete-time model can approximate its continuous time counterpart.

$\hat{u}_{am}(k), \tilde{u}_{am}(k)$	The inflow rate of equipped and unequipped travelers of mode m on link a during time interval k.
$\hat{u}^*_{am}(k), \tilde{u}^*_{am}(k)$	The arrival flow rate to exit queue of equipped and unequipped travelers of mode m on link a during time interval k.
$\hat{v}_{am}(k), \tilde{v}_{am}(k)$	The departure flow rate of equipped and unequipped travelers of mode m from link a during time interval k.
$\hat{q}_{am}(k), \tilde{q}_{am}(k)$	The vehicle numbers of equipped and unequipped travelers of mode m in the queue at time interval k.
$t_{am}(k)$	The travel time experienced by travelers of mode m entering into link a at time interval k.
s_a	The maximum exit flow rate of the bottleneck on link a(unit: passenger car number of hour).
Pcu_m	The passenger car equivalents parameter of mode m (here, denote Pcu as the unit of passenger car).

2.1 Link Dynamic Function

We extend single-mode deterministic point queue model proposed by Li (2000), Huang (2002), Han (2003) to multi-mode deterministic point queue model for considering interactions among multiple transportation modes. We don't consider the spillback effect of queue length explicitly, the reader can be referred to the work of Astarita (1996), Adamo (1999), Kuwahara (2001) and Hong et al. (2002c) for handling the spillback of congestion in a dynamic network simulation model. We assume the link is consisted of two parts. The first part is the running segment of the link that each mode of travelers can run according to each mode of free-flow velocity and don't interact with each other among the travelers of various modes. In other words, each mode of travelers reaches the position of the exit queue segment of link through the constant running time t_{am}. (It assume $t_{a1} < t_{a2} < \ldots < t_{aw}$, due to difference of the velocity of various modes under un-congested traffic conditions. For example, the velocity of car is higher than that of truck under un-congested traffic conditions). The second part is the exit queue segment (the vehicle of mode m is assumed to be a point without length). The queue delay experienced by travelers of mode m is caused by the limited link exit capacity (in *Pcu*), or the maximum link exit flow rate (in *Pcu/h*).
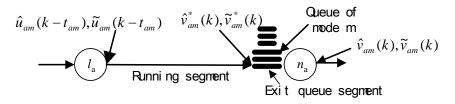

Figure 1. Link flow propagation conditions

The link flow propagation conditions are depicted in Fig.1. The equipped and unequipped travelers of mode m entering into link a during time interval $k-t_{am}$ arrive at the exit queue segment of link a during time interval k through the constant running time t_{am}. The arrival flow rates of equipped and unequipped travelers of mode m to the exit queue segment of link a during time interval k are $\hat{v}^*_{am}(k)$ and $\tilde{v}^*_{am}(k)$, respectively. The departure flow rates of equipped and unequipped travelers of mode m from the exit queue segment during time interval k are $\hat{v}_{am}(t)$ and $\tilde{v}_{am}(t)$, respectively. The link dynamic functions can be expressed as follows.

The running segment function:

$$\tilde{v}^*_{am}(k) = \tilde{u}_{am}(k - t_{am}), \forall a, k, m \tag{1}$$

$$\hat{v}^*_{am}(k) = \hat{u}_{am}(k - t_{am}), \forall a, k, m \tag{2}$$

The exit queue segment function:

$$\frac{\hat{q}_{am}(k) - \hat{q}_{am}(k-1)}{\delta} = \hat{v}^*_{am}(k) - \hat{v}_{am}(k), \forall a, k, m \tag{3}$$

$$\frac{\tilde{q}_{am}(k) - \tilde{q}_{am}(k-1)}{\delta} = \tilde{v}^*_{am}(k) - \tilde{v}_{am}(k), \forall a, k, m \tag{4}$$

Let $\hat{q}_{am}(k)$ and $\tilde{q}_{am}(k)$ be number of vehicles of equipped and unequipped travelers of mode m waiting in the queue on link a at time interval k, respectively. Equation (3) ((4)) expresses the marginal change of number of vehicles of equipped (unequipped) travelers of mode m is equal to the difference between the arrival flow rate of equipped (unequipped) travelers of mode m to exit queue segment and the departure flow rate of equipped (unequipped) travelers of mode m from exit queue segment on link a during time interval k.(Li,jun, 2000)

2.2 Link Exit flow function

We will give the following assumptions for deriving our link exit flow function:
1. The class-specific Pcu_m parameter that transforms the effect of mode m into passenger car equivalents is fixed under all traffic conditions.
2. The mixture of equipped and unequipped travelers of various modes is homogenous on the link.
3. The temporal and spatial interactions of equipped and unequipped travelers of various modes mainly appear in the exit queue segment of the link.

If the total queue vehicle number $q_a(k)$ on link a at time interval k (in *Pcu*, let $q_a(k) = \sum_m pcu_m(\hat{q}_{am}(k) + \tilde{q}_{am}(k))$ is equal to zero and the queue vehicle number of equipped and unequipped travelers of mode m for all k and a is nonnegative, $\hat{q}_{am}(k) \geq 0, \tilde{q}_{am}(k) \geq 0, \forall a, m, k$), then the queue vehicle numbers of equipped and unequipped travelers of mode m at time interval k must be equal to zero, $\hat{q}_{am}(k) = 0, \tilde{q}_{am}(k) = 0, \forall a, m, k$. Thus the following equations can be obtained according to equations (3) and (4).

$$\hat{v}_{am}(k) = \hat{v}_{am}^*(k) + \frac{\hat{q}_{am}(k-1)}{\delta}, \forall a, k, m \tag{5}$$

$$\tilde{v}_{am}(k) = \tilde{v}_{am}^*(k) + \frac{\tilde{q}_{am}(k-1)}{\delta}, \forall a, k, m \tag{6}$$

On the other hand, if $q_a(k) > 0$, due to the limited link exit capacity (in *Pcu/h*), among all vehicles that hope to exit link a during time interval k, $v_a(k)$ (in *Pcu*)

$$(v_a(k) = \sum_m pcu_m(\hat{v}_{am}^*(k) + \tilde{v}_{am}^*(k)) + \sum_m pcu_m(\hat{q}_{am}^*(k-1) + \tilde{q}_{am}^*(k-1))\Big/\delta) \tag{7}$$

The first term on the right side represents the arrival flow rate of total vehicles to the exit queue segment of link a during time interval k. The second term on the right side represents the total queue vehicle number at time interval k-1.), only a part can exit from link a, while the other part will form new queue at the exit queue segment of link a. According to the above assumptions and equations (3) and (4), The departure flow rates of equipped and unequipped travelers of mode m may be calculated as follows.

$$\hat{v}_{am}(k) = \begin{cases} \dfrac{\hat{v}_{am}^*(k) + \hat{q}_{am}(k-1)\big/\delta}{v_a(k)} \cdot s_a & if \quad v_a(k) \geq s_a \quad or \quad q_a(k) > 0 \\ \hat{v}_{am}^*(k) + \hat{q}_{am}(k-1)\big/\delta & otherwise \end{cases} \quad \forall a, k, m \tag{8}$$

$$\tilde{v}_{am}(k) = \begin{cases} \dfrac{\tilde{v}^*_{am}(k) + \tilde{q}_{am}(k-1)/\delta}{v_a(k)} \cdot s_a & if \quad v_a(k) \geq s_a \quad or \quad q_a(k) > 0 \\ \tilde{v}^*_{am}(k) + \tilde{q}_{am}(k-1)/\delta & otherwise \end{cases} \quad \forall a, k, m \tag{9}$$

