A Dynamic Model to Appraise Strategic Land-Use and Transport Policies

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This paper presents a framework for an ex ante appraisal of urban land use and transport strategies. At the core of this framework is a dynamic land use and transport interaction model. Objective functions and an optimisation routine are used to assess the effects of complex strategies formed by policy instrument combinations with different policy levels possible. The main objective of the underlying study is to promote sustainable policies. Therefore the proposed framework considers either explicitly or implicitly all five domains of the STELLA focus group 4 (environment, safety, health, land use and congestion).

1. Introduction

This paper aims to present a framework to appraise long term urban planning strategies. It consists of two main items: a dynamic land-use/transport model (termed **S**ketch **P**lanning **M**odel, SPM) and an evaluation/optimisation procedure. Both are results of the EU funded 5th framework research project PROSPECTS (**P**rocedures for **R**ecommending **O**ptimal **S**ustainable **P**lanning of **E**uropean **C**ity **T**ransport **S**ystems).
Environment, Safety, Health, Land Use and Congestion are the domains of the STELLA Focus Group 4. All five of them are addressed by the framework presented here. Environment, safety and health aspects are part of a sustainable objective function used in the evaluation procedure. Changes in land use – location choices of residences and businesses – are output of the SPM. Congestion is covered on a coarse level by an area speed flow relationship in the land use/transport model. This paper sets out the general appraisal framework and the development of the SPM, covering its sub-models for land use and transport and their interaction. It then describes how policy instruments may be "optimised" making use of the objective function and an automated optimisation routine. The application

of the framework to produce optimal levels of given policy instruments is demonstrated with a case study. The conclusions and recommendations section is followed by references and an appendix which describes the mathematical formulations used in the appraisal framework.

2. The overall framework

A framework for strategic policy appraisal with respect to sustainable urban planning was developed by Emberger (1998) and applied by May et al., (2000) (figure 1). This appraisal framework work has been extended to cover sustainability and land use interactions (Minken et. al., 2003). The framework consists of four modules: policy instruments, a land use and transport model, objective functions and an optimisation method. A set of *n* different policy instruments and their associated values (section 4.1) form the input into the land use and transport model (section 3). Output indicators such as generalised costs, distance travelled by mode, local atmospheric emissions, CO_2 tonnes emitted and accidents are computed by the SPM. An objective function value is calculated from the relative changes of these indicators in comparison to a do nothing scenario (section 4.2). This procedure is repeated *n+1* times to produce an objective function vector for a given policy instrument matrix. The matrix and the vector are used to start a formal optimisation routine (section 4.3). The output of the optimisation routine is a suggestion for a new set of policy instrument levels which is again input to the land use/transport model. This procedure is repeated until a convergence criterion is fulfilled. The policy instrument combination which gives the highest objective function value so far, can be seen as the "optimal strategy", i.e. a solution which is near to an at least local optimum. Sensitivity tests and a re-start of the optimisation with a tighter convergence criterion might be used to check and refine the strategy. A fuller description of the framework and its different parts is given in (May et. al., 2003) and (Minken et. al., 2003).
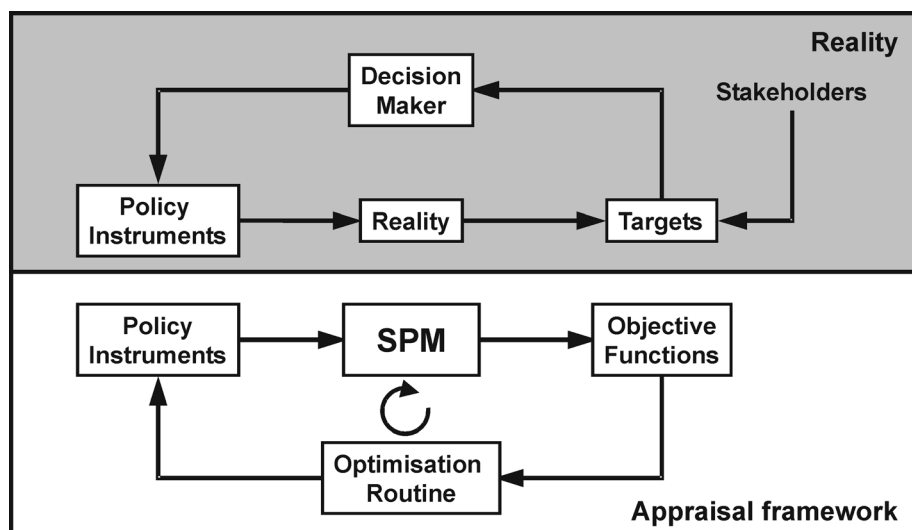

Figure 1: Framework for policy assessment (Emberger, 1998)

3. The dynamic land use/transport model SPM

The dynamic land use/transport model SPM is at the core of the policy appraisal framework. Sustainability is a long term, strategic objective and as such any land use changes should be used to inform policy decisions. However it is still common to see studies which use transport models only assuming that land use is constant. In contrast to this assumption land use and transport are parts of a dynamic system and linked by time lagged feedback loops. Although this has been recognised since the early Seventies (Wermuth, 1973), Land Use and Transport Integrated (LUTI) models are still rare. Therefore one of the objectives in the project PROSPECTS was the development of a dynamic LUTI model on a strategic level. The PROSPECTS consortium refers to this model as the Sketch Planning Model (SPM). The SPM broadly follows the concept of self-organising systems (Allen, 1997). Due to the strategic characteristic of the SPM the level of spatial aggregation is rather high[1], e.g. administrative districts on a municipal level are chosen as travel analysis zones. It is therefore appropriate to choose the functional urban region as the study area.
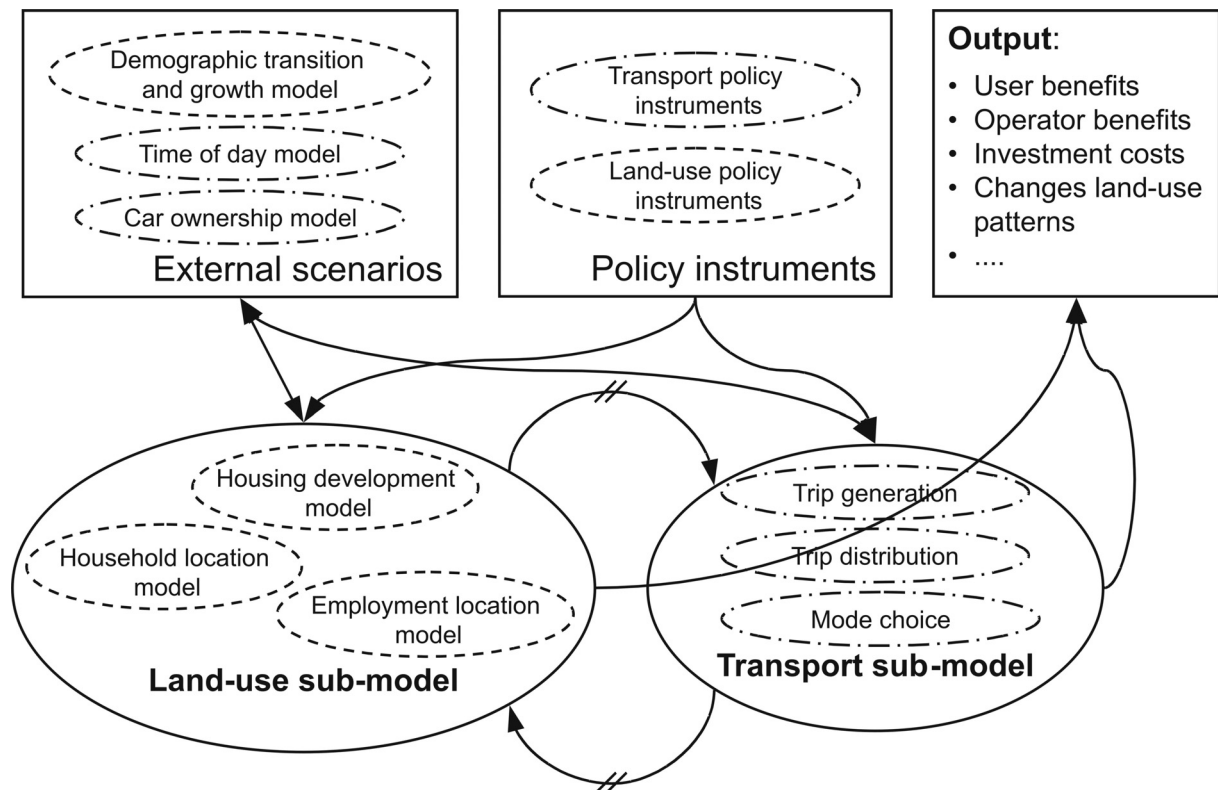

Figure 2: Development of the qualitative structure of the SPM using causal loop diagramming

The first stage of the SPM development was a qualitative analysis using causal loop diagramming (CLD).[2] Figure 2 shows the result of this initial process. The SPM can be

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1 The subdivision of the study area has to be done with great care to limit aggregation errors to **a** reasonable level.

2 For a description of CLD see e.g. (Emberger, Fischer 2000).

divided into two main sub-models: the land-use and the transport model. These two model parts are linked together with time lags. Input to the SPM comes from external scenarios and policy instruments. Outputs can be in the form of indicators or these can be adapted to form an objective function. The next step in the model development was a detailed qualitative description of the sub-models (figure 3 and figure 4) and the quantification of the relations found in the qualitative analysis (see in the appendix). During the development process the original structure had to be slightly changed, e.g. the time of the day model had to be moved to external scenarios.

