

The Effect of Crosswalks on Traffic Flow

Victor L. Knoop¹

Transport and Planning, Delft University of Technology, Netherlands.

Carlos F. Daganzo²

Graduate School in civil and environmental engineering, University of California, Berkeley.

In urban areas and especially in inner cities, pedestrians crossing the road considerably influence the road traffic flow. For political reasons, priority could be given to pedestrians. A larger number of crossings reduces the pedestrian load per crossing and facilitates both the pedestrian flow and the car flow; the ultimate case is a “cross anywhere” scenario. Earlier work shows that the road capacity decreases with the square of the pedestrian crossing time, hence a short crossing time is desired. Crosswalks can ensure pedestrians cross orthogonally, and thus quickly, and can thereby improve traffic flow. Moreover, a limited number of crosswalks is less stressful than a “cross anywhere” scenario for a car driver since (s)he only needs to expect crossing pedestrians at dedicated crosswalks. This paper studies the effect of the distances between crosswalk and road traffic capacity. The paper’s goal is finding a single formula or universal set of charts that can describe the effect of pedestrian crosswalks on traffic flow under virtually all scenarios (with long blocks). This type of result would obviate the need for simulations of specific situations when only a rough assessment of the effect of crosswalks is desired. Traffic flow for several distances between pedestrian crossings is simulated, and moreover, a non-constant inter-crosswalk spacing is considered. The simulation results can be used for other situations, using transformations and an interpolation recipe. Overall, the closer the crosswalks, the better the flow. However, spacings closer than approximately 25-50 meters do not add much. Speed of traffic under a broad array of pedestrian crossing scenarios is given.

Keywords: traffic flow theory, pedestrian crossings, crosswalks, road capacity.

1 Introduction

Many cities try to stimulate “active modes” for various reasons, e.g. environment, public health, or reducing car traffic. On urban roads, vehicular traffic interacts with pedestrians which cross the road. Crosswalks can be introduced, where pedestrians can safely cross, and have priority over vehicular traffic. A policy of stimulating pedestrian traffic will give rise to more crosswalks, since it will make walking more attractive. Crossing pedestrians will in these cases influence vehicular traffic, in particular decrease the road capacity and the vehicular speed.

Usually, crosswalks are suggested for a safe crossing of pedestrians. Researches suggest that a form of road design where all drivers need to negotiate about priorities could improve safety [Hamilton-Baillie \(2008\)](#). However, the “crosswalk everywhere” situation is different, since it gives priority to pedestrians, thereby limiting the flow and speed of vehicles, and requiring drivers to always drive carefully. Instead, a situation with a limited number of crosswalks could be more relaxing for drivers, since they would not need to pay as much attention to the pedestrians in the sidewalk.

¹A: Stevinweg 1, 2628CN Delft, The Netherlands T: +31152788413 F: +31152783179 E: v.l.knooo@tudelft.nl

²A: 416 McLaughlin Hall, Berkeley, California, USA T: +15106423853 F: N/A E: daganzo@berkeley.edu

Table 1. Threshold values on vehicular flow for applying a crosswalk, from De Leur and Wildenburg (2011)

Period	No crosswalk if flow below	Always crosswalk if flow above
Day	4000 veh	12500 veh
Average peak hour	320 veh/h	1000 veh/h
Volume near school start or end hours	80 veh/15 minutes	250 veh/15 min

This situation with crosswalks is hence considered in this paper.

We did not find anywhere where crosswalks are suggested other than for safety reasons. Guidelines in the Netherlands are only how the crosswalks should be designed. The Dutch highway capacity manual (CROW, 2017) indicates in chapter 12.11.10 and 12.11.1 (guidelines to design pedestrian crossings how the specific geometric design (horizontal and vertical curvature) can cause a decrease of speed. The guidelines recommend: (1) ensuring low vehicular speeds (85% of the vehicles has a speed of 50 km/h or lower), (2) a crossing distance of 5.5 meter at maximum, (3) sufficient visibility of pedestrians for motorists. On the issue when to create a crosswalk, the guidelines are not clear. Guidelines only indicate when a facility needs to be created to have pedestrians crossing the road safely, but they do not differentiate for pedestrian priority or not. In any of the following cases, a form of pedestrian crossing is recommended. The above-mentioned should be created if there are either (1) (a) short period(s) of high-volume of crossing pedestrians (near bus stops, or schools, or (2) long periods of crossing of individual pedestrians – as guideline, they mention approximately 100 crossings between 7am and 7pm, or (3) relatively high fraction of vulnerable pedestrians, e.g. elderly, children, handicapped.

Whereas this indicates when the issue of crossing pedestrians requires attention, it does not differentiate yet between crosswalks (by which we mean a facility with priority for pedestrians) or a marked pedestrian crossing (without priority for pedestrians). Guidelines for when to build a crosswalk are not in the highway capacity manual (Transportation Research Board, 2010), nor in the Dutch guidelines. De Leur and Wildenburg (2011) try to fill the gap and mention there should be crosswalks or traffic lights depending on (1) the presence of a cyclist crossing (no concrete recommendation) (2) the vehicular flow (see table 1 for values): a too high vehicular flow will without crosswalk cause too long waiting times and (3) the presence of other crosswalks. In relation to the other crosswalks, they mention 80 meters as reference for a crosswalk close by, but do not support that value.