Furthermore, substituting equations (8) and (9) into equations (3) and (4), the link queue vehicle numbers of equipped and unequipped travelers of mode *m* can be expressed too.

$$\hat{q}_{am}(k)$$
$$= \begin{cases} \hat{v}^*_{am}(k) \cdot \delta + \hat{q}_{am}(k-1) - \dfrac{\hat{v}^*_{am}(k) \cdot \delta + \hat{q}_{am}(k-1)}{v_a(k)} \cdot s_a & if \quad v_a(k) \geq s_a \quad or \quad q_a(k) > 0 \\ 0 & otherwise \end{cases} \quad \forall a, k, m \tag{10}$$

$$\tilde{q}_{am}(k)$$
$$= \begin{cases} \tilde{v}^*_{am}(k) \cdot \delta + \tilde{q}_{am}(k-1) - \dfrac{\tilde{v}^*_{am}(k) \cdot \delta + \tilde{q}_{am}(k-1)}{v_a(k)} \cdot s_a & if \quad v_a(k) \geq s_a \quad or \quad q_a(k) > 0 \\ 0 & otherwise \end{cases} \quad \forall a, k, m \tag{11}$$

2.3 Link Travel Time Function

The queue delay experienced by the travelers of mode *m* is depended on the link total queue vehicle number, $q_a(k)$ (in *Pcu*). The travelers of mode *m* entering into link *a* during time interval *k* will spend a constant running time t_{am} on link *a*. Then they will reach the exit queue segment of link *a* during time interval $k + t_{am}$ and the total queue vehicle length at time interval $k+t_{am}$ is $q_a(k +t_{am})$. Hence, The link delay experienced by the travelers of mode *m* entering into link *a* during time interval *k* can be given as

$$d_{am}(k) = \frac{q_a(k + t_{am})}{s_a} \quad \forall a, k, m \tag{12}$$

The total travel time over link *a* for the travelers of mode *m* entering into link *a* during time interval *k* is the sum of constant running time and the delay experienced by the travelers of mode *m*

$$t_{am}(k) = t_{am} + d_{am}(k) = t_{am} + \frac{q_a(k + t_{am})}{s_a} \quad \forall a, k, m \tag{13}$$

Till now, we can consider single-mode point queue model as a special case of the above model formulated in this paper. If only one vehicle type (truck or car) appear on a link, let the link inflow rate of the other vehicle types be 0, and then the above model is equivalent to single-mode point queue model proposed by Huang (2002).

2.4 Path Travel Time And Cost

The path travel time function can be given as $\{p=(a_1, a_2, \ldots, a_n)\}$

$$t^{rs}_{pm}(k) = t_{a_1 m}(k) + t_{a_2 m}(k + t_{a_1 m}(k)) + \cdots + t_{a_n m}(k + t_{a_1 m} + t_{a_2 m} + \cdots + t_{a_{n-1} m}), \forall rs, p, m, k \tag{14}$$

for short $t_{a_1m} = t_{a_1m}(k), \ t_{a_2m} = t_{a_2m}(k + t_{a_1m}(k))$.

Consider the schedule delay cost function as follow.

$$Sch_{sm}(k) = \begin{cases} \beta\left[t_s - \Delta_s - k - t_{pm}^{rs}(k)\right] & if \ t_s - \Delta_s > k + t_{pm}^{rs}(k) \\ \gamma\left[k + t_{pm}^{rs}(k) - t_s - \Delta_s\right] & if \ t_s + \Delta_s < k + t_{pm}^{rs}(k) \qquad \forall k, m \\ 0 & otherwise \end{cases} \tag{15}$$

Denote $\left[t_s - \Delta_s, t_s + \Delta_s\right]$ as the desired time interval of the travelers for arrival at the destination s in the network. Where $t_s - \Delta_s$ is the commuter's desired earliest arrival time of the travelers at the destination s, $t_s + \Delta_s$ is the desired latest arrival time of the travelers at the destination s. β and γ are the unit cost of schedule delay early and late of the travelers at the destination s, respectively.

Therefore, the generalized travel cost of a trip from origin r to destination s on path p for the travelers of mode m leaving origin r during time interval k is

$$c_{pm}^{rs}(k) = \alpha \cdot t_{pm}^{rs}(k) + Sch_{sm}(k) \tag{16}$$

Where α is a convention factor to transform the path travel time into travel cost. In accordance with the empirical results (small, 1982), we assume that $\gamma > \alpha > \beta$ holds.

3. Dynamic User Equilibrium Conditions

In most previous multi-class dynamic network models (Bin, 1996; Hong, 1996; Williams and Huang, 2004, etc), equipped travelers who may receive the real-time perfect traffic information are assumed to make travel choices in a deterministic dynamic user equilibrium manner. In reality, traffic information is rarely perfect. There are many difficulties in estimating the current traffic information and predicting the future traffic information in current traffic technology condition. For these reasons, we model the travel choice behavior of equipped travelers as following the principle of stochastic dynamic user equilibrium. The traffic information quality is reflected by the parameter of general travel cost perception variation. Better information qualities have lower travel cost variation. Unequipped travelers in making travel choices, in particular, route and departure time choices, according to the past experiences that don't capture better information than equipped travelers follow another stochastic dynamic user equilibrium with higher travel cost variation.

$\hat{f}_{pm}^{rs}(k), \tilde{f}_{pm}^{rs}(k)$	The inflow rate of equipped and unequipped travelers of mode m entering the path p between origin r and destination s during time interval k
$\hat{f}, \tilde{f}$	The set of $\{\hat{f}_{pm}^{rs}(k), \forall rs, p, k, m\}, \{\tilde{f}_{pm}^{rs}(k), \forall rs, p, k, m\}$.
$\hat{Q}_m^{rs}, \tilde{Q}_m^{rs}$	The demand of equipped and unequipped travelers of mode m between origin r and destination s

$\hat{P}_{pm}^{rs}(k), \tilde{P}_{pm}^{rs}(k)$ The proportion of equipped and unequipped travelers of mode m between origin r and destination s selecting path p and departure time k.

$\hat{\theta}_m$ The parameter representing travel cost variation of equipped travelers of mode m

$\tilde{\theta}_m$ The parameter representing travel cost variation of unequipped travelers of mode m

Equipped travelers of mode m follow the stochastic dynamic simultaneous route and departure time equilibrium (SUE-SRD), expressed as

$$\hat{f}_{pm}^{rs}(k) = \hat{P}_{pm}^{rs}(k) \cdot \hat{Q}_m^{rs} \qquad \forall rs, p, k, m \tag{17}$$

Where

$$\hat{P}_{pm}^{rs}(k) = \frac{\exp(-\hat{\theta}_m \cdot c_{pm}^{rs}(k))}{\sum_p \sum_k \exp(-\hat{\theta}_m \cdot c_{pm}^{rs}(k))} \qquad \forall rs, p, k, m \tag{18}$$

Where $\hat{\theta}_m$ is the parameter representing general travel cost perception variation of equipped travelers of mode m. A higher $\hat{\theta}_m$ means smaller general travel cost variation and better information quality.