3.1 Transport model

Previously developed strategic transport models Knoflacher et al., (2000), Pfaffenbichler and Emberger, (2001) were used as a basis for the development of the transport sub-model. Numerous studies and household surveys have shown that travel time budgets are stable as well over time as across cites, countries and even continents[3]. Therefore constant travel time budgets are seen as an appropriate concept to model trip generation in the SPM. The transport sub-model uses a simultaneous distribution to destinations and modes. This process uses friction factors based on generalised costs weighted by exponential functions (Walther et. al., 1997). The non-motorised modes pedestrian and bike are fully represented in the SPM. A simplified road and public transport network is used. In the current version of the SPM that means the network is aggregated to one link per origin-destination (OD) pair. The consequence is that there is no route choice stage in the SPM. The full qualitative description of the transport sub-model is given in figure 3. Supply and demand effects for road traffic are considered using area speed flow relationships (see in the appendix).

3.2 Land use model

The land use sub-model was developed based on research performed for the Viennese Municipal Department 18 "Urban Development and Planning" (Pfaffenbichler, 2001). The land use sub-model consists of a residential and a business/workplace location model. The land use sub-models are in general LOGIT or gravity type models. The ratio of the exponential function value of utilities and dis-utilities of an alternative to the sum of all alternatives is used to distribute a potential quantity demanded to different locations. The detailed equations of the land use sub-model are included in the appendix.

The residential location model consists of four sub-models: a development model, a willingness to move out model, a willingness to move in model and a supply/demand redistribution model (see figure 4). The decision to develop new housing in a zone is influenced by the expected rent, the land price and the amount of land available for development. The outcome is the number of new housing units per zone which will be available after a time lag. Rent and land price depend on the demand-supply situation of a zone. The decision to move out and to move in depends on the share of green areas, the accessibility and the rent in a zone. If the demand in a zone could not be met by the supply of housing then the resulting over demand is re-distributed to other zones with over supply.

The basic structure of the workplace location model is similar to the residential sub model but consists of two sub-models: one for the production sector and one for the service sector.

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3 E.g. (Hupkes, 1982), (Marchetti, 1994), (Brög, Erl, 1999), (Schafer, 2001).

The study area wide workplace development has to be defined as an exogenous scenario. The SPM calculates the amount of space available for business use and allocates the total potential of re-allocating and newly developed businesses to the different locations.
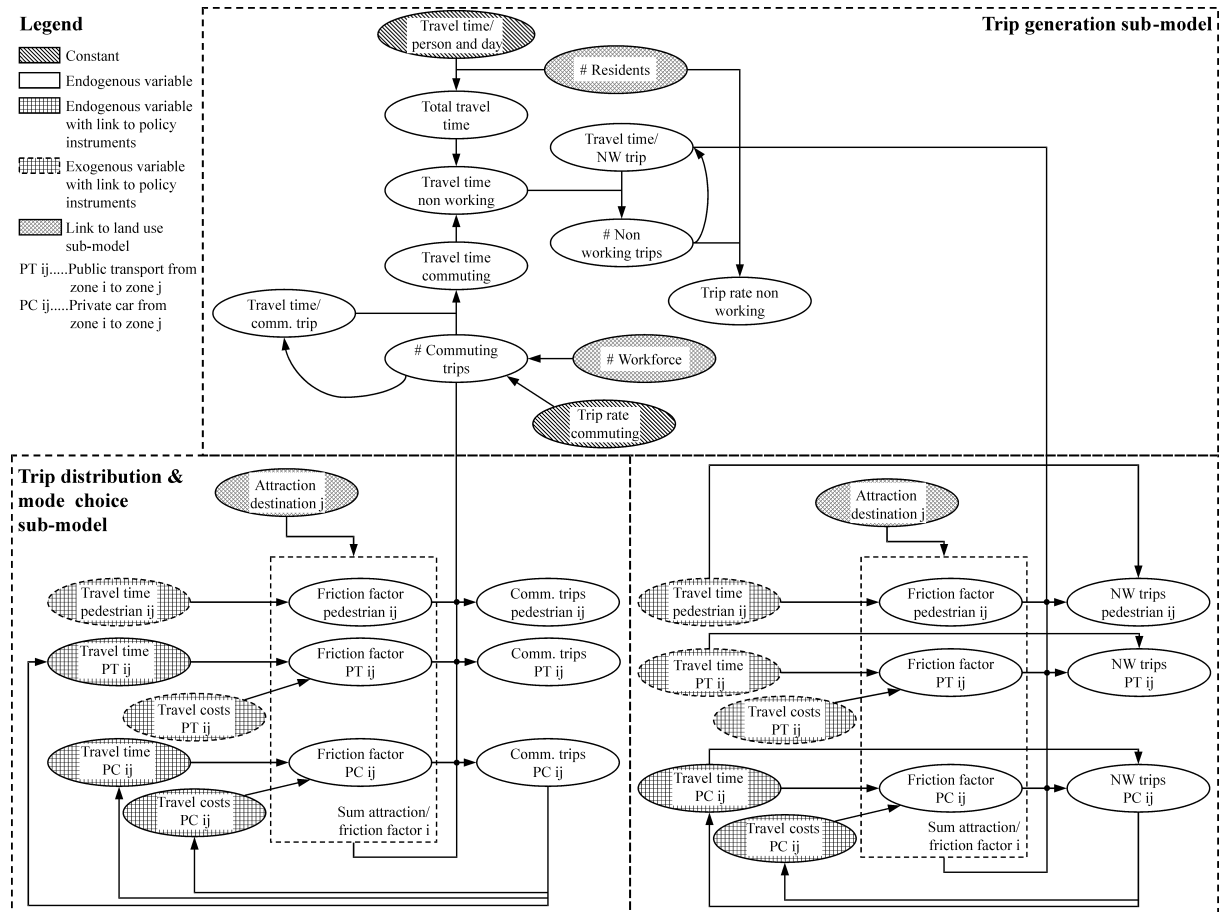

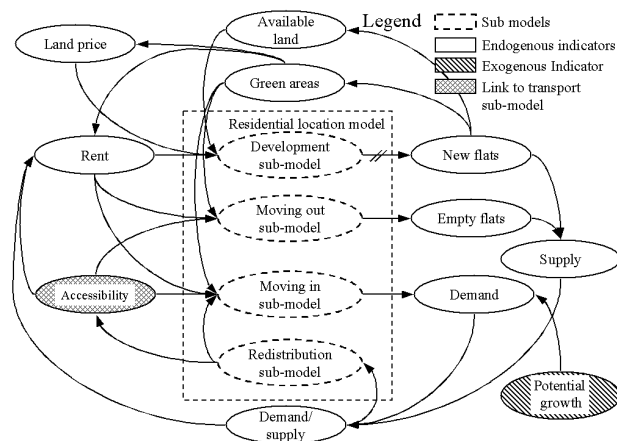*Figure 3: Sub-system diagram transport model*

Figure 4: Sub-system diagram residential location model

3.3 Links between the land use and transport sub-models

The links between the sub-models are shown in figure 5. Accessibility is one of the outputs of the transport sub-model. Accessibility in the year *n* is used as an input into the location models in the year *n+1*. Workplace and residential location is an output of the land use model. The number of workplaces and residents in each zone in the year n is used as attraction and potential in the transport model in the year *n+1*. The SPM iterates in a time lagged manner between the transport and the land use sub-model over a period of 30 years. I.e. a single SPM run consists of 30 iterations.
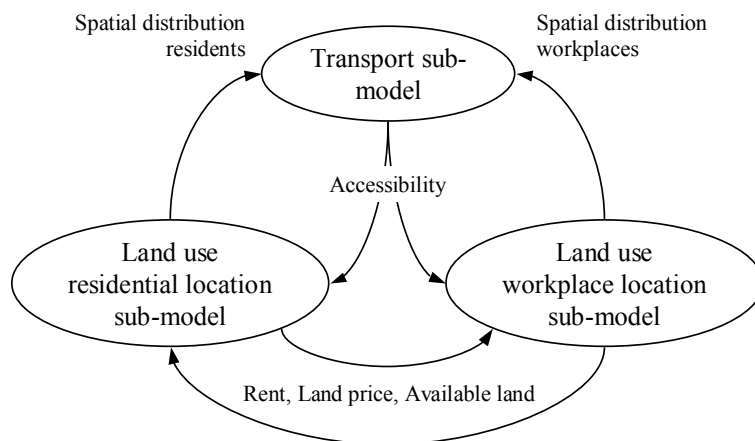

Figure 5: Link between the transport and the land use sub-model of the SPM

4. Finding the optimal "second-best" strategy

Given a broad set of policy instruments and associated levels, numerous combinations are possible. To find the best strategy the use of automated optimisation methods was suggested (SAMI, 2000). The application of area-wide policy instruments results in second-best strategies as opposed to "first-best" strategies which assume that social marginal costs can be charged across all modes by some future GPS based system.

4.1 Policy instruments

The instruments identified in PROSPECTS covered a wide range of possibilities (May et. al., 2001). The formal optimisation process will lend itself well to optimisation of strategic instruments which form the basis of an overall package or plan. Strategic instruments can be considered as those instruments which are expected to have a significant impact upon indicators and objectives, or which impact upon a significant area of the urban region. Furthermore most strategic instruments have some level which may be varied e.g. a price which can be optimised.

The policy instruments may consist of differing types of instruments as suggested below:

- Continuous overall policy variables are policy variables that are used to change the relative overall level of an instrument applied to the whole of the study area or a

significant part thereof. Examples would include changes in the relative level of the fuel tax, parking charges in different zones by the same percentage, changes in uniform tolls around a cordon, uniform changes in public transport fares and frequencies.

- Discrete policy variables are binary variables which describe an instrument as either used or not used (on/off). Whether to implement a large road investment project is one example of a discrete instrument, i.e. the investment project is either implemented or not implemented. Some discrete instruments introduce an associated continuous variable and the dimension of the problem increases, e.g. different cordon locations may be considered as discrete options within the optimisation process with the charge as an associated variable.

