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Optimizing performance-based mechanisms in road management: an agency theory approach

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This paper develops a model based on the agency theory to analyse road management systems that employ a mechanism of performance indicators to establish the payment for the contractor. The base assumption is the asymmetric information between a principal (Public Authorities) and an agent (contractor) and the risk aversion of the latter. It is assumed that the principal may only measure the agent's performance indirectly and by means of certain performance indicators that may be verified by the authorities. In this model it is assumed there is a relation between the efforts made by the agent and the performance level measured by the corresponding indicators, although there may be dispersions between both variables that give rise to a certain degree of randomness in the contract. An analysis of the optimal mechanism was made on the basis of this model and in accordance with a series of parameters that characterize the economic environment and the particular conditions of road infrastructure. As a result of the analysis, the incentive mechanism should include a fixed component and a payment according to the obtained performance level. The higher the risk aversion of the agent and the greater the marginal cost of public funds, the lower the impact of this performance-based payment. By way of conclusion, the system of performance indicators should be as broad as possible but should avoid those indicators that encompass greater randomness in their results.

Keywords: *incentive, performance risk, public procurement, road management.*

1. Introduction

Numerous countries have developed diverse road management systems over recent decades that have gradually replaced direct management by local and central government. The majority of these formulas are based on a contract that establishes the relation between the authorities and a company entrusted with certain aspects of road management. The scope of the services rendered by the contractor varies and may include the maintenance of the asset, traffic and road management and, in some cases, the funding and construction of a new road infrastructure and its ensuing management on completion. The contract formulas used may also vary from standard maintenance contracts to integral management contracts, concessions or other types of Public-Private Partnerships (PPPs).

It is becoming more common to base these different contract formulas on a payment to the agent that is associated with the obtained quality level, which is, in turn, measured by a series of indicators that have become increasingly well defined over recent years (Delgado Quiralte *et al.*, 2007; Harding *et al.*, 2010; Federal Highway Administration, 2011). This approach acts as an incentive system to ensure the correct performance of the contractor and the optimization of resources employed in road maintenance and operation.

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Contract formulas of this nature may be analysed within the general framework of agency theory, which has undergone considerable development since it was first established back in the 1970's. This theory is an attempt to explain the agency relationship whereby one party (the principal) delegates certain tasks to another party (the agent), and where this relation is regulated by a contract or by similar means (Jensen and Meckling, 1976). Agency theory has been applied to many different fields of activity ever since its origins (Harris and Raviv, 1978). Among the first authors to study the relationship between a public authority and an agent, were Loeb and Magat (1979) and Baron and Myerson (1982), who focused on the problem of the regulation of a natural monopoly under conditions of asymmetric information between the authorities (principal) and the company providing the asset or service (agent). Laffont and Tirole (1993) worked on systematizing this area of economic analysis, and Iossa and Martimort (2009), Mu *et al*. (2010) and González-Díaz and Montoro-Sánchez (2011), specialized in the field of transport. In this paper we contribute to these earlier studies by being the first to apply the agency approach to the design of a system of quality indicators in road management. As indicated above, the use of this type of indicators has had a boost in recent years, and there is some expertise in the definition and calibration of these systems. In section 4 of this paper, are included some examples of indicators that have been used in road concession contracts. However, there is no systematic study of the principles that should be taken into account to design an efficient set of quality indicators, and in this paper we aim to contribute a theoretical model that can be applied to the road sector. For example, one problem that can be found in many road contracts is the excess in the number of quality indicators used to control the activity of the contractor. This paper shows that, under certain circumstances, the use of redundant indicators may, in fact, reduce the overall efficiency.

The agency problem arises under conditions of asymmetric information between the principal and the agent when there is a conflict of interest and where it is difficult or expensive for the principal to verify what the agent is actually doing (Eisenhardt, 1989). In these cases, the research conducted on the principal-agent relationship was based on the specifications of a series of assumptions regarding the possible means by which the principal may provide incentives to the agent to bring their respective objectives into line. On the basis of these assumptions and by a method of deduction, it was then possible to find a solution to the problem of contract optimization. One of the main aspects of the agency relationship is the attitude towards risk of the contracting parties. In agency theory, it is generally assumed that the agent is risk-averse and the principal is risk-neutral or, at the very least, that the agent is more averse to risk (Eisenhardt, 1989; Dewatripont and Legros, 2005; Sadka, 2007).