The logit-based SUE-SRD of equipped travelers of mode m can be expressed as

$$\hat{C}_{pm}^{rs}(k, \tilde{f}, \hat{f}^*) \begin{cases} = \hat{C}_{m,\min}^{rs} & if \ \hat{f}_{pm}^{rs*}(k) > 0 \\ > \hat{C}_{m,\min}^{rs} & if \ \hat{f}_{pm}^{rs*}(k) = 0 \end{cases} \qquad \forall rs, k, p, m \tag{19}$$

$$\sum_p \sum_k \hat{f}_{pm}^{rs*}(k) = \frac{\hat{Q}_m^{rs}}{\delta} \qquad \forall rs, m \tag{20}$$

$$\hat{C}_{pm}^{rs}(k, \tilde{f}, \hat{f}^*) = c_{pm}^{rs}(k) + \frac{1}{\hat{\theta}_m} \ln \frac{\hat{f}_{pm}^{rs*}(k)\delta}{\hat{Q}_m} \qquad \forall rs, k, p, m \tag{21}$$

$$\hat{f}_{pm}^{rs*}(k) \geq 0 \qquad \forall rs, k, p, m \tag{22}$$

Where, $\hat{C}_{m,\min}^{rs}$ is the minimum perceived unit travel cost of equipped travelers of mode m between origin r and destination s, $\hat{C}_{m,\min}^{rs} = \min\{\hat{C}_{pm}^{rs}(k,\cdot), \forall p, k\}$. $\hat{C}_{pm}^{rs}(k,\cdot)$ in equation (21) is perceived unit travel cost incurred by equipped travelers of mode m entering path p between origin r and destination s during time interval k (see Bin, 1996; Huang and Williams, 2004 etc). Equation (20) represents the flow conservation of equipped travelers of mode m between origin r and destination s and equation (22) represents the non-negative of all path inflow rates.

For each mode of equipped travelers and for each origin-destination (OD) pair, the perceived path travel costs experienced for all equipped travelers of mode m, regarding of departure

times, is equal and minimum, and less than (or equal to) the perceived path travel costs for equipped travelers of mode m on any unused route.

The above SUE-SRD equilibrium condition of equipped travelers of mode m can be expressed by a finite dimensional variational inequality formulation.

Find a vector $\hat{f}^* \in \Omega$ if and only if it satisfy

$$\sum_{rs}\sum_{p}\sum_{k}\sum_{m}\hat{C}_{pm}^{rs}(k,\tilde{f},\hat{f}^*)(\hat{f}_{pm}^{rs}(k)-\hat{f}_{pm}^{rs*}(k)) \geq 0 \qquad \forall \hat{f} \in \hat{\Omega} \tag{23}$$

Where $\hat{\Omega}$ is a closed convex.

$$\hat{\Omega} = \left\{ \hat{f} \Big| \sum_{p}\sum_{k}\hat{f}_{pm}^{rs}k) = \frac{\hat{Q}_{m}^{rs}}{\delta}, \hat{f}_{pm}^{rs}(k) \geq 0, \forall rs, m \right\} \tag{24}$$

The treatment of unequipped travelers of mode m is identical. Without loss of generality, one may write:

$$\tilde{f}_{pm}^{rs}(k) = \tilde{P}_{pm}^{rs}(k) \cdot \tilde{Q}_{m}^{rs} \qquad \forall rs, p, k, m \tag{25}$$

Where

$$\tilde{P}_{pm}^{rs}(k) = \frac{\exp(-\tilde{\theta}_{m} \cdot c_{pm}^{rs}(k))}{\sum_{p}\sum_{k}\exp(-\tilde{\theta}_{m} \cdot c_{pm}^{rs}(k))} \qquad \forall rs, p, k, m \tag{26}$$

Where $\tilde{\theta}_{m}$ expresses the travel cost perception variation of unequipped travelers of mode m that can be interpreted as their familiarity of the network condition or the past experiences.

The above SUE-SRD equilibrium condition of unequipped travelers of mode m can be expressed by a finite dimensional variational inequality formulation too.

Find a vector $\tilde{f}^* \in \Omega$ if and only if it satisfy

$$\sum_{rs}\sum_{p}\sum_{k}\sum_{m}\tilde{C}_{pm}^{rs}(k,\tilde{f}^*,\hat{f})(\tilde{f}_{pm}^{rs}(k)-\tilde{f}_{pm}^{rs*}(k)) \geq 0 \qquad \forall \tilde{f} \in \tilde{\Omega} \tag{27}$$

where $\tilde{\Omega}$ is a closed convex.

$$\tilde{\Omega} = \left\{ \tilde{f} \Big| \sum_{p}\sum_{k}\tilde{f}_{pm}^{rs}k) = \frac{\tilde{Q}_{m}^{rs}}{\delta}, \tilde{f}_{pm}^{rs}(k) \geq 0, \forall rs, m \right\} \tag{28}$$

3.1 The Composite VI Formulation

The composite VI problem that integrates the VI (23) with VI (27) is equivalent to the above user equilibrium conditions (17) and (25). The composite VI model can be formulated as follows:

Find a vector ($\hat{f}^* \in \hat{\Omega}, \tilde{f}^* \in \tilde{\Omega}$) that is a multi-mode, multi-class stochastic dynamic user equilibrium pattern if and only if it satisfies the VI problem.

$$\sum_{rs}\sum_{p}\sum_{k}\sum_{m}\tilde{C}_{pm}^{rs}(k,,\hat{f}^*,\tilde{f}^*)(\tilde{f}_{pm}^{rs}(k)-\tilde{f}_{pm}^{rs*}(k))+\sum_{rs}\sum_{p}\sum_{k}\sum_{m}\hat{C}_{pm}^{rs*}(k,\hat{f}^*,\tilde{f}^*)(\hat{f}_{pm}^{rs}(k)-\hat{f}_{pm}^{rs*}(k))\geq 0$$

$$\forall \hat{f} \in \hat{\Omega}, \forall \tilde{f} \in \tilde{\Omega} \qquad\qquad\qquad (29)$$

It has been noted that the aforementioned model is mainly used for evaluating the impacts of ATIS during normal peak hour periods for commuter trips. Therefore, some of the assumptions adopted in the model may be appropriate under certain circumstances such as under recurrent congestion conditions without spillback queue. Additionally, the above multinomial logit model for modeling travelers' simultaneous path and departure time choice behaviors is a very simplistic model that may give unrealistic result of prediction since they neglect the impacts of path overlap. In further studies, a general C-logit, PS-logit and Probit model are used.

4. Algorithm

The optimal solution to variational inequality (29) can be found in the framework of the diagonalisation method (Ran and Boyce, 1996; Han, 2003; Chen, 1998). Before describing the diagonalisation method in detail, we will see how we can perform stochastic dynamic network loading, which is essential to find feasible link flow patterns. Note that this paper particularly develops a stochastic dynamic network loading method considering the logit-based route and departure time choices. The logit-based route and departure time choice function can be written as follows:

$$P_{pm}^{rs}(k)=\frac{\exp(-\boldsymbol{\theta}\cdot c_{pm}^{rs}(k))}{\sum_{p}\sum_{k}\exp(-\boldsymbol{\theta}\cdot c_{pm}^{rs}(k))} \qquad \forall rs,p,k,m \qquad\qquad (30)$$

4.1 Dynamic Stochastic Network Loading Method

In this section, stochastic dynamic network loading algorithm for the logit-based route and departure time choice is proposed. This network loading algorithm is similar to the algorithm proposed by Dial's STOCH for stochastic static network assignment (Sheffi, 1985) and the algorithm proposed by Ran's DYNASTOCH for stochastic dynamic network assignment(Ran and Boyce, 1996). In this study, we consider only the logit model for stochastic dynamic simultaneous route/departure time choice. The algorithm maintains the structure of the DYNASTOCH algorithm, so only deals with reasonable routes, and assigns the demand between OD pair *rs* to the link of the network according to the actual link travel cost.(denote the stochastic dynamic network loading method as SRD-DYNASTOCH)