- Other dimensions. These basic variables can be given other dimensions in space, by time of day and by other instrument specific attributes. For example pricing instruments can be given different levels in the peak and off-peak as suggested by marginal cost pricing. Parking charges can vary by time of day, duration of stay and by zone within a city. Property taxes may vary according to zone and use of floor-space.

Continuous policy instruments can in the most general case be applied at any level in any one year (t = 0,...,30). Thus, for a single instrument there could be 30 different levels in a single SPM run. As an example: For finding the optimal public transport fare and frequency levels in the peak and off peak period an optimisation problem in 120 dimensions would have to be solved. To optimise these two types of instruments is already a challenging and time consuming task. As the goal was to formulate strategies consisting of more different types of instruments, it was decided to cut the dimensions of the optimisation problem down. Therefore the variability of instruments over the evaluation period of an SPM run was limited. For the present study the concept of "policy profiles" was introduced. Policy instrument levels were allowed to be specified (and later optimised) for two points in time, t_A the implementation year and t_L the long run year (figure 6). Thus we need to specify the year of implementation t_A and the number of years until a long run value is to be expected.

The vector of levels on instruments in the short-term year are denoted X_A and levels on instruments in the long-term year are denoted X_L. The levels on instruments in intermediate years can be determined by interpolating between the instrument levels in year t_A and t_L while the level is then assumed constant for any year after the long run year as depicted in figure 6. The long run year is chosen such that any time-lagged responses in the model have taken full effect by the year t_H which is taken to be the final horizon year of the evaluation period.
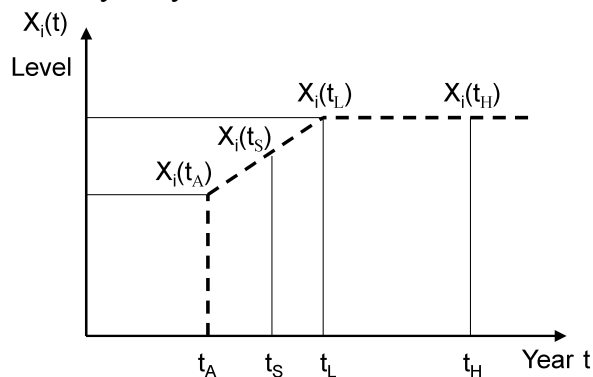

Figure 6: Instrument profile for the continuous instruments $X_i(t)$

4.2 Objective functions

One possible definition of the overall objective of sustainability is given in (May et. al., 2003) p. 12:
A sustainable urban transport and land use system

- provides access to goods and services in an efficient way for all inhabitants of the urban area,
- protects the environment, cultural heritage and ecosystems for the present generation, and
- does not endanger the opportunities of future generations to reach at least the same welfare level as those living now, including the welfare they derive from their natural environment and cultural heritage.

Seven relevant sub-objectives to achieve sustainability can be defined (May et. al., 2003) p. 13:

economic efficiency,
liveable streets and neighbourhoods,
protection of the environment,
equity and social inclusion,
safety,
contribution to economic growth and
intergenerational equity.

Optimisation requires a quantifiable objective function to be maximised (or minimised). The objective function used in the appraisal framework presented here is an attempt to take into account these seven sub-objectives of sustainability. The basic idea behind the objective function was developed by Minken (1999). Further developments were made in different research projects and case studies (May et. al., 2000), (Knoflacher et. al., 2000), (Minken et. al., 2003). The core of the definition of sustainability applied here is sub-objective seven intergenerational equity. The objective function is a linear combination of the net present value over a 30 year period and the annual net benefit of the last year of this period. This approach is seen as an approximation for the sub-objective of intergenerational equity. The last year is constrained to satisfy certain environmental and financial requirements so as to represent as far as possible the welfare of future generations. For the mathematical description and more details see in the appendix and (Minken et. al., 2003).

The first sub-objective is dealt with by discounted net present value. For the sub-objectives two to six, indicators have been defined. For some of the years, there may be targets on some of the indicators, or there may be other constraints on their levels. It is not assumed that the indicators that go into the constraints cannot be used in the objective function, neither is it assumed that all indicators need to be included in the objective function. Some may be used only as constraints or be kept out from the optimisation altogether.[4] The evaluation period is taken to be 30 years though the sustainability issues relate to an even longer term.

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4 E. g.: No appropriate indicator for the objective liveable street was found at the SPM spatial aggregation level.

4.3 The optimisation method

The method applied within the appraisal framework presented here is based on the downhill simplex method in multi-dimensions (Nelder and Mead, 1965). It solves a multidimensional minimisation, i.e. finding the minimum of a function of more than one independent variable. The method requires only function evaluations, not derivatives. The method is applied to the SPM thus allowing an automated optimisation. A more detailed description is given in the appendix.

5. Vienna case study

The Vienna SPM uses its municipal districts as analysis zones. To illustrate the level of aggregation: the number of residents per zone varies between 18,000 to about 150,000. The area of a zone varies between about 1 km² to about 100 km². The Vienna SPM was calibrated to observed mode split in the base year 2001, development of housing between 1991 and 1998 and changes in population between 1991 and 2001. The 2001 census data are not yet available for workplaces. Therefore the business and workplace sub-model was only calibrated towards some estimates. As an example results of an optimisation using the Vienna SPM and the policy instruments public transport fares, public transport frequency and parking charges are presented below. Today parking charges have to be paid in the central business district 1 and the adjacent districts 2 to 9 and 20. In this case study additional parking charging is implemented in the districts 10 to 19.

To start an optimisation the SPM user has to specify the implementation and long run years, the upper and lower bounds of the policy instruments and which constraints, if any, should be considered in the objective function. The results presented here used the present value of finance and CO_2 emissions as constraints. To achieve the reductions defined in the Kyoto-targets for greenhouse gas emissions in the last year and to avoid a negative present value of finance were used the targets. I.e. policy instrument combinations violating these targets were penalised. An initial set of policy instrument values used as input vectors is calculated by the optimisation routine (see figure **7**). The SPM calculates the behavioural changes and the objective function values for each policy instrument vector. These are used to suggest new policy instrument values. As the instruments can vary by the period of time and in the implementation and the long run year the total number of variables to be optimised in this example is 12. In figure **7** the thick line with diamonds show the convergence of the constrained objective function. The thin black lines show the development of the corresponding policy instrument values. The convergence criterion was fulfilled after about 180 SPM runs. The tolerance chosen in this optimisation was 0.1. The total computing time for this optimisation was about 3 hours.
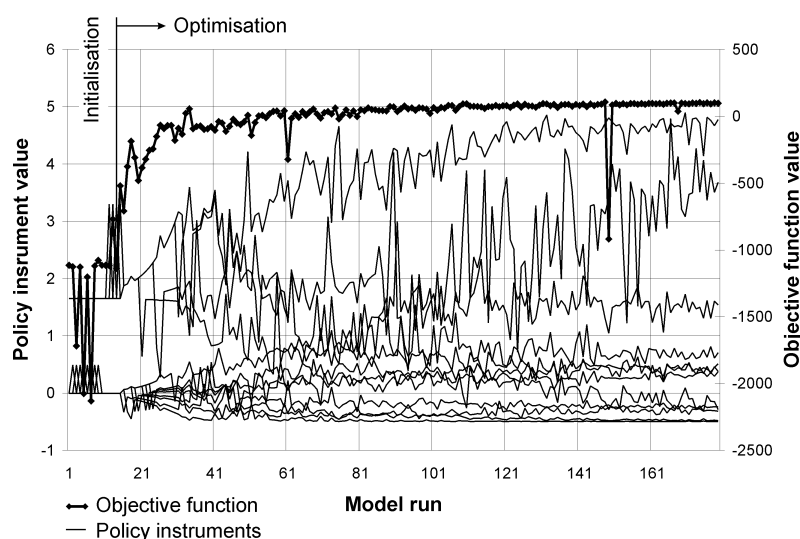
Figure 7: Convergence of the optimisation process

The following table 1 shows the results of an optimisation with a tolerance of 0.01 using the constrained objective function. All instruments were first introduced in year 5 (implementation year) and then linearly changed until year 15 (long run year). In general the policy is to reduce fares towards the lower limit of –50% by the long run year, to decrease frequencies in the peak period and to increase it in the off-peak. Long term parking charges are increased significantly while short term charges are increased only slightly. The resulting objective function value for this policy instrument combination is about 112 million Euro. Figure 8 shows the corresponding undiscounted net benefits for users and operators. In the year 2006 the net benefit to society is negative due to the investments in additional public transport frequency in the off peak period. Afterwards the yearly benefits increase until 2016 and are stable with a slight decreasing tendency in the following years.