Note that these are not necessarily crosswalks at an intersection. Mid-block crosswalks have been studied by Zhao et al. (2017), focussing on the pedestrian crossing time and speed. From empirical observations of several crosswalks, they find pedestrians have a more or less constant speed while crossing of 1-1.1 m/s. Combining this with the guideline of at most 5.5 meter crossing width, a pedestrian will typically block the road for 5 seconds. Yang et al. (2016) present guidelines for the design of a mid-block crossing: they perform a comparison on signalised and unsignalised intersections for several junctions, using a multi-criteria analysis, taking into account delays, emissions, life-cycle costs and safety. Whereas these studies indeed indicates the need for crosswalks, a proper discussion on the amount of crosswalks is lacking.

Earlier work (Daganzo and Knoop, 2016) showed that a “cross anywhere” scenario, where any part of the road could be considered a crosswalk, would be best for cars and pedestrians in terms of travel times. The reasoning for this is as follows. Conceptually, the larger the inter-crosswalk distance D , the more pedestrians there are per crosswalk. Since the capacity of a road stretch is determined by the capacity of its bottleneck, the crosswalks determine the capacity. On a road with more than one bottleneck, the capacity is determined by the bottleneck with the lowest capacity, being the crosswalk with the highest pedestrian load. Therefore, for the capacity, not the total amount of pedestrian crossings or crossing pedestrians matter, but the flow of pedestrians per crosswalk. This number can be decreased by creating more crosswalks, which is hence beneficial for the capacity (and hence speed) of road traffic, and also for the pedestrians which have a shorter

distance towards the closest crosswalk. For throughput and travel times, the optimum for both car drivers and pedestrian hence seems to be a “cross anywhere” situation. That problem is discussed in [Daganzo and Knoop \(2016\)](#) and an equation for road capacity for this situation was presented.

First of all, it might cause pedestrians to walk slowly or cross not perpendicular to the road, which increases crossing time, which in turn has a large (quadratic) effect on the capacity. Crosswalks can enforce perpendicular crossing if they are not placed next to each other. So, therefore it is better to have crosswalks rather than have crosswalks everywhere. Moreover, the stress for car drivers might be too high, which have to account for pedestrians crossing at any time. Crosswalks can also overcome this shortcoming. This paper studies this situation of the yet unknown resulting vehicular traffic flow under mid-block unsignalized pedestrian traffic. The gap addressed in this study is hence: to which extent will car traffic be influenced by pedestrians crossings at crosswalks, and which role plays the inter-crosswalk distance in this?

A series of simulations will show speed and capacity curves. Some general conclusions are drawn on the interaction effects between cars and pedestrians on crosswalks. More importantly for practice is that the given curves, along with the transformation equations in the paper, can describe the traffic flow for a wide variety of problems.

Assumptions in this paper are that the time instants that the pedestrians approach the road are independently of each other (hence their headway distribution is exponential), and the locations are homogeneously distributed over the length of the road. This happens for instance in a city center where a road separates two sides of a shopping street, and people want to visit shops at both sides of the road. Moreover the curves will be created for vehicles following Newell’s simplified car-following model ([Newell, 2002](#)). The free-flow speed, the closest headway, and the jam spacing are the three parameters of the model and the simulations with the transformations will provide results for all parameter settings.

The remainder of the paper is set up as follows. First, section 2 simplifies the problem so it can be described with few variables. Then, simulations are done, testing the influence of these variables on the traffic stream. The setup of these simulations is explained in section 3. Then, section 4 presents and interprets the results of the simulation runs. Section 5 shows how the results of this paper can be applied to solve this problem for cases with different characteristics. The appendix provides curves useful when applying this work in practice. Finally, section 6 discusses the results and presents some conclusions.

2 The pedestrian crossing problem

This section discusses the problem and its fundamental properties. First, a methodological justification is provided in section 2.1 before section 2.2 introduces simplifying symmetries and scalings.

2.1 Methodological justification

If vehicles would not need physical space or crosswalks are very far apart, traffic can flow whenever there are gaps in the flow of pedestrians. This problem has been studied in the 60’s and 70’s. For instance, the problem is related to the waiting time to enter a major road, i.e. the time it takes until a suitable gap arises ([Hawkes, 1968](#)). ([Daganzo, 1977](#)) expanded, and introduced a difference for a required gap between a first vehicle and subsequent vehicles, which could in our case allow for introducing the pedestrian crossing time. However, the main problem is that once crosswalks become closely separated, the space to queue between the crosswalks is too limited. The same problem arises also with closely spaced intersections, and is then referred to as short block problem.

Variational theory ([Daganzo, 2005](#)) can be used to overcome this problem capturing explicitly the space that queued vehicles occupy. However, for this particular problem the analytical derivations and formulas become too complicated to be useful. Therefore, for this problem, we resort to simulation.

The assumptions in this paper bring limitations. First of all, the road traffic flow assumptions, Newell’s simplified car following model is not as “advanced” as other models. However, for average behavior, it performs well, yielding on the aggregate level an empirically observed relationship between flow and speed. Besides, all car-following models would need a location-specific calibration, without which the results would be void. In case the Newell model is used, results can be generated for a general case, and afterwards, these be transformed to the situation at hand (with the right calibrated values for traffic and pedestrians) *without redoing the simulations*.

That same arguments holds for other methods models describing the individual action of pedestrians. One of the options would be to use a microscopic model for the pedestrian. An option would be to use social force model, (Helbing and Molnar, 1995), or this model generalised for wayfinding in continuous space and time in Nomad, (Hoogendoorn and Bovy, 2004)). This would allowing the pedestrians to move freely. For an extensive overview of simulation models, we refer to (Duijes et al., 2013). Just like for car traffic simulation, microscopic simulation of pedestrians should be adapted to the specific situation at hand, and cannot be generalised to a generic case. Therefore, in this paper we choose a more generalisable way, where pedestrians are introduced as temporal blockings independent of the cars.