In the present work, we developed a model based on the assumption that the agent (contractor) will normally be averse to risk, although we have also examined the implications in the case that the agent is risk neutral. The incentive provided by a payment system based on performance levels is restricted in the model due to the randomness of the contract. This randomness is due to the fact that the authorities cannot directly measure the effort made by the contractor to render the service and may only do so by means of specific performance indicators that may be observed and verified by the authorities. This may well give rise to certain discrepancies between the efforts made by the contractor and the effective results obtained from the performance indicators and which, in turn, affects the payment received by the contractor. A good example is the case of road accident and fatality indices that are employed in many PPP contracts. The results obtained from these indicators effectively depend on numerous random factors and only to a certain degree on the performance of the contractor.

The problem of information asymmetry as described in the previous paragraph leads to a case of *moral hazard* (Laffont and Tirole, 1993). The main idea is that the application of contract incentives to the agent conflicts with their risk aversion and leads to a duality of objectives by, 1) paying the productive work of the agent and, 2) efficiently assigning the risks, and thereby preventing the optimal outcome obtained in an environment of complete information (Holmström and Milgrom, 1991). In this context, an important issue is the so-called informativeness principle. This principle was first formulated by Blackwell (1953), and has been developed within contract theory by Holmström (1979), Shavell (1979) and Grossman and Hart (1983), among others. According to this principle, a contract should maximize the precision of performance indicators used to evaluate the agent. In turn, the maximization of precision would imply incorporating all the informative signals on the agent´s performance and filter out irrelevant noise (Chaigneau *et al.*, 2014). As will be seen, the results of this paper are, to a large extent, based on this principle.

More recently, the agency theory approach has been adopted by Dewatripont and Legos (2005), Martimort and Pouyet (2008) and Iossa and Martimort (2012, 2015), who fundamentally geared their studies towards the comparative analysis of public-private partnerships with respect to more traditional public procurement. These authors consider a PPP to be basically defined by the bundling of diverse tasks within one single contract between the public authorities and the agent: and particularly, the design and construction of a certain infrastructure together with its maintenance and the operation of the corresponding service once the infrastructure is up and running. Under traditional procurement methods, these milestones are separated and assigned to different agents. The most relevant conclusion reached by the above authors is that, under certain circumstances, PPP contracts may be more efficient than traditional public procurement since the efforts of the agent during the construction stage are positively reflected in the agent's results during the operation stage. As such, it is necessary to design a suitable system of incentives that allow the agent to internalise the benefits derived from the greater build quality of the infrastructure. In an optimal situation this would then lead to the greater allocation of risks to the agent under a PPP than under a traditional contract.

Unlike these previous studies, the model considered in the present work is applicable to both a PPP and to any other type of road management contract where the contractor's remuneration is based on the quality level obtained. The incentive system to the contractor should, at the outset, combine payment based on performance levels and payment in accordance with the number of users, as indicated by Sánchez Soliño (2012). However, there has been a shift in attitude in recent years in favour of the retention of the demand risk by the authorities (Abdel Aziz, 2007). This tendency has been very marked in the United Kingdom, where contracts since the early part of the 2000s carried out under the Private Finance Initiative (PFI) framework were based on performance-based payment systems (Standard & Poor's, 2003). This method has become even more widespread following the 2008 financial collapse which led to the reluctance of the majority of financial corporations to fund any project of this nature that incorporates a substantial transfer of the demand risk to the agent. In this paper, we shall therefore assume a contract where no demand risk is transferred to the contractor.

The following section includes a description of the model employed and the base assumptions of the incentive system to the contractor. The object of the analysis was to optimize the parameters defining the payment system to the contractor from the perspective of welfare. These parameters depend on a series of factors and the influence of each of these is the subject of detailed study. In section 3 we analyse the implications of correlation between different performance indicators. Section 4 includes an empirical study of the criteria applied in the design of the performance indicator system in a wide range of road concessions in Spain and compare these to the efficiency criteria analysed earlier in the model proposed in this work. Finally, the conclusions contain a summary of the main results and a series of recommendations to be taken into account in the design of the contracts.

2. Formulation of the model

We shall presume that a certain authority (the principal) is responsible for the management of a certain road and decides to delegate the corresponding functions by way of a contract with a specific company (agent). In most cases, these functions will include the maintenance of the road infrastructure and the corresponding installations, the necessary services for correct traffic operation and, when applicable, the necessary investment to extend or improve the infrastructure.

In the model developed below, the road is taken to be free for users though the model may readily be applied to the case of toll roads. The objective function of the principal is that of social welfare and one the public authority will attempt to maximize through the optimization of the parameters defining the remuneration mechanism of the contract, as described further on. Therefore, we assume that the public authority behaves as a benevolent regulator, and leaving aside any self-interest of the principal, an assumption that is commonly used in public regulation literature (see, for example, Laffont and Tirole, 1993).