Table 1: Policy instrument combination resulting from the optimisation of a constrained objective function Vienna

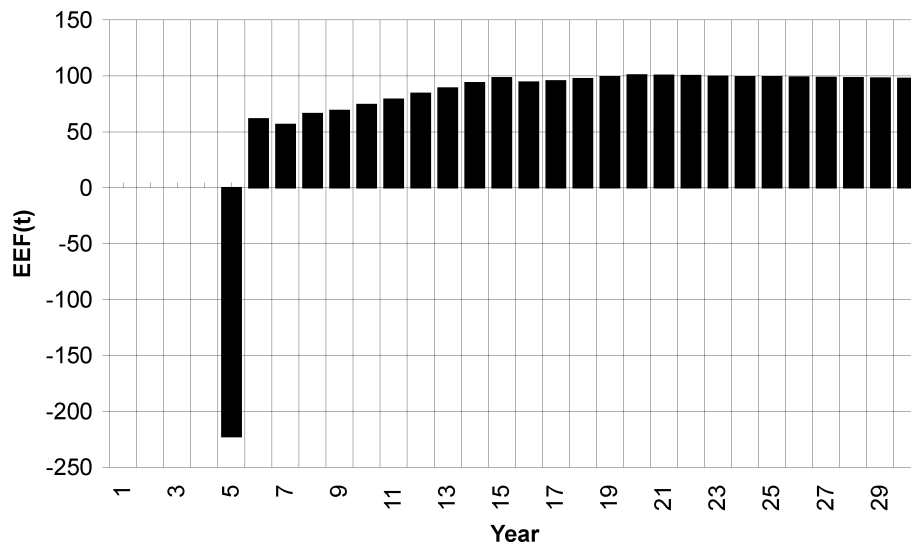| Policy instrument	Period	Implementation year	Long run year
		2006	2016
PT Fare	Peak	-8%	-47%
	Off peak	+6%	-50%
PT Frequency	Peak	-15%	-36%
	Off peak	+46%	+46%
Parking charges	Long term parking	2.9 €	4.2 €
	Short term parking	0.7 €	0.8 €

Figure 8: Development of the yearly EEF

The mode split development for the policy instrument combination given in table 1 is shown in figure 9. In the implementation year a sharp fall in the share of car and a sharp increase in the share of public transport could be seen in the peak period as well as in the off peak period. Due to raising the parking charges further towards the long run year public transport shares continue to increase. Due to the growth in the number of residents in combination with land use effects car share increases slightly in later years. This result is at the expense of the slow modes pedestrian and bicycle. The share of slow modes is increasing in the implementation year but afterwards decreasing over the whole of the remaining period.
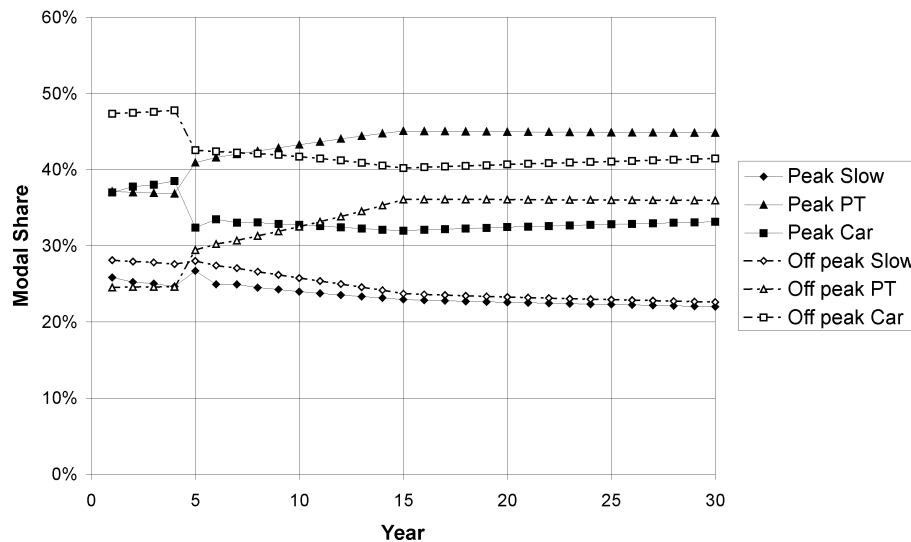

Figure 9: Mode split development Vienna SPM optimisation example

Figure 10 shows the difference in the number of residents in each zone between the strategy found in the optimisation and the do nothing scenario over the 30 year evaluation period. The extension of the parking charges to the zones around the city centre leads to an increase in population in these zones.
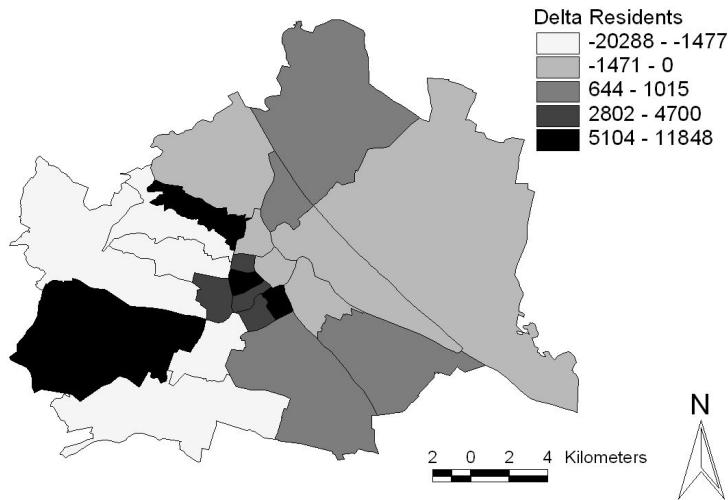
Figure 10: Changes in residential development for 30 years constrained optimisation strategy – do nothing Vienna

In figure 11 the development of the CO2 emissions is shown. The dotted line shows CO2 emissions in the do-nothing scenario. The increase is caused by a growth in population and changes in the land use pattern. The sum of the black and the grey area shows the development of CO_2 for the best performing strategy found in the optimisation. The black area shows CO_2 emissions of additional public transport frequency. The grey area shows CO_2 emissions from other traffic. As can be seen the strategy meets the target on CO_2 emissions in the final year which was one of the constraints imposed in this example. The other constraint was to create a positive present value of finance.

Figure 11: Development CO2 emissions

6. Conclusions and Recommendations

This paper described the strategic land use and transport interaction model SPM which was developed in the project PROSPECTS. An important feature of the SPM is its dynamic nature. The evolution of processes over time can be observed. User benefits and other indicators can be calculated for each single year over the whole evaluation period. This is particularly useful in d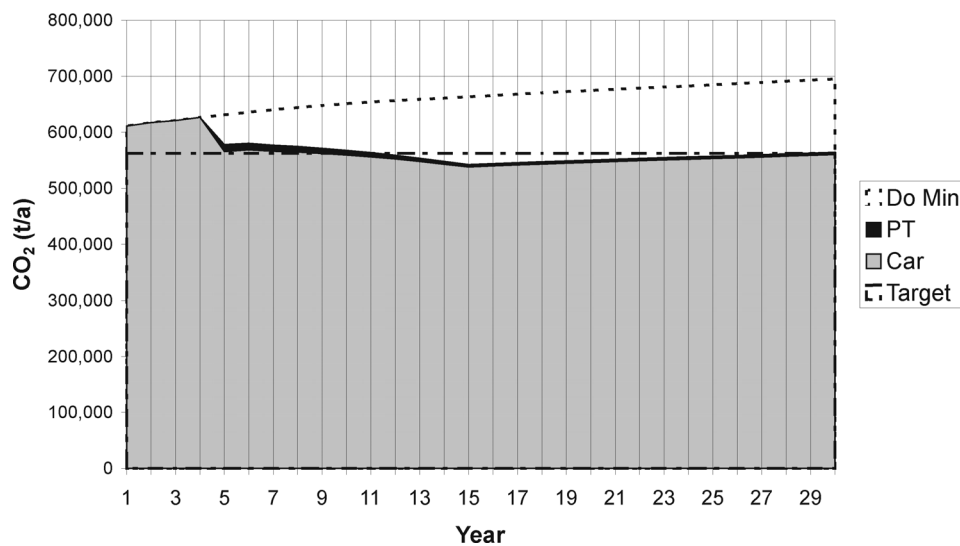etermining the trends in the longer term of indicators of sustainability such as the trend in CO_2 emissions. In the case study presented here we have seen that CO_2 emissions are reduced significantly but as exogenous growth continues emissions raise again and may violate targets of sustainability in years beyond the evaluation period.

We have demonstrated the application of an automated optimisation procedure in combination with a simplified policy profile over the evaluation period. The simplified policy profile is linear between implementation and long run years and constant afterwards. For future research the investigation of more complex policy profiles might be of interest. It would be possible to optimise the shape of the profile itself to find optimal implementation paths for particular instruments. Barriers to implementation could be eased or lifted by finding appropriate policy profiles.

This paper is mainly a description of a methodology which was developed in PROSPECTS and as such does not allow for many policy conclusions to be made. Nevertheless the results of the presented optimisation for Vienna are reasonable. As expected public transport fares were suggested to be decreased significantly. Public transport frequency in Vienna is already quite high especially in the peak period. Increases would be very expensive. Therefore no increase in the peak period was expected while there is still potential in the off peak period. In accordance with this decreases in peak and increases in off peak frequency are suggested by the appraisal framework. With respect to the peak frequency result it has to be mentioned that the current SPM does not take into account overcrowding effects in public transport. It is planned to include this improvement in the development of the next SPM generation. Increases in parking charges were also expected for a sustainable solution. The effect of the parking charges applied in the Vienna case study is rather similar to an area licensing scheme. A comparison with experience in road charging in other cities indicates that the magnitude of the suggested parking charges is reasonable.

Many useful potential improvements of the SPM have been identified. The resources did not allow to implement all of them. Issues for future research and development are e.g.: public transport overcrowding effects, a more detailed representation of policy instrument costs and investments, an internal time of the day sub-model, greater dis-aggregation of demand by person type, an interface to an assignment model to improve the representation of supply. Nevertheless we have shown that the SPM and the appraisal framework is an appropriate and useful tool to formulate sustainable land use and transport strategies. One promising future field of use is to "separate the grain from the chuff". I.e. strategic policy areas can be marked out for a further investigation with spatially more detailed land use and transport interaction models.