In order to reflect the effect of the schedule delay cost in the method, we extend the original network to include the dummy link with schedule delay cost, $c^{s's}(k)=sch_{sm}(k),\forall m$ as shown in Fig.2.(link *a*=(*i,j*), Node *i* is head point of link a, Node *j* is end point of link a)
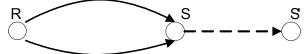

Figure 2. The Extended network Structure

Step 1: Calculation of link likelihood

Compute the minimum actual travel cost $\pi_{is}(k)$ for travelers departing node i during time interval k. calculate the link likelihood, $L_{(i,j)}(k)$, for each link (i,j) during each time interval k:

$$L_{(i,j)}(k) = \begin{cases} \exp(\theta[\pi_{is} - \pi_{js}(k + t_{(i,j)}(k)) - c_{(i,j)}(k)]) & if \quad C_o^{rs} > C_o^{js} \\ 0 & otherwise \end{cases} \quad i \in r \tag{31}$$

$$L_{(i,j)}(k) = \begin{cases} \exp(\theta[\pi_{is}(k) - \pi_{js}(k + t_{(i,j)}(k)) - c_{(i,j)}(k)]) & if \quad C_o^{is} > C_o^{js} \\ 0 & otherwise \end{cases} \quad i \notin r \tag{32}$$

Where equations (31) and (32) express the calculation way of the link likelihood when the head node i of link (i,j) is and isn't the origin r, respectively. The difference between equations (31) and (32) is π_{rs} and $\pi_{is}(k)$.

$\pi_{is}(k)$ The minimum travel cost from i to s by departing the node i during time interval k.

π_{rs} The minimum path travel cost from origin r to destination s for all departure times. $\pi_{rs} = \min\{c_p^{rs}(k), \forall p, k\} \quad \forall rs$

C_o^{is} The ideal travel cost from i to s when there is no flow in the network.

$t_{(i,j)}(k)$ The link travel time experienced by the travelers entering into link (i,j) during time interval k

$c_{(i,j)}(k)$ The link travel cost experienced by the travelers entering into link (i,j) during time interval k

Step 2: backward pass

By examining all nodes j in ascending sequence with respect to $\pi_{is}(k)$ from the destination s, calculate $w_{(i,j)}(k)$, the link weight for each link (i,j) during each time interval k:

$$w_{(i,j)}(k) = \begin{cases} L_{(i,j)}(k) & if \quad j = s \\ L_{(i,j)}(k) \cdot \sum_{(j,k) \in A(j)} w_{(j,k)}(k + t_{(j,k)}(k)) & otherwise \end{cases} \tag{33}$$

Where $A(j)$ is the set of links starting from node j, When the origin r is reached, stop.

Step 3: forward pass

Consider all nodes i in descending sequence with respect to $\pi_{is}(k)$, starting with the origin r. when each node i is considered during each time interval k, compute the inflow to each link (i,j) during each time interval k using the following formula:

$$v_{(i,j)}(k) = \begin{cases} q^{rs} \cdot \dfrac{w_{(i,j)}(k)}{\displaystyle\sum_{k}\sum_{(i,k)\in A(i)} w_{(i,k)}(k)} & if \quad i = r \\[2em] \left\{ \displaystyle\sum_{(k,i)\in B(i)} u_{(k,i)}(k) \right\} \cdot \dfrac{w_{(i,j)}(k)}{\displaystyle\sum_{(i,k)\in A(i)} w_{(i,k)}(k)} & otherwise \end{cases} \tag{34}$$

Where, $B(i)$ is the set of links ending at node i. When the destination s is reached, stop.
The flow generated by the algorithm is equivalent to a logit-based flow independent route/departure time assignment between each OD pair, given the reasonable route set is fixing in order to produce a convergence solution. The SRD-DYNASTOCH method for many to many OD is similar to the method proposed by Han (2003).
The proofs of the algorithm see Appendix A.

4.2 Diagonalisation Method

Here, we propose the diagonalisation method to solve multi-mode and multi-class stochastic dynamic simultaneous route/departure time equilibrium problem. The method is similar with that of Ran (1996), Chen (1998) and Han (2003). The method consists of the outer and inner iterations, outer iteration includes the updating estimation of actual link travel time or link inflow rate, inner iteration calculates the link inflow updating direction and the auxiliary link inflow rate by the method of successive averages. The processes of algorithm are stated as follows.

Step 0   Initialization: Set outer iteration counter i=1, perform 2*m stochastic dynamic network loading (SRD-DYNASTOCH) for the given demand $\hat{Q}_m^{rs}, \tilde{Q}_m^{rs}$ according to free flow travel cost, find initial link inflow rate $\hat{u}_{am}^i(k), \tilde{u}_{am}^i(k)$.

Step 1   Inner iteration (MSA).

Step 1.0   Initialization: set inner iteration counter j=1, $\hat{u}_{am}^j(k) = \hat{u}_{am}^i(k), \tilde{u}_{am}^j(k) = \tilde{u}_{am}^i(k)$.

Step 1.1   Calculate link travel time and travel cost $t_{am}^j(k), c_{am}^j(k)$ by using $\hat{u}_{am}^j(k), \tilde{u}_{am}^j(k)$.

Step 1.2   Direction finding: perform 2*m stochastic dynamic network loading(SRD-DYNASTOCH) for the given demand $\hat{Q}_m^{rs}, \tilde{Q}_m^{rs}$, according to current actual link travel time and travel cost $t_{am}^j(k), c_{am}^j(k)$. This generates auxiliary link flow $\hat{u}_{am}^{j*}(k), \tilde{u}_{am}^{j*}(k)$.

Step 1.3   Move: update flow pattern as:
$\hat{u}_{am}^{j+1}(k) = \hat{u}_{am}^j(k) + \lambda^j(\hat{u}_{am}^{j*}(k) - \hat{u}_{am}^j(k))$
$\tilde{u}_{am}^{j+1}(k) = \tilde{u}_{am}^j(k) + \lambda^j(\tilde{u}_{am}^{j*}(k) - \tilde{u}_{am}^j(k))$

Step 1.4   (convergence test of inner iteration) If

$$\Psi_s = \frac{\sqrt{\sum_m \sum_a ((\hat{u}_{am}^{j+1}(k) - \hat{u}_{am}^j(k))^2 + (\tilde{u}_{am}^{j+1}(k) - \tilde{u}_{am}^j(k))^2)}}{\sum_m \sum_a (\hat{u}_{am}^j(k) + \tilde{u}_{am}^j(k))} \leq \gamma .$$

(γ is a predetermined tolerance) or j is equal to a given number, then stop; otherwise, go to step 1.1 and set $j=j+1$.

Step 2　　　convergence test of outer iteration: if $\left| \hat{u}_{am}^{i+1}(k) - \hat{u}_{am}^i(k) \right| + \left| \tilde{u}_{am}^{i+1}(k) - \tilde{u}_{am}^i(k) \right| \leq \varepsilon$ or i is equal to a given number, stop; otherwise, go to step 1 and set $i=i+1$.

The step size λ^j is a predetermined value, we set $\lambda^j = 1/j$,or $\lambda^j = 1$ .in order to maintain correct flow propagation constraint, we calculate new link flow patterns by directly updating link choice probability or use pure network loading (Han, 2003).