The social benefits obtained from the operation of the road, including the surplus of users, depend on the quality levels offered and which, in turn, can be measured by a series of indicators, taken to be independent from each other. In order to simplify the statement, we shall presume that the social benefits rise linearly with quality, in accordance with the following equation:

$$S = S_o + \sum_{i=1}^{n} a_i q_i \tag{1}$$

Where S is the total social benefit, a_i are constants greater than zero and q_i represent the levels obtained in a series of n road quality dimensions. These quality levels depend to a certain extent on the efforts made by the contractor, as we shall see below. S_0 represents the minimum social benefit obtained at a quality level that we shall conventionally establish at zero in each dimension. This reflects the fact that public authorities generally demand certain minimum conditions before allowing the public to use the road.

As indicated, the quality levels q_i may be related to the efforts (referred to as e_i) made by the agent to improve service performance in each of the quality dimensions. An essential element of our model is that this relation between the quality obtained and the efforts made by the contractor presents a statistical noise, or random component, to the extent that:

$$q_i = e_i + \varepsilon_i \tag{2}$$

This formulation can be found, for example, in Martimort and Pouyet (2008). Each variable ε_i represents this random component which is taken to have a normal distribution, with a mean equal to zero and a standard deviation equal to σ_i. This means that the public authority cannot directly observe the efforts made by the agent and may only estimate this indirectly by the values obtained for the quality indicators employed. However, these values do not only depend on the effort e_i, but also on other unpredictable factors that are grouped in the said random variables.

As such, in our model the agent acts under risk conditions due to the randomness of the variables q_i and it is similarly assumed that the agent is not neutral to these risks. This thinking follows the general assumptions adopted in agency theory (Eisenhardt, 1989). Regarding the authorities, however, it is reasonable to assume that risk is sufficiently diversified on account of the large number of projects and services for which they are responsible, and which subsequently makes them risk neutral.

The agent performs the road management duties and subsequently incurs certain costs that may be expressed as follows:

$$C = C_0 + \sum_{i=1}^{n} \psi_i(e_i) \tag{3}$$

Where C represents the cost of producing the services rendered by the agent and C_0 is a fixed cost term incorporating the cost derived from any necessary initial investment for rendering the service. This fixed cost term may be interpreted as the minimum cost necessary to keep the road in operation with a minimum level of quality. The variable cost terms are, in turn, a function of the levels of efforts e_i by the agent, and where $\Psi_i'(e_i) > 0$ and $\Psi_i''(e_i) > 0$. By this, we are assuming that the cost functions strictly increase with the efforts e_i and that these cost functions are also strictly convex.

Regarding remuneration, it is assumed that the agent will receive an amount from the authority which, in accordance with our model, depends on the quality levels q_i. The remuneration is given by a linear equation, following the incentive schemes proposed by Holmström and Milgrom (1991):

$$t(q_i) = \alpha + \sum_{i=1}^{n} \beta_i q_i \tag{4}$$

In this expression, t is the amount paid by the authorities to the contractor and α and β_i are fixed parameters established in the contract. As may be seen, the agent will receive a fixed sum (given by the parameter α), regardless of the quality provided, together with a variable amount that depends on the quality verified by the authorities. Since the authorities cannot directly observe the efforts made by the agent, an incentive system based on the achieved quality is then established. However, the agent's revenue function includes the random variables defined above and the greater the incentives that be established the greater the bearing of these variables. Our problem is then to determine the value of the α and β_i parameters, established in the contract, that will maximise the social utility, when taking into account the trade-off between the agent's efforts and their risk aversion

When taking into account the random component of the agent's remuneration, the expected value would be as follows:

$$E[t] = \alpha + \sum_{i=1}^{n} \beta_i e_i \tag{5}$$

While the variance in revenue received by the agent, and specified as σ_t^2, would be as follows:

$$\sigma_t^2 = \sum_{i=1}^{n} \beta_i^2 \sigma_i^2 \tag{6}$$

In addition to the social welfare function, which is defined later on, it is also necessary to consider the restrictions established on the basis of the agent's utility function. This function is given by the following equation (Laffont and Tirole, 1993):

$$Ur = E[t] - C - r\sigma_t^2 \tag{7}$$

In this equation, Ur is the utility of the agent, E[t] the expected value of the payment made by the principal, and C the cost of producing the service. Here, r is a parameter that represents the agent's attitude to risk and, when expressed in other terms, Ur would then be the certainty equivalent of the returns expected by the agent.