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Appendix

A1. The sketch planning model

Nomenclature
The nomenclature shown in equation (1) was used as far as possible in the following sections. The right hand side lower case indices refer to SPM zones. The index i always refers to the source of an action, the index j always refers to the destination of an action. The upper case indices on the right hand side refer to modes in the transport sub-model and to sub-models and actions in the land use sub-model (domiciles, residents, workplaces, moving in or out). The uppercase indices on the left hand side refer to additional information like different cost components. The index in brackets refers to iteration numbers ("years") within a single SPM run ($0 <= t <= 30$).

$$^{p}X_{ij}^{m}(t) = f(p, m, i, j, t) \tag{1}$$

The transport sub-model
The equations **(2) to (10)** are describing the trip distribution and mode choice part of the SPM transport sub-model.

$$T_{ij}^{m} = \left[P_i * \frac{A_j / f\left(t_{ij}^{m}, c_{ij}^{m}\right)}{\sum_{mj} A_j / f\left(t_{ij}^{m}, c_{ij}^{m}\right)} \right]_{peak} + \left[P_i * \frac{A_j / f\left(t_{ij}^{m}, c_{ij}^{m}\right)}{\sum_{mj} A_j / f\left(t_{ij}^{m}, c_{ij}^{m}\right)} \right]_{off\ peak} \tag{2}$$

T_{ij}^{m} Number of trips by mode m from source i to destination j
P_i Production of trips at source i
A_j Attraction of zone j as destination
t_{ij}^{m} Travel time by mode m from i to j (min)
c_{ij}^{m} Travel costs **for a trip** by mode m from i to j (€)
$f(t_{ij}^{m}, c_{ij}^{m})$ Friction factor **for a trip** by mode m from i to j (min)

The friction factors used in the SPM were developed within a long term research programme of the Institute of Transport Science, Aachen University of Technology (Walther et. al., 1997). Their soundness was verified with several case studies in German cities. The basic form of the time dependent component of the friction factors used in the SPM is shown in equation (3). The basic form of the cost dependent component is shown in equation (4). The two components are added to give the total friction factor.

$$f\left(t_{ij}^{m}\right) = t_{ij}^{m} * e^{t_{ij}^{m}} \quad \text{(Walther et. al., 1997) p. 16} \tag{3}$$

$$f\left(c_{ij}^{m}\right) = \frac{c_{ij}^{m}}{\alpha * Inc_i} \quad \text{(Walther et. al., 1997) p. 24} \tag{4}$$

α Factor willingness to pay
Inc_i Household income in zone i (€/min)

As an example figure 12 and equations (5) to (10) show the detailed calculation of the friction factor for the mode private car.
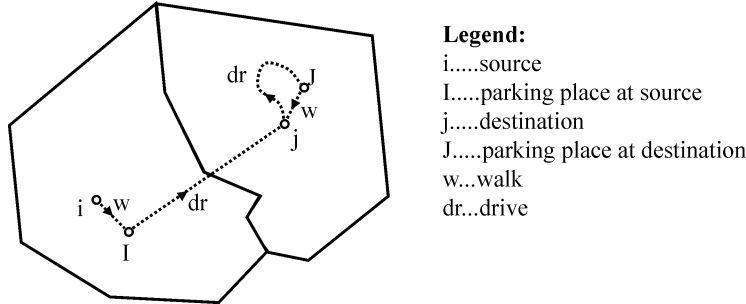
Figure 12: Components of a car trip from source i *to destination* j *in the SPM*

$$f\left(t_{ij}^{PC}, c_{ij}^{PC}\right) = (t_{iI}^{PC,w} * SV_{iI}^{PC,w} + t_{Ij}^{PC,dr} + t_{jJ}^{PC,ps} * SV_{jJ}^{PC,ps} + t_{Jj}^{PC,w} * SV_{Jj}^{PC,w}) * SV_{ij}^{PC} + {}^{k}Z_{ij}^{PC} \tag{5}$$

$t_{iI}^{PC,w}$	Walking time from source i to parking place I (min)
$SV_{iI}^{PC,w}$	Subjective valuation factor walking time from source i to parking place I
$t_{Ij}^{PC,dr}$	Driving time from parking place I to destination j (min)
$t_{jJ}^{PC,ps}$	Parking place searching time at destination j to find parking place J (min)
$SV_{jJ}^{PC,ps}$	Subjective valuation factor parking place searching time at destination j to find parking place J
$t_{Jj}^{PC,w}$	Walking time from parking place J to destination j (min)
$SV_{Jj}^{PC,w}$	Subjective valuation factor walking time from parking place J to destination j
SV_{ij}^{PC}	Aggregated subjective valuation factor private car for origin-destination pair ij
${}^{k}Z_{ij}^{PC}$	Impedance to travel by car from i to j caused by cost component k (cost components are fuel costs, other operating costs, parking charges and road charges) (min)

$$SV_{iI}^{PC,w} = 1.0 \text{ (Walther et. al., 1997) p. 25} \tag{6}$$

$$SV_{jJ}^{PC,ps} = 2.0 + 10^{-4} * e^{0.8 * t_{jJ}^{PC,ps}} \text{ (Walther et. al., 1997) p. 25} \tag{7}$$

$$SV_{Jj}^{PC,w} = 2.0 + 10^{-4} * e^{0.8 * t_{Jj}^{PC,w}} \text{ (Walther et. al., 1997) p. 25} \tag{8}$$

$$SV_{ij}^{PC} = 0.8507 * (1 - 0.7318 * e^{-0.1879 * D_{ij}^{PC}}) \text{ (Walther et. al., 1997) p. 25} \tag{9}$$

D_{ij}^{PC}	Travel distance from i to j by car (km)

$${}^{k}Z_{ij}^{PC} = \frac{{}^{k}c_{ij}^{PC}}{{}^{k}\alpha * Inc_{i} * o^{PC}} \tag{10}$$

${}^{k}c_{ij}^{PC}$	Costs of costs component k for a car trip from i to j (€/trip)
${}^{k}\alpha$	Factor willingness to pay for cost component k; 0.43 for fuel and other running costs, 0.769 for parking and road charging costs; Source: (Walther et. al., 1997) p. 24
Inc_{i}	Household income in zone i (€/min)
o^{PC}	Car occupancy rate

The SPM uses an area speed flow relationship in the peak period. The speed flow relationship is described in the equations (11) and (14). It uses the following principle form (Singh, 1999):

$$V_{ij}^{PC,pk} = \frac{V_{ij}^{PC,fr}}{1 + \beta * \left(DF_{ij}^{PC,pk}\right)^{\alpha}} \tag{11}$$

$V^{PC,pk}{}_{ij}$ Speed by car from *i* to *j* during the peak period [km/h]
$V^{PC,fr}{}_{ij}$ Free flow speed by car from *i* to *j* [km/h]
$DF^{PC,pk}{}_{ij}$ Demand factor road from *i* to *j* during the peak period
α, β Constant factors

Equation (11) is used in the SPM to calculate the speed flow relationship as follows. Two speed matrices are needed as SPM input: average speed in the start year and free flow speed in the base year. These matrices are required to calculate initial demand factors (12).

$$DF_{ij}^{PC,pk}(0) = \sqrt[\alpha]{\frac{V_{ij}^{PC,fr}(0) - V_{ij}^{PC,pk}(0)}{\beta * V_{ij}^{PC,pk}(0)}} \tag{12}$$

$DF^{PC,pk}{}_{ij}(0)$ Demand factor road from *i* to *j* during the peak period in iteration *0*
$V^{PC,fr}{}_{ij}(0)$ Free flow speed by car from *i* to *j* in iteration *0* [km/h]
$V^{PC,pk}{}_{ij}(0)$ Speed by car from *i* to *j* during the peak period in iteration *0* [km/h]

In the next iterations *t* the demand factors are recalculated as follows.

$$DF_{ij}^{PC,pk}(t) = DF_{ij}^{PC,pk}(0) * x * y * \frac{\sum_{i} T_{ij}^{PC,pk}(t-1) + \sum_{j} T_{ij}^{PC,pk}(t-1)}{\sum_{i} T_{ij}^{PC,pk}(0) + \sum_{j} T_{ij}^{PC,pk}(0)} \tag{13}$$

$T^{PC,pk}{}_{ij}(t)$ Number of peak period trips by car from source *i* to destination *j* in iteration *t*
x, y   Factors to take SPM internal development of new road capacity into account; see equation (14)

The demand factors from (13) are used in equation (11) to re-calculate the values $V^{PC,pk}{}_{ij}(t)$ in the speed matrices. The SPM summarises over all sources and destinations. I.e. *DF* changes with the number of all incoming and outgoing trips.

If there are zones with a low share of developed areas in the base year, then the land use sub-model might develop a quite significant amount of new domiciles and workplaces in these zones. Especially if there are high growth rates assumed. If this is the case, the number of trips will rise significantly. If the basic assumptions for speed flow are kept constant, an unrealistic drop in average car speed will occur. Therefore the factors *x* and y (14) are used to lower the basic demand factor (13) if growth in the number of workplaces and/or residents in a zone is higher than a user defined threshold. Lowering the demand factor is equivalent to additional road infrastructure capacity. The basic demand factor also changes if the policy instruments "road capacity changes" or "road infrastructure investments" are applied.

$$\text{If } \frac{N_j^R(t) - N_j^R(0)}{N_j^R(0)} > C \text{ then } x = \frac{N_j^R(0)}{N_j^R(t)} \text{ else } x = 1$$

$$\text{If } \frac{N_j^{WP}(t) - N_j^{WP}(0)}{N_j^{WP}(0)} > C \text{ then } y = \frac{N_j^{WP}(0)}{N_j^{WP}(t)} \text{ else } y = 1$$

(14)

$N_j^R(t)$ Number of residents in zone j in the iteration t
$N_j^{WP}(t)$ Number of residents in zone j in the iteration t
C Threshold value

Accessibility as output of the transport sub-model and input in the land use sub-model is calculated as follows.

$$^{WP}Acc_i^m(t) = \sum_j N_j^{WP}(t) * \left[10^{-4} * \left(t_{ij}^m(t)\right)^2 - 0.0183 * t_{ij}^m(t) + 0.75\right]$$

(15)

$$^{C}Acc_i^m(t) = \sum_j N_j^R(t) * \left[10^{-4} * \left(t_{ij}^m(t)\right)^2 - 0.0183 * t_{ij}^m(t) + 0.75\right]$$

(16)

$^{WP}Acc_i^m(t)$ Accessibility of working places by mode m from zone i in iteration t
$N_j^{WP}(t)$ Number of workplaces in zone j in iteration t
$^{C}Acc_i^m(t)$ Accessibility of costumers by mode m from zone i in iteration t
$N_j^R(t)$ Number of residents in zone j in iteration t
$t_{ij}^m(t)$ Travel time by mode m from zone i to zone j in iteration t

The weighting factors in equation (15) and (16) are derived from a study made by the Institute for Transport Planning and Traffic Engineering, Vienna University of Technology on behalf of the Viennese Municipal Department 18 "Urban Development and Planning" (Knoflacher,1997).