5. Examples

In this section, the above algorithm finds the solution of multimode and multi-class stochastic dynamic simultaneous route/departure time equilibrium problem and we perform only one iteration in the inner iteration of the diagonalisation method according to Sheffi's (1985) advice. For stochastic dynamic simultaneous route/departure time network loading, we use the SRD-DYNASTOCH method in order to perform a logit-based flow assignment.
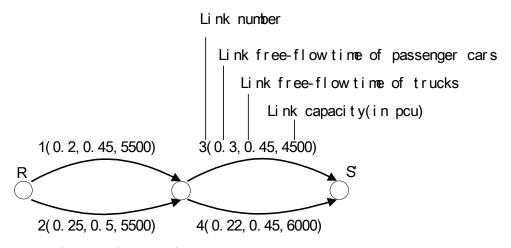

Figure 3. Example Network

Here we assume there are two transportation modes in the network; one is passenger car, and another is truck. The example network, shown in Fig.3, consists of 3 nodes, 4 links and one OD pair. All free flow travel times and link exit capacities are also given in this figure. The passenger car equivalents parameters of car and truck are $Pcu_1=1$ and $Pcu_2=2$,respectively. Other input data are: $\alpha=6(\ \$ /h) \beta=4(\ \$ /h), \gamma=22(\ \$ /h)$, $\Delta_s=0.25h$, $t_s=9.0h$, set T be from 5 to 12.a.m.and K=600, $\delta=0.6min$. Here we assume the perception parameters of each mode of equipped (unequipped) travelers are $\hat{\theta}_1 = \hat{\theta}_2 = 0.1$ and $\tilde{\theta}_1 = \tilde{\theta}_2 = 0.05$ respectively

The demands of car and truck are $Q_1^{rs} = 15000$ (persons) $Q_2^{rs} = 12000$ (persons), respectively. The demands of equipped and unequipped of car (truck) are $\hat{Q}_1^{rs} = \tilde{Q}_1^{rs} = 50\% \cdot Q_1^{rs}$, $\hat{Q}_2^{rs} = \tilde{Q}_2^{rs} = 50\% \cdot Q_2^{rs}$. Here, it is further assumed that the vehicle occupancy is 1 person per vehicle (car and truck).
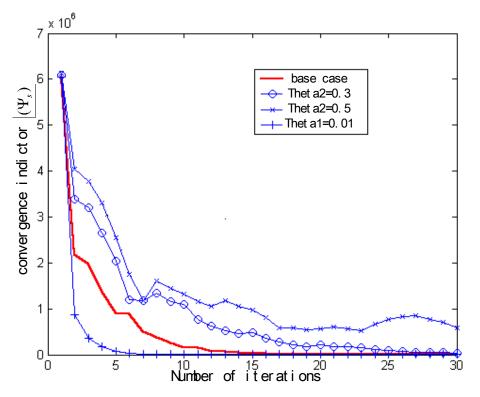

Figure 4. The convergence characteristics of the solution algorithm

Firstly, we check the effectiveness of the proposed approach for solving the multi-mode, multi-class dynamic user equilibrium problem. The above parameters can be regarded as basic ones.. We will investigate the consequence of the algorithm convergence when each time changing one parameter and remaining others with the same as in the base case. Fig.4 gives the convergence performance of the algorithm in some cases differentiating from $\hat{\theta}_m, \tilde{\theta}_m$ (m=1,2) values. We can see the algorithm convergences rapidly, particularly in the first several iterations when the values of $\hat{\theta}_m$ and $\tilde{\theta}_m$ are small. however, when $\hat{\theta}_m$=0.5, we can not find the convergence performance of the algorithm. The convergence solution of logit-based stochastic assignment cannot be found if $\hat{\theta}_m$ and $\tilde{\theta}_m$ are too large (e.g. Han, 2000; Hyman, 1969).

The inflow rates of equipped (unequipped) travelers of car (truck) are shown on links 1, 2, 3 and 4. In figures 5, 6, 7 and 8. we can find that equipped travelers of car (truck) are more concentrated on the limit departure times than unequipped travelers of car (truck) due to the fairly perfect traffic information. The formation and dissipation of the queues are depicted on some links in figure 9. There are many queues on links 1 and 2. However there are few on

link 4 and none on link 3 since the exiting rates of their upstream links are less than or equal to their exit capacities.

Now we check the ATIS impacts on each mode of travelers and system performance with respect to the ATIS market penetration and the information quality of equipped travelers $\hat{\theta}_m$.

Total market penetration of two modes is is $\eta = \dfrac{\hat{Q}_1^{rs} + \hat{Q}_2^{rs}}{Q_1^{rs} + Q_2^{rs}}$, the market penetration of car is

$\eta_1 = \dfrac{\hat{Q}_1^{rs}}{Q_1^{rs}}$, and the market penetration of truck is $\eta_2 = \dfrac{\hat{Q}_2^{rs}}{Q_2^{rs}}$. For simplicity, we assume

$\eta = \eta_1 = \eta_2$. The value of general travel cost perception parameter $\tilde{\theta}_m$ of unequipped travelers of car (truck) is fixed as 0.01. The demands of car and truck are $Q_1^{rs} = 15000$ and $Q_2^{rs} = 15000$, respectively.

Figures 10 and 11 depict the individual average travel costs of equipped and unequipped travelers of car (truck) against the total market penetration η and the travel cost perception parameter of equipped travelers of car (truck) $\hat{\theta}_m$ ($\hat{\theta}_m$=0.05, 0.1, 0.15 as shown in the legend). It is shown that the average travel cost of equipped travelers of car (truck) is higher than that of unequipped travelers of car (truck) at the different market penetration and the value of $\hat{\theta}_m$. It implies that using ATIS always benefits equipped travelers of different modes if we neglect the cost for purchasing the ATIS device and using the information system. The average travel costs of equipped and unequipped travelers of car (truck) are ascending with the increase of market penetration and the value of $\hat{\theta}_m$. We can find the average travel cost saving of equipped travelers of car (truck) compared with unequipped travelers of car (truck) is marginally descending when the market penetration is above 30%. It is shown that impacts of ATIS upon equipped and unequipped travelers of car (truck) are negative under many conditions.

This could be explained as follows. With the increase of the market penetration and the value of $\hat{\theta}_m$, equipped travelers of car (truck) that influence the ability of the travelers and traffic system are superior to unequipped travelers of car (truck). It can be seen from Fig.5 and 6 that equipped travelers of car (truck) choose the range of departure times and routes narrower than that of unequipped travelers of car (truck) since receiving the more traffic information. On the other hand, A greater number of equipped travelers of car (truck) may select the best alternatives (from their individual point of view) and consequently equipped travelers of car (truck) will tend to concentrate on the same routes during the same departure times. Thus, higher levels of traffic congestion could potentially be generated by more information and higher market penetration. Finally the benefits of equipped travelers of car (truck) could be reduced.

It can be seen from Figs.10 and 11 that the average travel cost of equipped travelers of car (truck) become small and the average travel cost of unequipped travelers of car (truck) become smaller with the increase of the value of $\hat{\theta}_m$ when the market penetration is less than 40%. And we can find the change of the system total travel cost and total travel time is little in Figs.12 and 13. In other words, when the market penetration is small, equipped travelers of car (truck) receiving more perfect traffic information can obtain more benefits. However, when the market penetration is more than 40%, the results turn upside down with the increase

of the value of $\hat{\theta}_m$. Not only the average travel costs of equipped and unequipped travelers of car (truck) ascend, but also the average travel cost saving of equipped travelers of car (truck) versus unequipped travelers of car (truck) descends and it can be seen from Figs.12 and 13 the system total travel cost and total travel time increase rapidly. In other words, when the market penetration is high, the equipped travelers of car (truck) for receiving more perfect traffic information can obtain smaller benefits and system traffic conditions might be exacerbated. The results give us an alarm that the bad results could be induced if the more perfect traffic information is provided to equipped travelers of car (truck) when market penetration is higher than a value.