In the case of the risk aversion of the agent, this would imply an r value strictly greater than zero, while in the case of risk neutrality this would suppose an r equal to zero. In our model it is assumed that the agent will normally be risk averse, though we shall also study the implications arising in the case that r = 0. We shall exclude the possibility that the agent is a risk lover, and therefore:

$$r\sigma_t^2 \geq 0 \tag{8}$$

On developing the equation (7), when considering (3) and (5), we then obtain:

$$Ur = \alpha + \sum_{i=1}^{n} \beta_i e_i - C_0 - \sum_{i=1}^{n} \Psi_i(e_i) - r\sigma_t^2 \tag{9}$$

However, in order for the agent to carry out their activity, they will have to obtain a utility greater or equal to the alternative of not carrying out any. If we take this latter alternative to represent a utility equal to zero, the agent's participation is then dependant on the following:

$$Ur \geq 0 \tag{10}$$

The social welfare function which should be maximised is thus formed by the benefits for society as a whole minus all costs incurred in making this service available and including, among other factors, the cost derived from the risk aversion of the agent. In this way, the social welfare function may be given by:

$$W_s = E[S] - C - r\sigma_t^2 - \lambda E[t] \tag{11}$$

In this equation Ws is the social welfare that is to be maximised and the variable S represents the social benefit given by (1), and as such:

$$E[S] = E[S_o + \sum_{i=1}^{n} a_i q_i] = E[S_o + \sum_{i=1}^{n} a_i (e_i + \varepsilon_i)] = S_o + \sum_{i=1}^{n} a_i e_i \tag{12}$$

Equation (11) includes the excess burden of public funds which is given by the term λE[t], where λ is a parameter (attributed with a value greater than zero) that characterises the tax system of the country in question. The value (1 + λ) is normally referred to as the marginal cost of public funds, a concept that incorporates various aspects, including the distortion introduced by the tax system in the decisions of the economic agents and the cost of tax administration. In other words, we are then supposing that the disutility to taxpayers inflicted by levying an additional monetary unit shall be equivalent to (1 + λ) monetary units, where the value of λ depends on the institutional framework of each country and the tax provisions used to obtain additional public funds. This weighting of public funds is not taken into account in the majority of cost-benefit analysis and it is, instead, implicitly established that λ = 0. However, Laffont and Tirole (1993) considered it reasonable to establish a value λ = 0.3 for the American economy and Kleven and Kreiner (2003) estimate λ values of between 0.09 and 0.80 for different OECD countries when considering proportional tax increases for the tax system as a whole. In all events, this concerns values that are by no means negligible. A detailed study of the concept of the marginal cost of public funds may be found in Dahlby (2008).

When considering (3), (5), (6) and (12), and when substituting certain terms by their values, the social welfare equation may then be expressed as follows:

$$W_s = S_0 + \sum_{i=1}^{n} a_i e_i - C_0 - \sum_{i=1}^{n} \psi_i(e_i) - r\sum_{i=1}^{n} \beta_i^2 \sigma_i^2 - \lambda\left(\alpha + \sum_{i=1}^{n} \beta_i e_i\right) \tag{13}$$

However, if we take into account equation (9), the social welfare function can then be written as:

$$W_s = S_0 + \sum_{i=1}^{n} a_i e_i - (1+\lambda)[C_0 + \sum_{i=1}^{n} \psi_i(e_i) + r\sum_{i=1}^{n} \beta_i^2 \sigma_i^2] - \lambda U_r \tag{14}$$

When resolving the problem of maximising this function, it is necessary to take into account, in addition to the restriction (10), the restrictions imposed by the maximisation of the objective function of the agent. The agent's efforts e_i should then comply with the following first-order conditions:

$$\frac{\partial Ur}{\partial e_i} = \beta_i - \psi_i'(e_i) = 0$$

That is to say:

$$\psi_i'(e_i) = \beta_i \tag{15}$$

The problem may then be laid out in the following terms:

$$Max_{(Ur, ei, \beta i)}\{S_0 + \sum_{i=1}^{n} a_i e_i - (1+\lambda)[C_0 + \sum_{i=1}^{n}\psi_i(e_i) + r\sum_{i=1}^{n}\beta_i^2\sigma_i^2] - \lambda U_r\} \tag{16}$$

subject to restrictions (10) and (15).

The procedure to obtain the solution to the preceding problem is detailed in Appendix A, and the results can be summarized as follows:

$$Ur = 0 \tag{17}$$

$$a_i - (1+\lambda)[\psi_i'(e_i^*) + 2r\sigma_i^2\psi_i'(e_i^*)\psi_i''(e_i^*)] = 0 \tag{18}$$

$$\beta_i^* = \psi_i'(e_i^*) \tag{19}$$

Where the maximum condition is as follows:

$$-(1+\lambda)[\psi_i''(e_i^*) + 2r\sigma_i^2(\psi_i''(e_i^*))^2 + 2r\sigma_i^2\psi_i'(e_i^*)\psi_i'''(e_i^*)] < 0 \tag{20}$$

It should be noted that in accordance with the base assumptions of the model and for a risk neutral or risk averse agent (that is to say, with $r \geq 0$), it is sufficient that $\Psi'''(e_i^*) \geq 0$ in order to comply with the maximum condition.