We recognize that a typical city block also includes arriving flows at its ends from pedestrians that have origins and destinations elsewhere. Therefore, in order to keep the number of scenarios at a minimum, while isolating the problem of interest, we will assume that our block is of infinite length.

Future work should consider finite blocks and all boundary effects they introduce. The resulting curves can be used as approximate guideline for long shopping streets. Indeed, a microscopic simulation of the exact case at hand (other distribution of pedestrians for example) would be the only option, but not generalisable for other cases.

2.2 Problem characteristics

This problem has various characteristics. The first and third column of table 2 show the most relevant variables and their meaning. The problem is defined by the properties of the car traffic, pedestrian traffic and crosswalk location.

Table 2. The symbols and their meaning. Capitals denote dimensional variables and lowercase letters dimensionless variables. Boldface letters indicate properties of the road under crossing pedestrians.

Symbol		Meaning
Dimensional	Dimensionless	
V_f	$v_f = 2$	Free flow speed
K_j	$k_j = 1$	Jam density
Q_o	$q_o = 1$	Road capacity without crossing pedestrians
Q	q	Vehicular flow with crossing pedestrians
Q_o	q_o	Road capacity with crossing pedestrians
K_o	$k_o = 1/2$	Critical density
W	$w = 2$	Wave speed
Y	$y = 1/2$	Newell’s wave trip time between two successive vehicles
S_o	$s_o = 2$	Critical spacing
S_j	$s_j = 1$	Jam spacing
F	$f/2$	Pedestrian flow per unit of time per unit of road length
T	$\tau = 1$	Pedestrian crossing time
M_D	μ_D	Mean distance between two crosswalks
Σ_D	σ_D	Standard deviation of the distance between two crosswalks
K	k	Vehicular density

For the car traffic, we assume a triangular fundamental diagram, which means the driving characteristics are defined by the variables $\{V_f, K_j, Q_o, K_o, W\}$. Three of these can be chosen freely. The density on the road is an external variable, for which we want to see the influence on the speed.

The pedestrian traffic with flow rate Q is assumed to be homogeneously distributed over space, and the headways of the pedestrians are exponentially distributed. The duration of a pedestrian crossing is the second variable for the pedestrians. The locations of the crosswalks are determined by two variables: the average inter-crosswalk distance, and the standard deviation thereof.

In total, hence, there are 8 variables influencing the traffic flow: 3 for the fundamental diagram, 1 for the car density, 2 for the pedestrian crossings, 2 for the pedestrian flow. With scaling (dimensional analysis) and slanting axes, several dimensions of the problem can be eliminated. We differentiate in notation between dimensional and dimensionless variables: capitals denote dimensional variables and matching lower case letters scaled, dimensionless variables. In particular, we have three units (time, space, vehicle number); moreover, we can slant the axes. These can be used to eliminate 4 variables. For more information, see (Daganzo and Knoop, 2016; Laval and Chilukuri, 2016). The eliminated parameters are:

1. The duration of a pedestrian crossing can be scaled with time, choosing $\tau = 1$
2. The capacity can be scaled with the unit of vehicle number, $q_0 = 1$
3. The jam density can be scaled with the unit of distance, $k_j = 1$
4. The wave speed (in relation to the free flow speed) can be scaled by skewing the fundamental diagram and the matching trajectories, $w = v_f = 2$. This also yields $k_o = 1/2$, $s_o = 2$ and $y = 1/2$.

Section 5 at the end of the paper gives a recipe to implement these scalings and includes an example. The scaling leaves four degrees of freedom in defining a situation: the pedestrian flow F , the average inter-crosswalk distance M_D , the variation of the inter-crosswalk distance Σ_D , and the density K . The pedestrian flow is expressed as flow per unit of space and unit of time. The pedestrian flow shall be expressed in terms of the dimensionless variable $f = FV_fT^2$. Because $v_f = 2$, in the new coordinate system $f = 2F$, or $F = f/2$. Table 3 shows an overview of the tested values for the tested parameters. For the pedestrian flow f , we consider values from 0.001 to 0.3, measured in pedestrians per pedestrian crossing time per distance covered at free flow speed in a pedestrian crossing time, representing a very low and a very high flow. The real-world interpretation of this is given based on an example of free flow speed $V_f = 30$ km/h and a pedestrian cross time $T = 5$ s. We find $F = f / (T^2V_f) = 5184$ pedestrians/h/km. Since all of them will block the road for 5 seconds, higher flows are unrealistic to have without traffic lights.

For the mean inter-crosswalk distance, we consider values from just over half the distance a vehicle travels in a pedestrian crossing time, which is a short distance (25 meters in the case above), to 22 times this distance (2 kilometers). If the inter-crosswalk distance gets larger, it is hardly worthwhile to consider the spatial effects of this crosswalk in one system – the system gets very large. Moreover, there high risk of people crossing at other places than at the crosswalks (jaywalking). For the standard deviations of the crossing distance, we choose values between 0 (equal spacing) and 0.3. Very high standard deviations will lead to quasi random crosswalk locations. For the density we spanned the density range in steps of $1/40$.

Table 3. Values for the parameters

Parameter	Values								
f	0.001	0.002	0.005	0.01	0.02	0.05	0.1	0.2	0.3
μ_D	.56	1.1	2.2	5.6	11	22			
σ_D	0	0.1	0.2	0.3					
k	(0), 1/40, 2/40, ..., 38/40, 39/40, (40/40)								

The aim of the paper is to consider the effect of all possible parameter combinations on both the road's capacity and the vehicle speeds for different traffic densities. Since a fairly exhaustive set of dimensionless parameter combinations were simulated, the paper's results comprehensively cover virtually all real situations that can arise in practice. How these results can be used for solving

problems in dimensional units is explained in section 5.