Figures 12 and 13 depict system total travel cost and total travel time with respect to the market penetration and the value of $\hat{\theta}_m$. It can be seen from figures 12 and 13 that system total travel cost and total travel time increase with the increase of the market penetration and the value of $\hat{\theta}_m$, when the market penetration is higher than 30%. In other words, at most levels of market penetration, the ATIS is most likely to generate negative effect on the transport network.
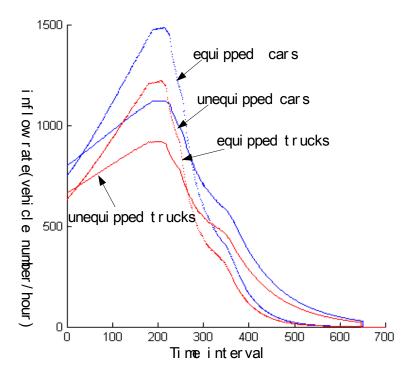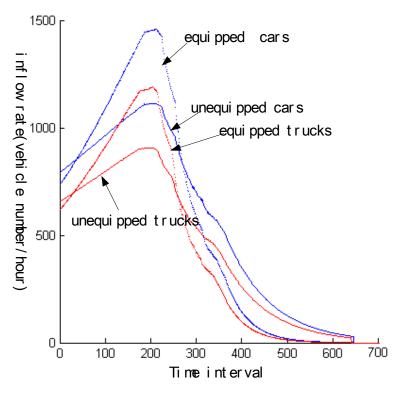

Figure 5. Link 4's inflow rate of truck and car

Figure 6. Link 2's inflow rate of truck and car

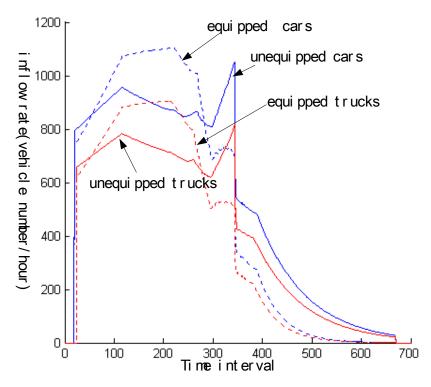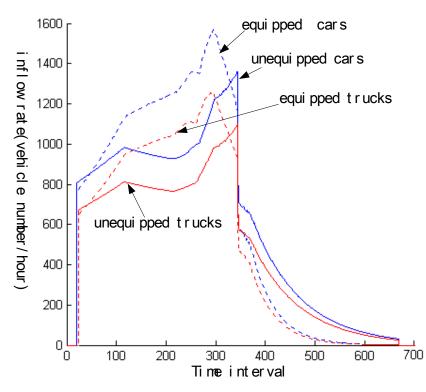*Figure 7. Link 3's inflow rate of truck and car*

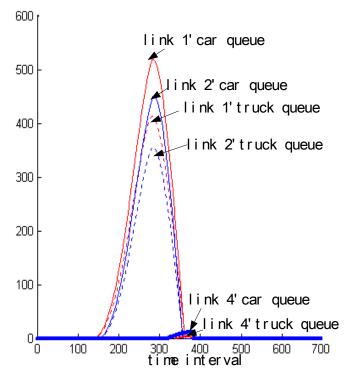*Figure 8. Link 4's inflow rate of truck and car*
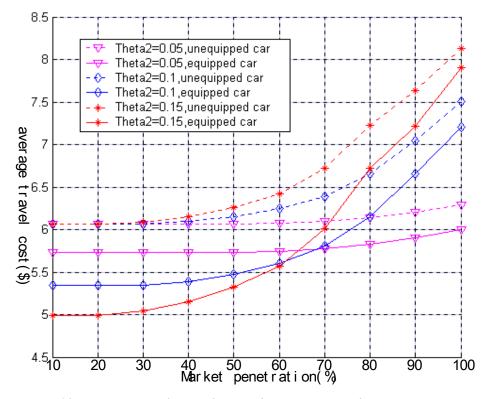

Figure 9. Link queue vehicle of truck and car

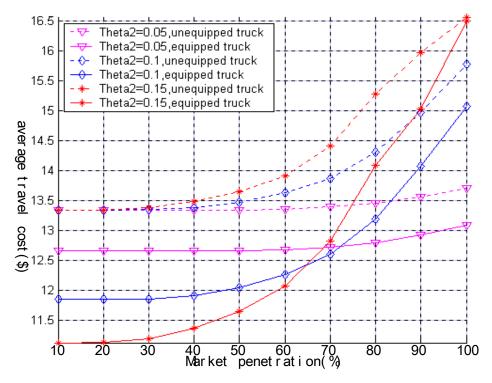*Figure 10. Average travel cost of car with respect to market penetration*

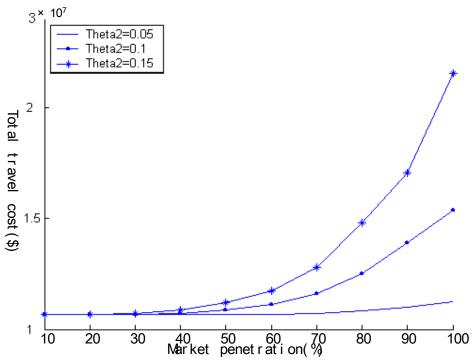*Figure 11. Average travel cost of truck with respect to market penetration*

Figure 12. Total travel cost with respect to market penetration

Figure 13. Total travel time with respect to market penetration

6. Conclusions

This paper presents a formulation and solution algorithm for multi-mode, multi-class dynamic user equilibrium problem, in order to assess the impacts of ATIS in recurrent congestion networks with queues. Suppose each mode of equipped travelers follows stochastic dynamic simultaneous route and departure time user-equilibrium with small travel cost perception variation, each mode of unequipped travelers in making travel choices according to the past experiences follows stochastic dynamic simultaneous route and departure time user-equilibrium with high travel cost perception variation. A diagonalisation algorithm based on stochastic dynamic network loading for logit-based simultaneous route and departure time is proposed. Finally a numerical example is presented to demonstrate the ATIS impacts on each mode of individual average travel cost, total travel cost, total travel time etc at the different market penetrations and the values of equipped traveler's travel cost perception variation.

In further studies: 1. The calibration of the model parameters such as travel cost perception variation of equipped and unequipped travelers, value of time etc. 2. Considering multi-mode dynamic network model with physical queue.

References

Abdel.Aty, M.A., Kitamura, R. and Jovanis, P. (1997). Using stated preference data for studying the effect of advanced traffic information on drivers' route choice. *Transportation Research,* C(5)-1, pp. 39-50.

Adamo, V., Astarita, V., Florian, M., Mahut, M. and Wu, J.H., (1999). Modelling the spill-back of congestion in link based dynamic network loading models: a simulation model with application. *Proceedings of the 14th International Symposium on Theory of Traffic Flow*, pp.555－573.

Al-Deek, H.M., Khattak, A.J. and Thananjeyan, P. (1998). A combined traveler behavior and system performance model with advanced traveler information systems. *Transportation Research-B,* 32, pp. 479-493.