In order to simplify the exposition, while retaining the general application of the main results of the model, it is possible to consider a specific cost function that satisfies the condition of being strictly positive in its first and second derivatives and greater or equal to zero in its third derivative. In economic literature it is common to take the following type of quadratic cost functions:

$$\Psi_i(e_i) = k_i e_i^2/2$$

Where the parameter k_i represents a constant scale factor for each effort. Examples of authors using quadratic cost functions include Arrow and Radner (1979), Gibbons (1998), Rob and Zemsky (2002), Socorro (2007) and Martimort and Pouyet (2008).

The solution obtained with these cost functions is then as follows:

$$e_i^* = \frac{a_i}{(1+\lambda)k_i(1+2k_i r\sigma_i^2)} \qquad \beta_i^* = \frac{a_i}{(1+\lambda)(1+2k_i r\sigma_i^2)} \tag{21}$$

And from which, and when taking into account equation (17):

$$\alpha^* = C_0 + r\sum_{i=1}^{n}(\beta_i^*)^2\sigma_i^2 - \sum_{i=1}^{n}\frac{(\beta_i^*)^2}{2k_i} \tag{22}$$

In this case the maximum condition is always observed, as $\Psi_i'''(e_i^*) = 0$.

In this way we then obtain the values of the parameters of equation (4) that optimize the contract in terms of social welfare. It may be noted that in an optimal contract it would generally be necessary to include a fixed payment α^*, regardless of the quality of the service and one that would largely depend on the fixed cost term C_0. However, the value of α^* could, in theory, be negative which would imply the payment of a fixed sum by the agent to the principal.

In this model the values of the parameters β_i^*, with a risk neutral or risk averse agent, will always be greater or equal to zero, as will the value of the agent's efforts. The values of β_i^* (and subsequently the efforts made by the contractor) will be seen to drop with any increase in the risk aversion of the contractor, the variance σ_i^2 or the marginal cost of public funds. On the contrary, the values of β_i* increase when the social benefits, represented by a_i, are high.

An important result of this model is that all the dimensions of quality that contribute a positive value in equation (1) should be included in the payment mechanism of the contract. As may be observed in equation (21), if $a_i > 0$, then β_i* will always be greater than zero. This result does not hold when the quality indicators are not independent, as it will be seen below.

It may be noted that in a situation of complete information of the authorities and where these may directly observe the efforts made by the contractor, this would then imply that $\sigma_i^2 = 0$ in equation (21). In this case the value of the contractor's efforts would be established as:

$$e_i{*} = \frac{a_i}{(1+\lambda)k_i} \tag{23}$$

When the contractor is risk averse, this value will always be higher than that obtained under the assumption of asymmetric information between principal and agent. However, when the contractor is risk neutral (r=0), the result obtained will always be as that given by equation (23).

3. Redundant performance indicators

In the previous section we assumed that the performance indicators employed are independent of each other. However, in practice, there is often a correlation between two or more indicators. In this section we shall tackle this question, though for reasons of simplicity the analysis shall be restricted to the case where it is taken that the agent will make an effort "e" that will only have an effect on two interrelated performance indicators. At the same time, each indicator is only affected by this sole effort e.

In this way, we would then obtain:

$$q_1 = e + \varepsilon_1 \tag{24}$$

$$q_2 = e + \varepsilon_2 \tag{25}$$

Where q_1 and q_2 represent the levels obtained in indicators 1 and 2 respectively, e the level of effort made by the contractor and ε_1 and ε_2 are random variables with distributions N(0, σ_1) and N(0, σ_2) respectively. These random variables are taken to be positively correlated, with a correlation coefficient ρ ($\rho \geq 0$), and we will assume that $\sigma_2 > \sigma_1$.

We can assume that the expected social benefit would be given by the following equation, in accordance with the notation employed above:

$$E[S] = S_o + ae \tag{26}$$

The cost incurred by the contractor, when taking a variable cost $\Psi(e) = ke^2/2$, would be given by:

$$C = C_0 + ke^2/2 \tag{27}$$

The contractor's remuneration would be given by:

$$t(q_1, q_2) = \alpha + \beta_1 q_1 + \beta_2 q_2 \tag{28}$$

Where α, β_1 and β_2 are the contract parameters to be optimized. By way of restriction, it is assumed that performance incentives are null or positive, that is to say, $\beta_1 \geq 0$ and $\beta_2 \geq 0$.