3 Simulation setup

This paper studies the results numerically using simulations. For the sake of understandability of the magnitude of various elements, we present the case study in real world, dimensional coordinates. The results will be presented in section 4 in dimensionless coordinates.

A closed loop road is considered as this allows to control for density and isolate the effect of the crosswalks without getting interference with the effect of intersections. We choose a length of $L=16$ km (a long distance) to mimic an infinitely long road.

For the vehicles, Newell's simplified car-following model is used (Newell, 2002), with $V_f=9$ m/s, $S_j=9$ m and $Q_o=1800$ veh/h. (Note this is a symmetrical fundamental diagram.) For under-critical densities, the speed is equal to the free flow speed. For over-critical densities, the equilibrium speed could be calculated based on the density and the fundamental diagram:

$$V = q(k)/k = \frac{Q_o - W(K - K_o)}{K} \quad (1)$$

Initially, all vehicles are placed with equal spacing. If the spacing is smaller than the critical spacing, they move at the equilibrium speed for that spacing, derived from equation 1

For the pedestrians, their headway is generated from an exponential distribution, with a mean of $\lambda = FL$. The first pedestrian arrives after 10 s (avoiding potential problems with this pedestrian arriving within the Newell wave trip Y), and iteratively the arrival time for a pedestrian is derived from arrival time of the previous pedestrian and its headway. The location of the arrival of a pedestrian is generated from a uniform distribution $[0,L]$. The spacing between one pedestrian crossings and the next is chosen from a normal distribution with a mean M_D and a standard deviation Σ_D . Pedestrians are then assigned to the pedestrian crossing closest to their location. The simulation is run for 750 minutes, in order to have sufficient independent measurements. All pedestrians have a crossing time T of 10 seconds. Previously, we found that the average crossing time is important, and variations have only a minor effect (Daganzo and Knoop, 2016).

The flow is computed using Edie's definitions (Edie, 1965) over a one-minute interval over the complete road length. For each condition, 750 one-minute observations are made for the flow. The first 100 measurements are ignored for to warm up of the simulation. The speed is computed as the quotient of the flow and the density. For determining the capacity, the maximum of the flow over the densities is taken, which, given the symmetrical fundamental diagram in the simulation, is found in all cases halfway the density range.

To simulate all combinations of the (dimensionless) variables in table 3, we adapt the flow of the pedestrians, the average inter-crosswalk distance, and its standard deviation.

4 Discussion of results

The simulations of section 3 were run and the results expressed in dimensionless units. In this section we discuss the results using a representative sample of scenarios. We will first discuss the capacities and then the speeds. How the results should be interpreted for a practical case at hand, is discussed later, in section 5. The simulation results in full, which are needed for application in practice, are shown in the appendix (appendix A), since these can be skipped without losing the storyline.

4.1 Capacities

Figure 1 shows the capacities as function of the inter-crosswalk μ_D distance. It also includes the

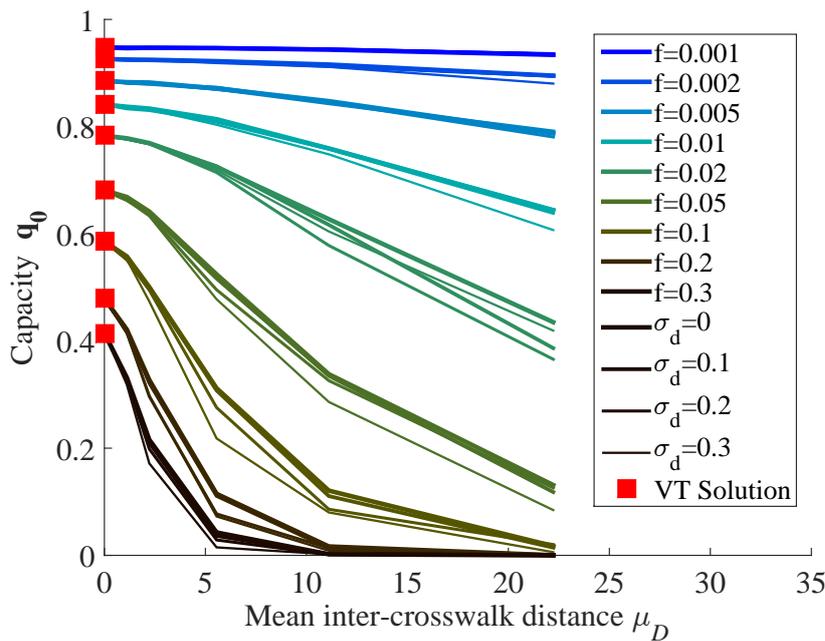


Figure 1. Capacities for different inter-crosswalk distances. The capacity decrease as function of the increasing mean distance between two crosswalks. Different colors indicate different pedestrian flows; different widths indicate different variations in inter-crosswalk distance. The red squares for $\mu_D=0$ indicate the variational theory (VT) solution for continuous crossings from Daganzo and Knoop (2016).

capacity for a cross-anywhere situation, as estimated in Daganzo and Knoop (2016):

$$q_o = 1 / \left(1 + \sqrt{8/\pi} \sqrt{f} + 1.27f + 0.35f^{2/3} \right) \quad (2)$$

In the graph, the value is represented by a square for an inter-crosswalk distance 0. The good match strongly suggest that the simulation was properly implemented. The curves starting from each square have the same f and only differ in σ_D .