Astarita, V. (1996). A continuous time link model for dynamic network loading based on travel time function. *Proceedings of the 13th International Symposium on Theory of Traffic Flow*, pp. 79–102.

Barcelo, A.J.L., Ferrer, R. (1999). Simulation assisted design and assessment of vehicle guidance systems. *International Transactions in Operational Research,* 6, pp. 123-143.

Ben-Akiva, M., De Palama, A., Kaysi, I., (1991). Dynamic network models and driver information systems. *Transportation Research* 25A, pp. 251-266.

Benneet, L. (1993). The existence of equivalent mathematical programs for certain mixed equilibrium traffic assignment problems. *European Journal of Operational Research* 71, pp. 177-187.

Bliemer, M.C.J. (1998). Multi-user class travel time functions. *Proceedings of the 6th Meeting of the EURO Working Group on Transportation*, Goteborg, Sweden.

Bliemer, M.C.J. (2001). *Analytical dynamic traffic assignment with interaction user-classes: theoretical advances and applications using a variational inequality approach*. Ph.D. thesis, Delft University of Technology, The Netherlands.

Bliemer, M.C.J., and Bovy., P.H.L (2003). Quasi-Variational Inequality Formulation of the Multiclass Dynamic Traffic Assignment Problem. *Transportation Research* B 37, pp. 501–519.

Bliemer, M.C.J., Castenmiller, R.J. and Bovy, P.H.L (2002). Analytical Multiclass Dynamic Traffic Assignment using a Dynamic Network Loading Procedure. *Proceedings of the 9th Meeting of the EURO Working Group on Transportation*, Bari.

Chen, Q. and Stauss, H.J. (1997). Evaluating traffic effects of a route guidance system by dynamic simulation. *Simulation practices and Theory,* 5, pp. 793-804

Chen, H.K. (1998). Dynamic travel choice models a variational nequality approach. Springer verlag Berlin. Heidelberg.

Dial, R.B., (1971). A probabilistic multipath track assignment model which obviates path enumeration. *Transportation Research 5* (2), pp. 83–111.

Emmerink, R.H.M., Nijkamp, P., Rietveld, P. and Axhausen, K.W. (1994). The economics of motorist information systems revisited. *Transportation Views* 14, pp. 363-388.

Emmerink, R.H.M., Axhausen, K.W., Nijkamp, P. and Rietveld, P. and (1995a). Effects if information in road transport networks with recurrent congestion. *Transportation* 22, pp. 21-53.

Emmerink, R.H.M., Axhausen, K.W., Nijkamp, P. and Rietveld, P., (1995b). The potential of information provision a simulated road transport network with non-recurrent congestion. *Transportation Research* C, pp. 293-309.

Kanafani, A., Al-Deek, H. (1991). A simple model for route guidance benefit. *Transportation Research,* 25B, pp. 191-202.

Han, S. (2000). D*ynamic traffic assignment techniques for general road networks*. Ph.D thesis, Center for transport studies university college London.

Han, S. (2003). Dynamic traffic modeling and dynamic stochastic user equilibrium assignment for general road networks. *Transportation Research B* (37), pp. 225-249.

Harker, P.T. (1988). Multiple equilibrium behaviors in network. *Transportation Science* 22, pp. 39-46.

Hong K.Lo., Ran, B., Hongola, B., (1996). Multi-class dynamic traffic assignment model: Formulation and computational experiences. *Transportation Research Records* 1537, pp. 74–82.

Hong K.Lo and Szeto, W.Y. (2002a). A methodology for sustained traveler information's services, *Transportation Research B*, 36, pp.113-130.

Hong K.Lo and Szeto, W.Y. (2002b). Paradigms of modeling advanced traveler information services. *IEEE 5th conference on ITS*.

Hong K.Lo and Szeto, W.Y. (2002c). A cell-based variational inequality formulation of the dynamic user optimal assignment problem. *Transportation Research B*, 36, pp.421-443.

Hong K.Lo and Szeto, W.Y. (2004). Modeling advanced traveler information services:static versus dynamic paradigms. *Transportation Research Part B*, 38, pp. 495–515.

Huang, H.J. (2002). Modeling and solving the dynamic user equilibrium route and departure time choice problem in network with queues. *Transportation Research B*, 36, pp. 253-273.

Huang and Williams (2004). A multi-class dynamic user equilibrium model for queuing networks with Advanced traveler information systems. *International Journal of Mathematical Modelling and Algorithms*, publication.

Hyman, G.M. (1969). The calibration of trip distribution models. *Environment and planning*, 1, pp. 105-112.

Kuwahara, M. and Akamatsu, T. (1993). Dynamic equilibrium assignment with queues for a one-to-many OD pattern. *Proceedings of the 12th International Symposium on Transportation and Traffic Theory*, ed. C. F. Daganzo, pp. 185-204, Berkeley, CA. Elsevier, New York.

Kuwahara, M. and Akamatsu, T. (1997). Decomposition of the reactive dynamic assignment with queues for a many-to-many origin-destination pattern. *Transportation Research B,* 31, pp. 1-10.

Kuwahara, M. and Akamatsu, T. (2001). Dynamic user optimal assignment with physical queues for a many-to-many OD pattern. *Transportation Research B*, 35, pp. 461-479.

Li, J., Fujiwura, O. (2000). A reactive dynamic user equilibrium model in network with queues, *Transportation Research B*, 34, pp. 605–624.

Mahmassani, H.S. (1994). *Development and testing of dynamic traffic assignment and simulation procedures for ATIS/ATMS application*. Technical Report DTFH6 1-90-R-00074-FG. Center for transportation Research, The University of Texas at Austin.

Maher, M.J. and Hughes, P.C. (1995). Estimation of the potential benefits from an ATT system using a multiple user class stochastic user equilibrium assignment model. In *Applications of Advanced Technologies in Transportation Engineering*, eds Y.J. Stephanedes and F. Filippi, pp. 700-704.

Nagurney. A (2000). A multiclass, multicriteria traffic network equilibrium model. *Mathematical and computer modelling*, 32, pp. 393-411.

Nagurney, A. Dong, J. (2001). A multiclass, multicriteria traffic network equilibrium model with elastic demand. *Transportation Research B*, 36, pp. 445–469.

Ran, B., Lo, H.K., Boyce, D.E. (1996). A formulation and solution algorithm for a multi-class dynamic traffic assignment model. In: Lesort, J.-B. (Ed.), *Transportation and Traffic Theory*. Pergamon, Oxford, pp. 195–216.

Bin Ran, Der-Horng Lee, and Michael Seong-Il Shin (2002). New Algorithm for a Multiclass Dynamic Traffic Assignment Model. J*ournal of Transportation Engineering,* 128, Issue 4, pp. 323-335.

Reddy, P et al. (1995). Design of an artificial simulator for analyzing route choice behavior in the presence of information system. *Mathematical and Computer modelling*, 22, pp. 119-147.

Sheffi, Y. (1985). Urban transportation network: equilibrium analysis with mathematical programming methods.

Tsuji, H., Takahashi, R., Kawashima, H. (1985). A stochastic approach for estimating the effectiveness of a route guidance strategies. *Transportation Science*, 19, pp. 333–351.

Vandebona, U. and Upadhyay, P.K. (1997). Simulation modeling of route guidance concept. *Transportation Research*, record 1573, pp. 44-51.

Van Vuren T and Watling D.P. (1991). A multiple user class assignment model for route guidance. *Transportation research*, record 1306, pp. 22-31.