The land use sub-model
The development of new domiciles is described by the following equation:

$$\Delta D_j(t) = P^D(t-T) * \frac{a^D * \dfrac{R_j^D(t-T)}{LP_j(t-T)} + b^D}{\displaystyle\sum_j a^D * \dfrac{R_j^D(t-T)}{LP_j(t-T)} + b^D}$$

(17)

$\Delta D_j(t)$ Number of new built domiciles available on the market in zone j in the year t
$P^D(t-T)$ Quantity of new built domiciles demanded in the year t as perceived by the developer in the year $t-T$
T Time lag to plan and build domiciles
$R_j^D(t-T)$ Monthly rent or mortgage for a domicile in zone j in the year $t-T$ (€)
$LP_j(t-T)$ Price for land in zone j in the year $t-T$ (k€/m²)
a^D, b^D Parameter (derived from a regression analysis using observed data)

The willingness to move is calculated with the following equation:

$$N_j^{mv}(t) = P^{mv}(t) * \frac{a^{mv} * e^{b^{mv} * {}^{WP}Acc_j^{PC}(t) + c^{mv} * ShGr_j(t) + d^{mv} * R_j^D(t)}}{\sum_j a^{mv} * e^{b^{mv} * {}^{WP}Acc_j^{PC}(t) + c^{mv} * ShGr_j(t) + d^{mv} * R_j^D(t)}} \tag{18}$$

$N_j^{mv}(t)$ Number of residents moving from zone j in the year t

$P^{mv}(t)$ Potential of moving residents in the year t

${}^{WP}Acc_j^{PC}(t)$ Accessibility of working places by private car from zone j in the year t

$ShGr_j(t)$ Share of green land in zone j in the year t

$R_j^D(t)$ Monthly rent or mortgage for a domicile in zone j in the year t (€)

$a^{mv}, b^{mv}, c^{mv}, d^{mv}$ Parameter (derived from a regression analysis using observed data)

$$P^{mv}(t) = \frac{N^R(t)}{\Delta T^{mv}} \tag{19}$$

$N^R(t)$ Total number of residents in the study area in the year t

T^{mv} Average time living at the same residence

$$S_j^D(t) = \Delta D_j(t) * n_j^{HH}(t) + N_j^{mv}(t) \tag{20}$$

$S_j^D(t)$ Supply with living places in domiciles in zone j in the year t

$n_j^{HH}(t)$ Number of residents per household in zone j in the year t

$$S^D(t) = \sum_j S_j^D(t) \tag{21}$$

$S^D(t)$ Total supply with living places in domiciles in the year t

$$P^{in,d}(t) = \sum_j N_j^{mv}(t) + N^{gr}(t) \tag{22}$$

$P^{in,d}(t)$ Total quantity of living places demanded in the year t

$N^{gr}(t)$ Change in population in year t (natural growth & migration, can be positive or negative)

$$N^{gr}(0) = p^{gr}(0) * N^R(0) \tag{23}$$

$p^{gr}(0)$ Percentage change in population in year 0 (natural growth & migration, can be positive or negative)

$$DF^D(t) = \frac{P^{in,d}(t)}{S^D(t)} \tag{24}$$

$DF^D(t)$ Demand factor for domiciles in the year t

$$\text{If } DF^D(t) > 1 \text{ then } P^{in}(t) = \sum_j S_j^D(t) \text{ else } P^{in}(t) = P^{in,d}(t) \tag{25}$$

$P^{in}(t)$ Total demand for living places which can be satisfied in the year t

$$N_j^{in}(t) = P^{in}(t) * \frac{A_j^{in}(t) / f(Z_j^{in}(t))}{\sum_j A_j^{in}(t) / f(Z_j^{in}(t))} = P^{in}(t) * \frac{a^{in} * e^{b^{in} * {}^{WP}Acc_j^{PC,PT}(t) + ShGr_j(t) * (c^{in} * ShGr_j(t) + d^{in}) + e^{in} * R_j^D(t)}}{\sum_j a^{in} * e^{b^{in} * {}^{WP}Acc_j^{PC,PT}(t) + ShGr_j(t) * (c^{in} * ShGr_j(t) + d^{in}) + e^{in} * R_j^D(t)}} \tag{26}$$

$N_j^{in}(t)$ Demand for residing in zone j in the year t

$A_j^{in}(t)$ Attraction to move into zone j in year t

$f(Z_j^{in}(t))$ Friction factor to move into zone j in year t caused by impedance Z

$^{WP}Acc^{PC,PT}{}_j(t)$ Aggregated accessibility of working places from zone j in the year t
$ShGr_j(t)$ Share of green land in zone j in the year t
$R^D{}_j(t)$ Monthly rent or mortgage for a domicile in zone j in the year t (€)
$a^{in}, b^{in}, c^{in}, d^{in}, e^{in}$ Parameter (derived from a regression analysis using observed data)

Equation (27) shows the iterative process of the redistribution of over demand to zones with sufficient supply.

$Do\ until\ \forall\ N_j^{in}(t)\big|_k \le S_j^D(t)\big|_k$

$If\ N_j^{in}(t)\big|_k > S_j^D(t)\big|_k\ then$

$$P_j^{in}(t)\big|_{k+1} = N_j^{in}(t)\big|_k - S_j^D(t)\big|_k\ \ and\ \ \frac{A_j^{in}(t)}{f(Z_j^{in}(t))}\bigg|_{k+1} = 0\ \ else \tag{27}$$

$$P_j^{in}(t)\big|_{k+1} = 0\ \ and\ \ \frac{A_j^{in}(t)}{f(Z_j^{in}(t))}\bigg|_{k+1} = \frac{A_j^{in}(t)}{f(Z_j^{in}(t))}\bigg|_{k}$$

Next k

k Number of iteration in the redistribution process

After each year t the demanded quantity of domiciles and the rent and mortgage for domiciles are adapted according to the demand/supply situation (equation (28) and (29)).

$$P^D(t+1) = (P^D(t) + a) * (DF^D(t))^2 \tag{28}$$

$DF^D(t)$ Aggregated demand factor for domiciles in year t
a Small constant number to allow a recovery of demand in year $t+1$ if demand was 0 in year t

$$R_j^D(t+1) = R_j^D(t) * \frac{\alpha}{\beta + \gamma * e^{\left(-\delta * DF_j^D(t) * \frac{\sum_i T_{ij}^{PC}(t-1) + \sum_j T_{ij}^{PC}(t-1)}{\sum_i T_{ij}^{PC}(t) + \sum_j T_{ij}^{PC}(t)}\right)}} \tag{29}$$

$\alpha, \beta, \gamma, \delta$ Constant factors

If the study area begins to run short of land, land prices are rising (equation (30)).

$$LP_j(t+1) = LP_j(t) * e^{\left(\frac{ShGr_j(t-1)}{ShGr_j(t)} - 1\right)} \tag{30}$$

Unsatisfied external demand is cumulated (31) and stimulates building activities via the demand factor for domiciles.

$$N^{gr}(t+1) = p^{gr}(t+1) * N^{Res}(t) + (P^{in}(t) - S^D(t)) \tag{31}$$

The development of workplaces in the service sector is calculated as described below.

$$p_{mv}^{sv} = \frac{1}{T_{mv}^{sv}} \tag{32}$$

p_{mv}^{sv} Percentage workplaces service moving out every year

T_{mv}^{sv} Average number of years until a service business either changes location or goes bankrupt

$$\Delta N_j^{Sv,mv}(t) = N_j^{Sv}(t-1) * p^{Sv,mv} \tag{33}$$

$\Delta N_j^{Sv,mv}(t)$ Number of workplaces in the service sector moving out of zone *j* in year *t*

$N_j^{Sv}(t-1)$ Workplaces in the service sector in zone *j* in year *t-1*

$$\Delta N_j^{Sv,in}(t) = P^{Sv}(t) * \frac{e^{(a^{sv}+b^{sv}*AvLd_j(t)+c^{sv}*LP_j(t))}-1}{\sum_j e^{(a^{sv}+b^{sv}*AvLd_j(t)+c^{sv}*LP_j(t))}-1} \tag{34}$$

$\Delta N_j^{Sv,in}(t)$ Number of workplaces service sector moving into zone *j* in the year *t*

$AvLd_j(t)$ Available land in zone *j* in the year t (1 = average)