The expected remuneration of the contractor is then:

$$E[t] = \alpha + (\beta_1 + \beta_2)e \tag{29}$$

While the variance in the remuneration t would, in this case, be given by:

$$V[t] = V[\beta_1\varepsilon_1 + \beta_2\varepsilon_2] = \beta_1^2\sigma_1^2 + \beta_2^2\sigma_2^2 + 2\beta_1\beta_2 \operatorname{cov}(\varepsilon_1, \varepsilon_2)$$

Or when expressed differently:

$$\sigma_t^2 = \beta_1^2\sigma_1^2 + \beta_2^2\sigma_2^2 + 2\beta_1\beta_2\rho\sigma_1\sigma_2 \tag{30}$$

Where ρ is the correlation coefficient between both indicators.

The contractor's utility function, where we assume risk aversion ($r > 0$), would once again be given by:

$$Ur = E[t] - C - r\sigma_t^2 \tag{31}$$

On developing this equation, we then obtain:

$$Ur = \alpha + (\beta_1 + \beta_2)e - C_0 - k\frac{e^2}{2} - r\sigma_t^2 \tag{32}$$

And where the restrictions imposed by the contractor's utility function would now be:

$$Ur \geq 0 \tag{33}$$

$$\frac{\partial Ur}{\partial e} = \beta_1 + \beta_2 - ke = 0$$

$$e = \frac{\beta_1 + \beta_2}{k} \tag{34}$$

The social welfare function to be maximized is now as follows:

$$W_s = S_0 + ae - (1+\lambda)[C_0 + k\frac{e^2}{2} + r\beta_1^2\sigma_1^2 + r\beta_2^2\sigma_2^2 + 2r\rho\beta_1\beta_2\sigma_1\sigma_2] - \lambda U_r \tag{35}$$

And where the problem is subsequently reduced to:

$$Max_{(Ur,\, e,\, \beta 1,\, \beta 2)}\{\mathrm{W_s}\} \tag{36}$$

Subject to restrictions (33) and (34).

This problem can be solved following a similar procedure as the one detailed in Appendix A. Then, the first order conditions are as follows:

$$Ur = 0 \tag{37}$$

$$\frac{a}{k} - (1+\lambda)[\frac{\beta_1 * + \beta_2 *}{k} + 2r\sigma_1^2\beta_1 * + 2r\rho\sigma_1\sigma_2\beta_2 *] = 0 \tag{38}$$

$$\frac{a}{k} - (1+\lambda)[\frac{\beta_1 * + \beta_2 *}{k} + 2r\sigma_2^2\beta_2 * + 2r\rho\sigma_1\sigma_2\beta_1 *] = 0 \qquad (\text{with } \beta_2^* > 0) \tag{39}$$

$$\frac{a}{k} - (1+\lambda)[\frac{\beta_1 *}{k} + 2r\rho\sigma_1\sigma_2\beta_1 *] \leq 0 \qquad (\text{with } \beta_2^* = 0) \tag{40}$$

$$e* = \frac{\beta_1 * + \beta_2 *}{k} \tag{41}$$

It may be verified that for variance values and a correlation coefficient that meet:

$$\rho\sigma_2 \geq \sigma_1 \quad (42)$$

then the following values of the parameters will meet the previous first order conditions and optimize the social welfare function:

$$\beta_1{*} = \frac{a}{(1+\lambda)(1+2kr\sigma_1{}^2)} \quad (43)$$

$$\beta_2{*} = 0 \quad (44)$$

With:

$$e{*} = \frac{\beta_1{*}}{k} \quad (45)$$

As such, if the correlation coefficient is high and the variance associated with indicator 2 is significantly higher than that with indicator 1, one may then obtain an internal solution for parameter β_1 and a corner solution for parameter β_2. Thus the optimal solution is obtained by eliminating the performance indicator showing the greatest variance from the performance indicator system. In the event of low correlation coefficient values (ρ) it is then not possible to guarantee that this is the optimal solution and it may well be preferable to retain both indicators.

4. Analysis of the indicators employed in Spanish road concessions

The results obtained in equation (21) above have an interesting implication. As may be seen, the efforts made by the agent under an optimal contract depend on certain factors related to the general economic environment, such as risk aversion and the marginal cost of public funds. However, this also depends on a factor, the variance σ_i^2, which in turn depends on the correct design of the quality indicators employed in the contract. In this respect, the selected indicators should not have, as much as possible, any considerable disparity between the efforts made by the contractor and the quality level measured according to the corresponding indicator.