As expected, the flow decreases with pedestrian flow f and mean inter-crosswalk distance μ_D . They are always lower than the “cross everywhere case”. For $\mu_D = 0.56$, the lowest simulated value, the capacity is almost equal to the capacity of cross everywhere case with continuous intersections. For any other inter-crosswalk distance μ_D value lower than approximately 0.5 we hence expect to find flows equal to the flow in the “cross anywhere” situation. This means that an inter-crosswalk distance of 0.5 would yield a road capacity which is close enough so traffic operations do not change if there are more crosswalks. In real world coordinates, for urban urban traffic conditions, this means the closest intersections need to be placed is $M_D = 0.5V_fT$, or approximately 25-50 meters, depending on road width and free flow speed.

The decrease of capacity with increasing inter-crosswalk distance is higher for a higher pedestrian load. For low pedestrian loads, the capacity hardly decreases with a higher inter-crosswalk distance, since there is hardly any queuing, or interference of queues between two pedestrians – given low load, they are almost independent in choosing a location, even if there are few.

The figure also shows the impact of variation in inter-crosswalk distance. Although this effect is relatively minor, it can be seen that as the variation increases, the capacity decreases; this can be explained by the fact that the capacity is determined by the most restrictive crosswalk, i.e. the crosswalk with the highest pedestrian load. Since pedestrians take the closest crosswalk, the crosswalk with the highest pedestrian load is the crosswalk which is furthest from it’s neighbors. If the variation in crosswalk distance increases, there will be crosswalks with larger distances to their neighbors.

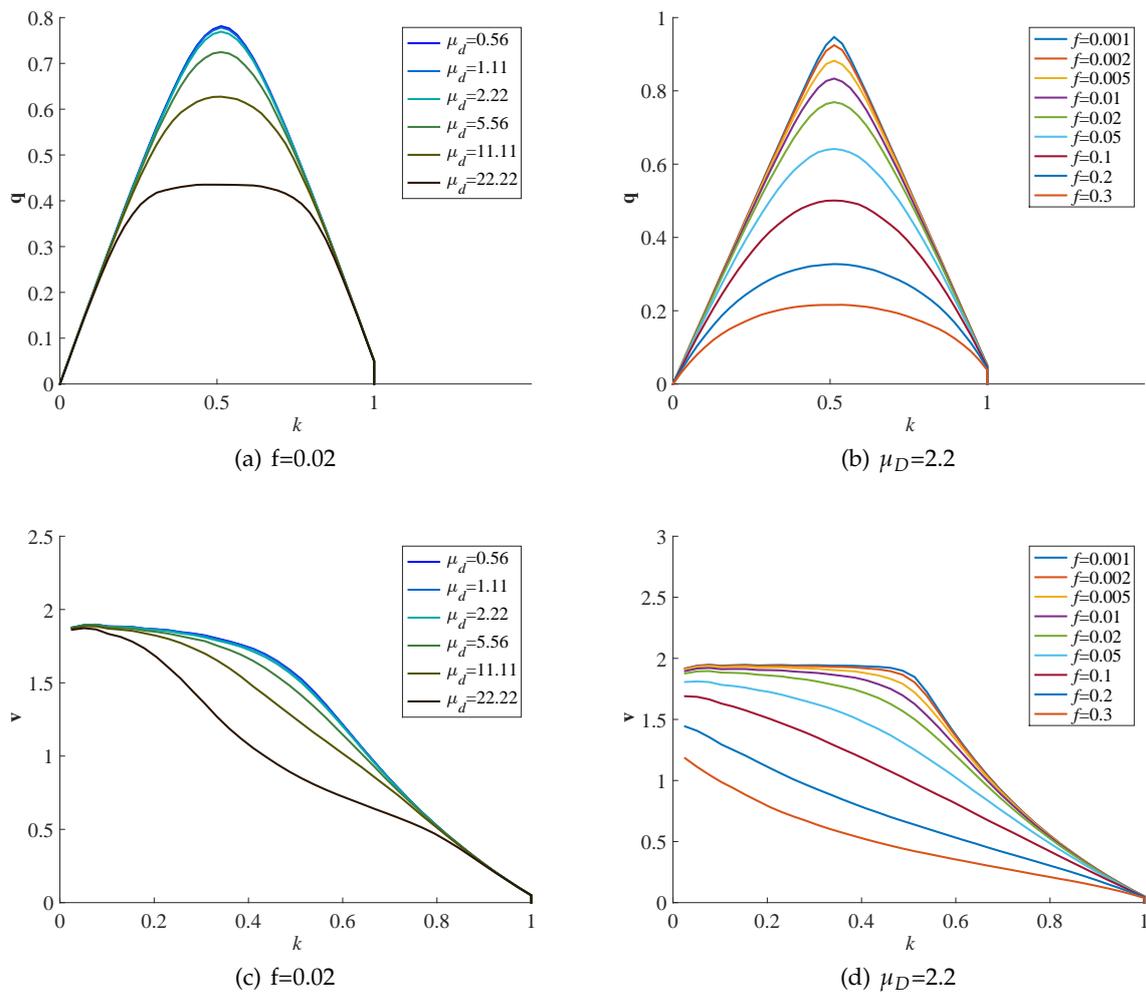


Figure 2. Flow and speed for combination of crosswalk distance and pedestrian flow; for all figures, $\sigma_D = 0$

4.2 Speeds

In this section we consider the speed for various densities. This can be shown in different ways, which are equivalent in information, but give different insights. Figure 2 shows (in the top row) the flow-accumulation graph, as well as (in the bottom row) the speed-accumulation graphs. The figures show the effect of the inter-crosswalk distance and the pedestrian flows for $\sigma_D = 0$. We show the figures for $f = 0.02$ for all μ_D , as well as for $\mu_D=2.2$ for all f .