Vythoulkas, P.C. (1990). A dynamic stochastic assignment model for the analysis of general networks. *Transportation Research-B,* 24, 453-469.

Wunderlich, K. (1998). A simulation-based assessment of route guidance benefits under variable network congestion conditions. *Mathematics computing and modeling*, 27, pp 87-101.

Wie, B.W., Tobin, R.L., Friesz, T.L., Bernstein, D. (1995a). A discrete time, nested cost operator approach to the dynamic network user equilibrium problem. *Transportation Science,* 29(1), pp. 79–92.

Yang, H. (1998). Multiple equilibrium behavior and advanced traveler information systems with endogenous market penetration. *Transportation Research,* 32B, pp. 205–218.

Yang, H. (1999). Evaluating the benefit of a combined route guidance and road pricing system in a network with recurrent congestion. *Transportation*, 26, pp. 299–322.

Appendix

Appendix A. Proof of the algorithm

We now prove that the algorithm does generate logit-based flow independent ideal stochastic dynamic simultaneous path and departure time choices between each OD pair. We note that each link likelihood $L_{(i,j)}(k)$ is proportional to the logit probability that link a=(*i,j*) is used during time interval *k* by a traveler chosen at random from among the population of trip-makers between origin *r* and destination *s*, given that the traveler is at node *i* during time interval *k*. The probability that a given path will be used is proportional to the product of all the likelihood of the links comprising this path. Suppose path *p* consists of nodes (*r, 1,2...n, s*) and links (*1,2...h*). Sub-path p1 includes (*1,2...n, s*) and links (*2...h*). The probability of traveler choosing path *p* and departure time *k* between origin *r* and destination *s* is $P_p^{rs}(k)$.

$$P_p^{rs}(k) = G \cdot \prod_{a \in p} \left\{ L_{(i,j)}(k) \right\}^{\delta_{ap}^{rs}} \tag{a1}$$

Where G is proportionality constant for each OD pair and the product is taken over all links in the networks. Here, $t = k + t_p^{ri}(k)$. The incidence variable δ_{ap}^{rs} ensures that $P_p^{rs}(k)$ include only those links in the pth path between origin r and destination s. Substituting the expression for the likelihood $L_{(i,j)}(k)$ in the above equation, the probability of choosing a particular efficient path-departure time pair becomes

$$\begin{aligned} P_p^{rs}(k) &= G \cdot \exp\left\{ \theta \cdot \left[\pi_{rs} - \pi_{js}(k + t_{(r,j)}(k)) - c_{(r,j)}(k) \right] \right\} \\ &\quad \cdot \prod_{a \in p_1} \exp\left\{ \theta \cdot \left[\pi_{is}(t) - \pi_{js}(k + t_{(i,j)}(k)) - c_{(i,j)}(k) \right] \cdot \delta_{ap_1}^{rs} \right\} \\ &= G \cdot \exp\left\{ \theta \cdot \left[\pi_{rs} - \pi_{js}(k + t_{(r,j)}(k)) - c_{(r,j)}(k) \right] \right\} \\ &\quad \cdot \exp\left\{ \theta \cdot \sum_{a \in p_1} \left[\pi_{is}(t) - \pi_{js}(k + t_{(i,j)}(k)) - c_{(i,j)}(k) \right] \cdot \delta_{ap_1}^{rs} \right\} \\ &= G \cdot \exp\left\{ \theta \cdot (\pi_{rs} - c_p^{rs}(k)) \right\} \end{aligned} \tag{a2}$$

The last equality results from the following summations:

$$\begin{aligned} &\pi_{rs} - \pi_{1s}(k + t_{(r,1)}(k)) + \sum_{a \in p_1} \left[\pi_{is}(k) - \pi_{js}(k + t_{(i,j)}(k)) \right] \cdot \delta_{ap}^{rs} \\ &= \pi_{rs} - \pi_{1s}(k + t_p^{r1}(k)) + \pi_{1s}(k + t_p^{r1}(k)) - \pi_{2s}(k + t_p^{r2}(k)) \\ &\quad + \vdots \\ &\quad + \pi_{ns}(k + t_p^{rn}(k)) - \pi_{ss}(k + t_p^{rn}(k)) \\ &= \pi_{rs} \end{aligned} \tag{a3}$$

and

$$\sum_{a \in p} c_a(k) \delta_{ap}^{rs} = c_1(k) + c_2(k + t_p^{r1}(k)) + \cdots + c_h(k + t_p^{rn}(k)) = c_p^{rs}(k) \tag{a4}$$

Since $\sum_p \sum_k P_p^{rs}(k) = 1$

The proportionality constant must equal

$$G = \frac{1}{\sum_p \sum_k \exp\left\{ \theta \cdot [\pi_{rs} - c_p^{rs}(k)] \right\}} \tag{a5}$$

Thus

$$P_p^{rs}(k) = \frac{\exp\left\{ \theta \cdot [\pi_{rs} - c_p^{rs}(k)] \right\}}{\sum_p \sum_k \exp\left\{ \theta \cdot [\pi_{rs} - c_p^{rs}(k)] \right\}} = \frac{\exp\left\{ -\theta \cdot c_p^{rs}(k) \right\}}{\sum_p \sum_k \exp\left\{ -\theta \cdot c_p^{rs}(k) \right\}} \tag{a6}$$

Above equation depicts a stochastic dynamic simultaneous path/departure time choice among the efficient paths connecting OD pair *rs*. The algorithm does generate a stochastic dynamic simultaneous path/departure time choice probability using actual path travel costs.

Now we try to prove the forward pass of the algorithm does generate the results of the logit flow assignment for simultaneous path/departure time choice. Firstly, we transform equation (a6) to the following equation.

$$
\begin{aligned}
f_p^{rs}(k) &= q^{rs} \frac{\exp\{-\theta \cdot c_p^{rs}(k)]\}}{\sum_p \sum_k \exp\{-\theta \cdot c_p^{rs}(k)]\}} = q^{rs} \cdot \frac{\sum_p \exp\{-\theta \cdot c_p^{rs}(k)]\}}{\sum_p \sum_k \exp\{-\theta \cdot c_p^{rs}(k)]\}} \cdot \frac{\exp\{-\theta \cdot c_p^{rs}(k)]\}}{\sum_p \exp\{-\theta \cdot c_p^{rs}(k)]\}} \\
&= q^{rs}(k) \cdot \frac{\exp\{-\theta \cdot c_p^{rs}(k)]\}}{\sum_p \exp\{-\theta \cdot c_p^{rs}(k)]\}}
\end{aligned}
\tag{a7}
$$

Where

$$
q^{rs}(k) = q^{rs} \cdot \frac{\sum_p \exp\{-\theta \cdot c_p^{rs}(k)]\}}{\sum_p \sum_k \exp\{-\theta \cdot c_p^{rs}(k)]\}} = q^{rs} \cdot \frac{\sum_{(i,k)\in A(i)} w_{(i,k)}(k)}{\sum_k \sum_{(i,k)\in A(i)} w_{(i,k)}(k)} \quad i \in r, \forall rs, k
\tag{a8}
$$

The demand between OD pair *rs* during time interval *k* assigns to the network according to the DYNASTOCH algorithm. The equation (a8) is substituted into the forward pass of the DYNASTOCH algorithm; the equation (35) can be obtained. The major difference(one is $\pi^{rs}(k)$, other is π^{rs})between the origin link 's link likelihood of the DYNASTOCH algorithm and this algorithm does not influence the calculation results.