When taking these results as the starting point it is possible to compare the theoretical criteria defining an optimal contract with the real application of quality indicators. To this effect, we shall employ examples of road concessions in Spain, taken from the work by Delgado Quiralte *et al.* (2007). In the majority of road concessions put out to contract in Spain since 2002 (from the time of the draft bill on Concession Law that was finally passed in May 2003), different quality indicators have been applied that partly determine the remuneration of the contractor, though in the majority of cases these have been combined with other payment mechanisms based on the number of users. Appendix B includes all the concessions considered in this study with an outline of some of the characteristics of these concessions. This includes a group of 25 road concessions put out to tender from 2002, all of which include performance indicator systems that affect, to a greater or lesser extent, the payment to the concessionaire.

The performance indicators employed vary from one concession to another. Some of these contain a very large number of indicators and for this reason table 1 only includes the most significant and more widely used indicators. The table includes a definition of each indicator and an ensuing appraisal (“high”, “medium” or “low”) of the dispersion between the possible result obtained for the indicator and the efforts made by the concessionaire to reach this result. The sole purpose of this exercise is to establish up to what extent the design of the performance indicator system follows the theoretical criteria of efficiency analyzed in this article. The majority of the concessions referred to have only been in operation for a short time and we subsequently do not have the necessary information to compare the quality levels that have really been obtained in the different concessions.

Table 1. Use of performance indicators in Spanish road concessions

Performance indicator	Definition of indicator	Assessment of dispersion between result and effort	No. of concessions where the indicator is applied
Road accidents	Accident index = No. of accidents with victims per year/Lx ADTx365 (L = length of road section; ADT= Average Daily Traffic)	High	25
Road fatalities	Fatality index = No. of fatalities in road accidents per year/LxADTx365	High	24
Congestion	Congestion index = No. of weighted hours per year in which the service level is E or F	High	20
Skid resistance	Sideways force coefficient (SFC)	Low	20
Longitudinal road smoothness	International Roughness Index (IRI)	Low	20
Road capacity availability	Application of deduction in the case of closure by the concessionaire of one or more lanes of the roadway	Medium	16
State of road markings	Rate of retroreflection of road markings	Low	11
State of road signs	Replacement response time	Low	11
Cleaning of roadway and verges	Correction response time	Low	11
Accident and incident response	Response time to incidents, signalling and repairs and replacements	Low	10

As may be seen, some of the most widely used indicators are those that tend to show a greater variation between the result effectively obtained and the effort made by the concessionaire to improve this quality aspect. This is particularly the case of the road accident and fatality indices that are employed in practically all the concessions that have been studied and which are also used in road management contracts in other countries (Rangel *et al.*, 2012, 2013).

5. Conclusions

As a result of the analysis conducted in this work, an incentive mechanism in road management should include both a fixed payment to the contractor and a set of payments established in accordance with the quality levels obtained. The optimal weights of quality indicators, together with the efforts made by the contractor, decrease with any increase in risk aversion, the σ_i^2 variance or the marginal cost of public funds.

The range of performance indicators employed in the contract should be as wide as possible and attempt to include all the quality dimensions that significantly influence the social welfare function. In this respect, it is to be expected that the agent will focus all their efforts on those quality dimensions that are specifically regulated, but there would be a high risk that certain quality aspects would be neglected merely on account of being omitted from the contract.

However, reference should be made to the results obtained in the present study regarding the selection of performance indicators. As may be seen in equation (21) above, the effort made by the agent under an optimal contract depends on a factor, the variance σ_i^2, which in turn depends on the correct design of the quality indicators employed in the contract. In this respect, the selected indicators should not have any considerable disparity between the efforts made by the contractor and the quality level measured according to the corresponding indicator. As such, all those indicators providing results with a high random component beyond the control of the contractor should not be overweighted.

Furthermore, the results of this work show that when there is a high correlation between two performance indicators, the optimal solution consists of a contractor retribution system that only employs the indicator whose random component shows the smallest variance.

These results are in line with the informativeness principle of contract theory mentioned in the introduction. An operative rule to apply this principle to the design of road management contracts could be the following, according to the results obtained in this work: the set of indicators should cover all the relevant dimensions of quality, but each kind of effort should be estimated only by one quality indicator (the one with the lowest variance), if the correlation between different indicators is high.

In practice, some of the most widely used indicators tend to show a great variation between the result effectively obtained and the effort made by the concessionaire to improve this quality aspect. This is particularly the case of the road accident and fatality indices that are employed in many road Public-Private Partnerships (Delgado Quiralte *et al.*, 2007; Rangel *et al.*, 2012). The results obtained from these indicators effectively depend on numerous random factors and only to a certain degree on the performance of the contractor. In addition, indicators such as the accident rate generally show a high correlation with indicators related to the correct maintenance of the different elements of the road (Wallman and Aström, 2001). Therefore, according to the model developed in this work, it would be ineffective to establish the contractor's payment mainly on these types of indicators such as the accident index, since this greatly reduces the social welfare on account of the risk aversion of the agent.