The MFDs seems symmetrical, which can be explained by the fact that in the case of the symmetric fundamental diagram, voids have the same equations of motion as vehicles: see [Daganzo and Knoop \(2016\)](#). All MFDs are presented in figure 3. That also shows that for higher inter-crosswalk distances and higher variation of the inter-crosswalk distance, the flows fluctuate more. This is an effect of the higher effect of the stochastic in these simulations: larger variations in inter-crosswalk distances will lead to some crosswalks with more pedestrians; especially for larger inter-crosswalk distances, these extra distances will lead to many more pedestrians, and hence have a higher effect on the flow.

Considering the shape of the MFDs, indeed, a higher μ_D give a lower flow. If figure 2(a) the shape of the MFD is remarkable: the flow reaches a near-maximum over a longer density range if the inter-crosswalk distance is larger. A similar phenomenon can be seen for the speed under various pedestrian loads, figure 2(d). For low pedestrian flows, the speed remains constant for a long density range. This is because the vehicles do not interact with each other in case of low number of

crossing pedestrians. If the speed is constant, that means that the only reason of speed reduction is if the vehicle interacts with a pedestrian – and not for further interactions with vehicles. The line $f = 0.3$ in this figure shows that for higher pedestrian flows, this interaction between the vehicles starts already for low densities. The consequence is that the MFD (figure 2(b)) is curved from the start.

5 Using the results: interpolation

The simulations have been performed for a specific shape of the fundamental diagram and a specific duration of the pedestrian crossing time. As stated in section 2.2, the charts of figure 3 can be used to obtain the flows and speeds of traffic for *any* triangular fundamental diagram, *any* pedestrian flow and *any* pedestrian crossing time. In this section we will show the steps to be followed. First, interpolation is shown with the transformation from dimensionless to dimensional coordinates. Due to the high level of abstraction of the transformation, we also include an example to clarify the workings of the transformations. This shows how the research outcomes should be used, and what the research outcomes can contribute for a specific case. The steps shown are generic, and the particular situation of the case can be generalised to any other situation.

5.1 How to estimate capacity and speed

The procedure to estimate the capacity and speed is simple:

1. Find the dimensionless parameters f , μ_D (and k if the traffic speed is desired) corresponding to the problem at hand by applying the transformations given below

$$f = FT^2V_f \quad (3)$$

$$\mu_D = \frac{M_D}{V_f T} \quad (4)$$

$$k = \frac{K}{K_j} + \delta q_r, \quad \text{with } q_r = \begin{cases} K/K_o & \text{if } K \leq K_o \\ \frac{K_j - K}{K_j - K_o} & \text{if } K > K_o \end{cases}, \quad \text{and } \delta = 1/2 - K_o/K_j \quad (5)$$

$$(6)$$

2. (To estimate capacity:) Choose the line from figure 1 that is closest to μ_D and read from the chart the corresponding (dimensionless) capacity q_o . The actual density can be computed by:

$$Q_o = q_o Q_o \quad (7)$$

3. (To estimate speed:) Choose the chart from this figure 3 which is closest to μ_D and choose the curve closest to f ; pick the point closest to k and read the (dimensionless) flow q . The actual flow is

$$Q = q Q_o \quad (8)$$

The speed can be obtained by using

$$V = \frac{Q}{K} \quad (9)$$

Equation 3 scales the pedestrian flow with the units of time and space, equation 4 scales the mean inter-crosswalk distance with the unit of space. Equation 5 scales and skews of the axes of the fundamental diagram, towards a symmetrical fundamental diagram with $k_j = 1$ and $k_o = 1/2$.

To obtain the speeds in a higher accuracy than reading the flow value of a single f and μ_D closest to the problem at hand, a two-dimensional interpolation can be done. A two-dimensional a cubic interpolation one accounts for the non-linearity of the results; in this case, two higher and two lower values for both the pedestrian flow f and the inter-crosswalk distance μ_D are used.

5.2 Numerical example

To show the working of the above transformations, this section will provide a numerical example. Consider the following practical problem:

- **Road properties:** $V_f=30$ km/h, $Q_o=1200$ veh/h, $K_j=125$ veh/km, $K_o = 1200/30 = 40$ veh/km
- **Pedestrians:** $T=5$ s, $F=1000$ peds/km/h
- **Crosswalks:** M_D 100 meters, regular spacing ($\Sigma_D=0$)
- **Interest:** Capacity under pedestrian crossing Q_o , or vehicular speed V at $K=20$ veh/km

First, we transform all variables in SI units. We thus have $F=1000/1000/3600=1/3600$ peds/m/s, and $V_f = 30/3.6$ m/s. Now, we perform the three steps laid out in section 5.1.

1. First, we obtain the dimensionless variables. Performing the transformation from 3 we obtain

$$f = FT^2V_f = \frac{1}{3600}5^230/3.6 = 0.057 \quad (10)$$

The inter-crosswalk distance is calculated as (equation 4)

$$\mu_D = \frac{M_D}{V_fT} = \frac{100}{30/3.65} = 2.4 \quad (11)$$

To obtain k (equation 5), we first find

$$Q_r = K/K_o = 20/40 = 1/2, \text{ and} \quad (12)$$

$$\delta = 20/125 - 1/2 = -.18 \quad (13)$$

Now, the dimensionless density is found as (equation 5)

$$k = K/K_j + 1/2 - (-0.18) = 0.25 \quad (14)$$

To check, one can see that the density K of 20 veh/km is half of the critical density. The scaled density k is also half of the scaled critical density $k_o = 1/2$, so $k = 0.25$.