$LP_j(t)$ Price for land in zone *j* in the year t (1 = average)

a^{sv}, b^{sv}, c^{sv} Parameter (derived from a regression analysis using observed data)

$$P^{Sv}(t+1) = N^{Sv}(t) * (p^{Sv}(t) - p^{Sv,mv}) \tag{35}$$

$P^{Sv}(t+1)$ Potential workplaces service sector in year *t+1*

$p^{Sv}(t)$ Percentage external change of workplaces service sector in year *t*

$$\Delta N_j^{sv}(t) = \Delta N_j^{Sv,in}(t) - \Delta N_j^{Sv,mv}(t) \tag{36}$$

$\Delta N_j^{Sv}(t)$ Change in workplaces in the service sector in zone *j* in year *t*

The development of workplaces in the production sector is calculated as described below.

$$p^{Pr,mv} = \frac{1}{T^{Pr,mv}} \tag{37}$$

$p^{Pr,mv}$ Percentage workplaces production moving out every year

$T^{Pr,mv}$ Average number of years until a production business either changes location or goes bankrupt

$$\Delta N_j^{Pr,mv}(t) = N_j^{Pr}(t-1) * p^{Pr,mv} \tag{38}$$

$\Delta N_j^{Pr,mv}(t)$ Number of workplaces in the production sector moving out of zone *j* in year *t*

$N_j^{Pr}(t-1)$ Workplaces in the production sector in zone *j* in year *t-1*

$$\Delta N_j^{Pr,in}(t) = P^{Pr}(t) * \frac{e^{a^{Pr}+b^{Pr}*AvLd_j(t)+c^{Pr}*LP_j(t)}-1}{\sum_j e^{a^{Pr}+b^{Pr}*AvLd_j(t)+c^{Pr}*LP_j(t)}-1} \tag{39}$$

$\Delta N_j^{Pr,in}(t)$ Number of workplaces production sector moving into zone *j* in the year *t*

$AvLd_j(t)$ Available land in zone *j* in the year *t* (1 = average)
$LP_j(t)$ Price for land in zone *j* in the year *t* (1 = average)
a^{Pr}, b^{Pr}, c^{Pr} Parameter (derived from a regression analysis using observed data)

$$P^{Pr}(t+1) = N^{Pr}(t) * \left(p^{Pr}(t) - p^{Pr,mv} \right) \tag{40}$$

$P^{Pr}(t+1)$ Potential workplaces Production sector in year t+1

$p^{Pr}(t)$ Percentage external change of workplaces production sector in year t

$$\Delta N_j^{Pr}(t) = \Delta N_j^{Pr,in}(t) - \Delta N_j^{Pr,mv}(t) \tag{41}$$

$\Delta N_j^{Pr}(t)$ Change in workplaces in the Production sector in zone j in year t

The SPM controls if the demand for new businesses and workplaces for both the service and the production sector can be met in a zone, i.e. if enough developable land is available. The over demand is redistributed to other zones if this it is possible. Otherwise the potential external growth is cut down.

A2. Objective function and optimisation framework

General description
The general form of the optimisation problem can be written as follows :-

$$\underset{\mathbf{X}(t)}{\text{Maximise}}\ OF[\mathbf{X}(t)] \tag{42}$$

$$OF[X(t)] = \sum_{t=0}^{30} \alpha(t) * \left[b(\mathbf{X}(t)) - c(\mathbf{X}(t)) - I(\mathbf{X}(t)) - \gamma(t) * g(\mathbf{X}(t)) \right] + \sum_{k \in M} \sum_{t=0}^{30} \mu_k(t) * y_k(t) \tag{43}$$

subject to constraints on some of the indicators of the form $\sum_{t=0}^{30} \mu_k(t) * y_k(t) \leq C_k$ or the form

$y_k(t) \leq C_k(t)$.

OF is the overall objective function[5] and the first term represents economic efficiency where:

X(t)   Vector of levels of policy instruments which can be used to maximise the objective function OF
b(X(t))   Sum of all benefits in year t (€)
c(X(t))   Sum of all costs in year t (€)
I(X(t))   Sum of capital investments in year t (€)
((t)   Shadow cost of CO_2 emission, reflecting national CO_2 targets for year t (€/t)
g(X(t))   Amount of CO_2 emissions in year t (t)
k   Represents the remaining indicators (k ∈ M)
$\mu_k(t)$ Weight in year t for indicator k (€/Unit of indicator k)
$y_k(t)$ Level of indicator k in year t (Unit of indicator k)
$C_k(t)$ Constraint/target for indicator i in year t (Unit of indicator)
C_k Overall constraint/target for indicator i (for instance, a financial constraint) (€)

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5 See (Minken et. al., 2003) p. 52

The annual cost and benefit terms are weighted by $\alpha(t)$. We use

$$\alpha(t) = \alpha * \frac{1}{(1+r)^t}$$

(44)

for all years between 0 and 29. Here, r is a (country specific) discount rate and α, the intergenerational equity constant, is a constant between 0 and 1, reflecting the relative importance of welfare at present as opposed to the welfare of future generations. So for these years, $\alpha(t)$ is an ordinary discount factor. For year 30,

$$\alpha(30) = \alpha \frac{1}{(1+r)^{30}} + (1-\alpha)$$

(45)

Application for the SPM case study
Equation (46) illustrates the policy instrument vector used in the framework presented here.

$$X(t) = \left[{}^p F^{PT}(t) \, {}^{op}F^{PT}(t) \quad {}^p Fr^{PT}(t) \quad {}^{op}Fr^{PT}(t) \quad ...... \quad {}^{st}P_j^{PC}(t) \quad {}^{lt}P_j^{PC}(t) \right]$$

(46)

$X(t)$	Policy instrument vector
${}^pF^{PT}(t)$	Public transport fare peak period (% change relative to do nothing)
${}^{op}F^{PT}(t)$	Public transport fare off peak period (% change relative to do nothing)
${}^pFr^{PT}(t)$	Public transport frequency peak period (% change relative to do nothing)
${}^{op}Fr^{PT}(t)$	Public transport frequency off peak period (% change relative to do nothing)
${}^{st}P_j^{PC}(t)$	Short term parking charge (€/stay)
${}^{lt}P_j^{PC}(t)$	Long term parking charge (€/stay)

Equation (47) illustrates the policy instrument profile for the example public transport peak fare.

If ${}^pF^{PT}(t) <$ then ${}^pF^{PT}(t) = 0$

If ${}^pF^{PT}(t) = t_{im}$ then ${}^pF^{PT}(t) = {}^pF_{im}^{PT}$

If ${}^pF^{PT}(t) > t_{im}$ and ${}^pF^{PT}(t) < t_{lr}$ then ${}^pF^{PT}(t) = {}^pF_{im}^{PT} + \dfrac{{}^pF_{lr}^{PT} - {}^pF_{im}^{PT}}{t_{lr} - t_{im}} * (t - t_{im})$

If ${}^pF^{PT}(t) >= t_{lr}$ then ${}^pF^{PT}(t) = {}^pF_{lr}^{PT}$

(47)

t_{im}	Implementation year; user defined
${}^pF_{im}^{PT}$	Public transport peak fare in the implementation year; user defined for a single SPM run, suggested by the optimisation routine in an optimisation
t_{lr}	Long run year; user defined
${}^pF_{lr}^{PT}$	Public transport peak fare in the long run year; user defined for a single SPM run, suggested by the optimisation routine in an optimisation

Benefits, costs, emissions and other indicator values required in equation (43) are calculated from SPM output values. The balance of yearly benefits and costs comes from two groups: the users and the operators (including government) of the land use and transport system.

$$b(X(t)) - c(X(t)) = \sum_k {}^k U(X(t)) + \sum_k {}^k O(X(t))$$

(48)

${}^kU(X(t))$ 	Balance of user benefits and losses for user group k in year t. User groups are pedestrians, cyclists, public transport and car users from the transport perspective and residents and businesses from the land use perspective.

${}^kO(X(t))$ 	Balance of operator benefits and losses for group k in year t. The groups are public transport operators, parking facility operators, toll operators, land and property owners and public authorities.

$${}^kU = {}^kU^T(t) + {}^kU^M(t)$$

(49)

${}^kU^T(t)$ 	Balance of monetarised user time benefits and losses for user group k in year t
${}^kU^M(t)$ 	Balance of monetary user benefits and losses for user group k in year t

As an example the balance of user benefits and losses from changes in travel time for the public transport user group is shown in equation (50).

$${}^{PT}U^T(t) = -\frac{1}{2} \sum_{ij} VoT * \left[t_{ij}^{PT}(t)\big|_{X(t)} - t_{ij}^{PT}(t)\big|_0 \right] * \left[T_{ij}^{PT}(t)\big|_{X(t)} + T_{ij}^{PT}(t)\big|_0 \right]$$

(50)

${}^{PT}U^T$ 	Balance of user time benefits and losses for public transport users in year t
VoT 	Value of time; specified by the SPM user (€/min)
$t_{ij}^{PT}(t)\big|_{X(t)}$ 	Travel time for a public transport trip from i to j in year t if the vector of policy instrument levels X_t is applied; output of the SPM (min)
$t_{ij}^{PT}(t)\big|_0$ 	Travel time for a public transport trip from i to j in year t in the do nothing scenario; input to the SPM (min)
$T_{ij}^{PT}(t)\big|_{X(t)}$ 	Public transport trips from i to j in year t if the vector of levels instrument levels X_t is applied; output of the SPM
$T_{ij}^{PT}(t)\big|_0$ 	Public transport trips from i to j in year t in the do nothing scenario output of the SPM

The balance of operating benefits and losses is a function of the trips made in the do something ${}^kT(X(t))$ and the do nothing scenario ${}^kT(0)$ and the policy instrument vector $X(t)$ applied in the do something scenario (51).

$${}^kO(t) = f\left({}^kT(t)\big|_{X(t)}, {}^kT(t)\big|_0, X(t)\right) + g(X(t))$$

(51)