2. The capacity can now be read from figure 1. At μ_D we read the $q_o=0.63$. Rescaling this (equation 7), will yield the capacity for the road under crossing pedestrians of

$$Q_o = q_oQ_o = 0.63 \times 1200 = 760 \text{veh/h} \quad (15)$$

3. For the speed, we consider the MFDs in figure 3, in particular figure 3(d). Reading the line for the closest pedestrian flow ($f = 0.05$) at the right density $k = 0.25$, we find $q = 0.42$. Applying equation 8 we obtain:

$$Q = qQ_o = 0.42 \times 1200 = 504 \text{veh/h} \quad (16)$$

The speed can be obtained by using equation 9

$$V = \frac{Q}{K} = 504/20 = 25 \text{km/h} \quad (17)$$

One can improve the result by performing a bi-cubic interpolation of the simulation results. We consider two lower and two higher pedestrian flows, i.e. $f=0.02$, $f=0.05$, $f = 0.1$ and $f = 0.2$. We also consider two lower and two higher inter-crosswalk distances: $\mu_D = 1.1$, $\mu_D = 2.2$, $\mu_D = 5.6$, and $M_D = 11.1$. The 16 flow values are read from the appropriate figures figure 3 and a cubic interpolation is done to find the flow value for $f = 0.057$ and $\mu_D=2.4$. This interpolation gives a scaled flow value of $q = 0.427$, a flow very close to the estimated value by choosing the nearest neighbor ($q = 0.42$, as mentioned above). As validation of the method a simulation is carried out with dimensionless variables $f = 0.057$ and $\mu_D=2.4$; this also yields a scaled flow value of $q = 0.42$. This shows the nearest-neighbor reading works accurate enough for this case.

6 Discussion and conclusions

This paper has shown how pedestrians on crosswalks influence the road traffic. The paper showed how by coordinate transformation the problem is only dependent on few parameters. The relative capacity (i.e., relative to non-interrupted capacity) depends on the pedestrian flow, the inter-crosswalk distance, and the variability of the inter-crosswalks distance pedestrian. All combinations within this parameter space have been simulated, and curves on capacity have been created. The study provides the curves which can be used for readout to obtain the traffic capacity and the traffic speed in practical cases, without the need to do any simulations.

The capacity increases with a decreasing distance between crosswalks. This increase is most noticeable for large pedestrian loads. Interestingly, for low pedestrian loads, speeds remain almost constant with increasing car density up to the critical density. In contrast, for high pedestrian loads, the speed declines with the pedestrian loads, also in the lower density regimes. We expect this to be due to the interactions between queues of vehicles: for a low number of pedestrians, the crossings are unrelated to each other, and the speed is determined by the number of pedestrians crossing. For higher pedestrian loads, a similar speed can be obtained. However, when density increases, vehicles will not only be stopped by pedestrians, but also by vehicles, thereby reducing the speed.

Most striking result is that intercrosswalk spacings of less than approximately 25-50 meters do not add any benefit. Compared to a “cross anywhere” scenario, the addition of crosswalks is beneficial to reduce crossing times because pedestrians speed up on crosswalks and they cross orthogonally. Personal observations in Berkeley show that when crosswalks are absent, pedestrians cross non-orthogonally.

The current study shows the effect of a homogeneous road, with equal pedestrian crossings (only different inter-crosswalk distance and variation thereof), and a homogeneous pedestrian crossing demand. Future research includes studies to the effect of a variation of pedestrian demand, and limiting the area of pedestrian crossing, showing the effect of block and inter-block crossings and the effect of shared space sections on through traffic.

Acknowledgement

Knoops contribution was co-funded by the Allegro project (Unraveling slow mode traveling and traffic: with innovative data to create a new transportation and traffic theory for pedestrians and bicycles), which is financed by the European Research Council and Amsterdam Institute for Advanced Metropolitan Solutions. Daganzo’s research was supported by UCCconnect Grant: 65A0529 TO 040.

References

- CROW (2017). CROW:ASVV [12.1.11 Voetgangers oversteekplaats (VOP) in gebiedsontsluitingsweg 50 km/h]. ASVV.
- Daganzo, C. F. (1977). Traffic delay at unsignalized intersections: Clarification of some issues. *Transportation Science*, 11(2):180–189.
- Daganzo, C. F. (2005). A variational formulation of kinematic waves: Solution methods. *Transportation Research Part B: Methodological*, 39(10):934 – 950. ISSN 0191-2615.
- Daganzo, C. F. and Knoop, V. L. (2016). Traffic flow on pedestrianized streets. *Transportation Research Part B: Methodological*, 86:211–222.
- De Leur, J. and Wildenburg, F. (2011). Wanneer wel een zebra, wanneer niet? nieuw kader voor oversteekvoorzieningen binnen de bebouwde kom. (in dutch: Crosswalk or not: new framework for crosswalks in urban area). In *Proceedings of the National Traffic Conference (Nationaal verkeerskundecongres)*.

- Duives, D. C., Daamen, W., and Hoogendoorn, S. P. (2013). State-of-the-art crowd motion simulation models. *Transportation research part C: emerging technologies*, 37:193–209.
- Edie, L. (1965). Discussion of traffic stream measurements and definitions. In *Proceedings of the Second International Symposium on the Theory of Traffic Flow*. OECD, Paris, France.
- Hamilton-Baillie, B. (2008). Shared space: Reconciling people, places and traffic. *Built environment*, 34(2):161–181.
- Hawkes, A. G. (1968). Gap-acceptance in road traffic. *Journal of Applied Probability*, 5(1):84–92.
- Helbing, D. and Molnar, P. (1995). Social force model for pedestrian dynamics. *Physical review E*, 51(5):4282.
- Hoogendoorn, S. P. and Bovy, P. H. (2004). Pedestrian route-choice and activity scheduling theory and models. *Transportation Research Part B: Methodological*, 38(2):169–190.
- Laval, J. A. and Chilukuri, B. R. (2016). Symmetries in the kinematic wave model and a parameter-free representation of traffic flow. *Transportation Research Part B: Methodological*, 89:168–177.
- Newell, G. (2002). A simplified car-following theory: a lower order model. *Transportation Research Part B: Methodological*, 36(3):195–205.
- Transportation Research Board, (2010). Highway Capacity Manual. Technical report.
- Yang, Z., Liu, P., Xu, X., and Xu, C. (2016). Multiobjective evaluation of midblock crosswalks on urban streets based on topsis and entropy methods. *Transportation Research Record: Journal of the Transportation Research Board*, (2586):59–71.
- Zhao, Y., Chen, Q., Qin, J., and Xue, X. (2017). Survey of pedestrians' crossing time at non-signalized mid-block street crossing. *Journal of Advanced Transportation*.

A Overview of simulation results

MFDs for all inter-crosswalk distances and pedestrian flows are shown in figure 3. Results for σ_D larger than zero are omitted due to space restriction and the limited effect of the variation on the results.

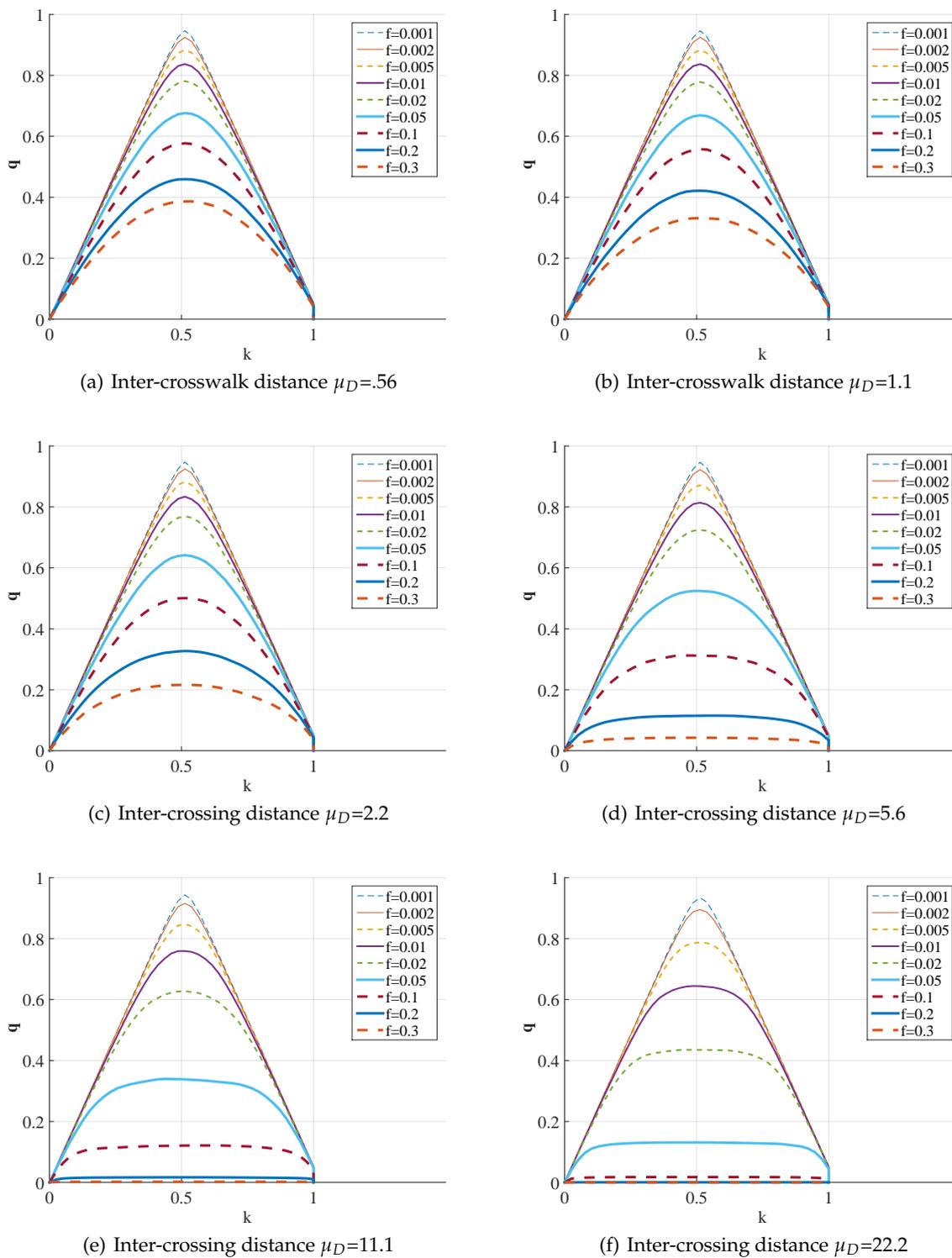


Figure 3. All MFDs