${}^kO(t)$ 	Balance of benefits and losses for the operator type k in year t

Equation (52) shows revenues from changes in fare levels as an example for the first type. An example for the second type are operating costs for additional public transport frequency (53).

$$f\left({}^{k}T(t)\big|_{X(t)}, {}^{k}T(t)\big|_{0}, X(t)\right) = \sum_{ij} F_{ij}^{PT}(t)\big|_{X(t)} * T_{ij}^{PT}(t)\big|_{X(t)} - F_{ij}^{PT}(t)\big|_{0} * T_{ij}^{PT}(t)\big|_{0}$$

(52)

$F_{ij}^{PT}(t)\big|_{X(t)}$ Public transport fares for a trip from i to j and year t when the vector of levels of policy instruments $X(t)$ is applied; output of the SPM (€/trip)

$F_{ij}^{PT}(t)\big|_{0}$ Public transport fares for a trip from i to j and year t in the do nothing scenario; input in the SPM (€/trip)

$T_{ij}^{PT}(t)\big|_{X(t)}$ Public transport trips from i to j in year t when the vector of levels of policy instruments $X(t)$ is applied; output of the SPM

$T_{ij}^{PT}(t)\big|_{0}$ Public transport trips from i to j in year t in the do nothing scenario; output of the SPM

$$g(X(t)) = {}^{Fr}o^{PT} * {}^{Fr}p^{PT}(t)$$

(53)

${}^{Fr}o^{PT}$ Operating costs for an additional percent of public transport frequency (k€/%)

${}^{Fr}p^{PT}(t)$ Percentage change in public transport frequency change year t; specified by the user for a single model run or the optimisation routine in an optimisation (%)

The emissions from car traffic are calculated using results from (Samaras, Ntziachristos, 1998). As an example the calculation of CO_2 emissions is shown in the equations (54) to (56).

$$^{CO_2}e_{ij}^{PC}(t) = a_2 * \left(V_{ij}^{PC}(t)\right)^2 + a_1 * V_{ij}^{PC}(t) + a_0$$

(54)

$^{CO_2}e_{ij}^{PC}(t)$ Specific carbon dioxide emissions of the mode car for a trip from i to j in year t (g/Vh-km)

$V_{ij}^{PC}(t)$ Average speed for a car trip from i to j in year t; depending of the applied policy instrument vector; output of the SPM (km/h)

a_n Parameters; Source: MEET project (Samaras, Ntziachristos, 1998)

$$^{CO_2}E^{PC}(t) = \sum_{ij} {}^{CO_2}e_{ij}^{PC}(t) * \frac{T_{ij}^{PC}(t) * D_{ij}^{PC}(t)}{o^{PC}} * \frac{365}{10^6}$$

(55)

$^{CO_2}E^{PC}(t)$ Carbon dioxide emissions of the mode car in year t (t/a)

$T_{ij}^{PC}(t)$ Trips by the mode car from zone i to zone j in year t; output o f the SPM

$D_{ij}^{PC}(t)$ Distance of a car trip from i to j in year t including search for parking place; output of the SPM (km)

o^{PC} Occupancy rate car (people per car); specified by the user

$$D_{ij}^{PC}(t) = \frac{t_{ij}^{PC,inVh}(t)}{60} * V_{ij}^{PC}(t)$$

(56)

$t_{ij}^{PC,inVh}(t)$ In vehicle time for a car trip from i to j and year t; output of the SPM (min)

$V_{ij}^{PC}(t)$ Average speed by car from i to j and year t; output of the SPM (km/h)

The emissions of public transport are calculated as follows.

$$^{CO_2}E^{PT}(t) = {}^{CO_2}E^{PT}(0) * \left(1 + {}^{Fr}p^{PT}(t)\right)$$

(57)

$^{CO_2}E^{PT}(t)$ Carbon dioxide emissions mode PT in year t (t/a)

$^{CO_2}E^{PT}(0)$ Carbon dioxide emissions mode PT in base year 0 (t/a); from statistical data

$^{Fr}p^{PT}(t)$ Percentage change in public transport frequency change and year t; specified by the user for a single model run or the optimisation routine in an optimisation (%)

A3. The downhill simplex method as applied in PROSPECTS

The core optimisation algorithm

The method applied within the PROSPECTS SPM is based on the downhill simplex method in multi-dimensions due to Nelder and Mead (1965). It solves a multidimensional minimisation, i.e. finding the minimum of a function (which is in our case $-OF$ (43)) of more than one independent variable (which is in our case $X(t)$). The method requires only function evaluations, not derivatives.

A simplex is the geometrical figure consisting, in N dimensions, of N+1 points (or vertices) and all their interconnecting line segments, polygonal faces etc. In two dimensions, a simplex is a triangle. In three dimensions it is a tetrahedron, not necessarily the regular tetrahedron.

In general the method is only interested in simplexes that are nondegenerate, i.e. which enclose a finite inner N-dimensional volume. If any point of a nondegenerate simplex is taken as the origin, then the N other points define vector directions that span the N-dimensional vector space.

The method requires an initial starting point, that is, an N-vector of independent variables. The algorithm is then supposed to make its own way downhill through the N-dimensional topography, until it encounters a (at least local) minimum.

The downhill simplex method must be started not just with a single point, but with *N+1* points, defining an initial simplex. If one of these points is taken to be the initial starting point X_0, then the other N points can be expressed as:

$$X_n^k = X_n^0 + \lambda_n * e_n$$

(58)

where *k* is the row number of the initial matrix (*0<=k<=N+1*) and *n* is the column number (*1<=n<=N*), where e_n is 1 if $k = n$ and otherwise 0, and where λ_n is a constant which is a guess at the problem's characteristic length or scale (λ_n could be different for each vector direction).

For example with 3 dimensions the initial simplex defined by equation (58) would be a tetrahedron made up as follows:

$$X = \begin{pmatrix} X_1^0 \ X_2^0 \ X_3^0 \\ X_1^1 \ X_2^1 \ X_3^1 \\ X_1^2 \ X_2^2 \ X_3^2 \\ X_1^3 \ X_2^3 \ X_3^3 \end{pmatrix} = \begin{pmatrix} X_1^0 & X_2^0 & X_3^0 \\ X_1^0 + \lambda_1 & X_2^0 & X_3^0 \\ X_1^0 & X_2^0 + \lambda_2 & X_3^0 \\ X_1^0 & X_2^0 & X_3^0 + \lambda_3 \end{pmatrix} \tag{59}$$

where:

X_1^0, X_2^0, X_3^0 Policy instruments 1 to 3

$\lambda_1, \lambda_2, \lambda_3$ Initial guesses at the scale of the simplex which depends upon the ranges considered for each measure.

The policy instruments to be optimised can be defined by the user along with feasible input ranges for each measure. The initial simplex is then generated automatically from the minimum and maximum for each measure as follows:

$$X_n^0 = X_n^{\min} + \frac{X_n^{\max} - X_n^{\min}}{3} \tag{60}$$

and

$$\lambda_n = \frac{X_n^{\max} - X_n^{\min}}{3} \tag{61}$$

where: X_n^{min}, X_n^{max} are the minimum and maximum values for policy measure X_n.

This is equivalent to assuming that the initial guess X_n^0 is one third of the feasible range and that the scale of the problem λ_n is also one third of the feasible range. This then ensures that the movement of the simplex is initially within the bounds of the problem as defined by the user. It also removes the onus of defining the initial simplex from the user and is easily generalised for *N* dimensions.

The downhill simplex method now takes a series of steps, most steps just moving the point of the simplex where the function is largest ("highest point") through the opposite face of the simplex to a lower point. These steps are called reflections, and they are constructed to conserve the volume of the simplex (hence maintain its nondegeneracy). When it can do so, the method expands the simplex in one or another direction to take larger steps. When it reaches a "valley floor", the method contracts itself in the transverse direction and tries to ooze down the valley. If there is a situation where the simplex is trying to "pass through the eye of a needle", it contracts itself in all directions, pulling itself in around its lowest (best) point. The routine name AMOEBA is intended to be descriptive of this kind of behaviour (Press et. al., 1990).

Re-parameterisation

To deal with upper and lower bounds on policy instruments re-parameterisation was used. Policy instruments X_n, (*n=1,…,N)* are economically interpretable and constrained between a

lower and an upper limit, $X_n^{\min} \le X_n \le X_n^{\max}$. Unconstrained optimisation with respect to X may give meaningless estimates that are beyond the limits. However, transformation of the parameters (policy instruments) with the re-parameterisation by (Vold et. al., 1999):

$$\xi(X) = \log((X - X^{\min})/(X^{\max} - X)) \text{,}$$
(62)

ensures that an original parameter p stays within its definition area during unconstrained estimation. Since $e^{\xi} = (X - X^{\min})/(X^{\max} - X)$, which is equivalent to $X * (e^{\xi} + 1) = e^{\xi} * X^{\max} + X^{\min}$, we have the unique inverse transformation:

$$p(\xi) = (X^{\max} * e^{\xi} + X^{\min})/(1 + e^{\xi})$$
(63)

Now, we can transform the maximisation problem to

$$\max_{\xi \in R_m} f(\xi)$$
(64)

and use an unconstrained optimisation algorithm to find

$$f(\hat{\xi}) = \max_{\xi \in R_m} f(\xi)$$
(65)

where the elements of the initial simplex defined by the section "The core optimisation algorithm" are transformed by

$$\xi_i(X_n^0 + \lambda_n \cdot e_n) = \log((X_n^0 + \lambda_n \cdot e_n - X^{\min})/(X^{\max} - X_n^0 + \lambda_n \cdot e_n))$$
(66)

It is guaranteed then that function evaluations at the final estimate and at the algorithmic search path are such that the values of the original parameters (policy instruments) are within their lower and upper limits.