The reason behind the generalized use of these types of indicators related to road accidents is that of the large social repercussions of these accidents. The introduction of accident and fatality indices among the quality indicators of a road management contract may transmit the idea of a public concern for traffic safety, but these are not necessarily the most efficient indicators. The results of this work may serve to revaluate the loss of efficiency resulting from the selection of quality indicators that generally tend to give random results and show that it is preferable to employ other indicators (related, for example, to road maintenance, signs and markings or response to incidents) that also affect safety but whose results depend more on the performance of the contractor.

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Appendix A. Solution of the optimization problem

Starting from equation (16), it may be observed that this social welfare function monotonically decreases with Ur, because it is assumed that λ > 0. A positive value of λ is crucial to guarantee the existence of a unique solution to this problem.

Thus the value of the variable Ur that maximizes the social welfare function will be the minimum compatible with restriction (10), i.e., Ur* = 0. Additionally, if we introduce the restriction (15) into the expression (16), the problem is then reduced to:

$$Max_{(e_i)}\{f(e_i)\}$$

Where the function $f(e_i)$ is the following:

$$f(e_i) = S_0 + \sum_{i=1}^{n} a_i e_i - (1+\lambda)[C_0 + \sum_{i=1}^{n} \psi_i(e_i) + r\sum_{i=1}^{n}(\psi_i'(e_i))^2 \sigma_i^2] \tag{46}$$

The first order conditions are the following:

$$\frac{\partial f(e_i^*)}{\partial e_i} = a_i - (1+\lambda)[\psi_i'(e_i^*) + 2r\sigma_i^2 \psi_i'(e_i^*)\psi_i''(e_i^*)] = 0 \tag{47}$$

Starting from equation (47) we can obtain the critical points e_i*. Taking into account equation (15), we can then obtain the parameters β_i* that optimize the contract:

$$\beta_i^* = \psi_i'(e_i^*) \tag{48}$$

The critical points will represent a maximum for the function f if the Hessian matrix H(f) is negative definite. In this case, the Hessian matrix is diagonal, so the maximum condition will be:

$$\frac{\partial^2 f(e_i^*)}{\partial e_i^2} < 0, \text{ for any i} \tag{49}$$

That is:

$$-(1+\lambda)[\psi_i''(e_i^*) + 2r\sigma_i^2(\psi_i''(e_i^*))^2 + 2r\sigma_i^2\psi_i'(e_i^*)\psi_i'''(e_i^*)] < 0 \tag{50}$$

It may be observed that, under the assumption of a risk neutral or risk averse agent (that is to say, with r ≥ 0), then it is sufficient that $\Psi'''(e_i^*) \geq 0$ in order to comply with the maximum condition (50).

Appendix B. Concessions examined

Concession	Type (Greenfield/ Brownfield)	Length of section or network (km)	Payment system to the concessionaire	Year of procurement
M-12 Airport access (Madrid)	Greenfield	9	Real tolls/quality indicators	2002
AP-7 Cartagena-Vera Motorway	Greenfield	114	Real tolls/quality indicators	2003
AP-36 Ocaña-La Roda Motorway	Greenfield	148	Real tolls/quality indicators	2003
AP-41 Madrid-Toledo Motorway	Greenfield	60	Real tolls/quality indicators	2003
AP-7 Alicante Ring-road	Greenfield	28.5	Real tolls/quality indicators	2003
AP-46 Málaga-Las Pedrizas Motorway	Greenfield	24.5	Real tolls/quality indicators	2005
A-1 Santo Tomé del Puerto-Burgos	Brownfield	146	Shadow tolling/quality indicators	2007
A-2 Madrid-R2	Brownfield	57	Shadow tolling/quality indicators	2007
A-2 km. 62- Guadalajara/Soria provincial boundary	Brownfield	76	Shadow tolling/quality indicators	2007
A-2 Guadalajara/Soria provincial boundary-Calatayud	Brownfield	95	Shadow tolling/quality indicators	2007
A-2 Calatayud-Alfajarín	Brownfield	107	Shadow tolling/quality indicators	2007
A-3 Madrid/Cuenca provincial boundary - Cuenca/Albacete provincial boundary	Brownfield	141	Shadow tolling/quality indicators	2007

A-3 La Roda-Bonete	Brownfield	94	Shadow tolling/quality indicators	2007
A-3 Bonete-Alicante	Brownfield	111	Shadow tolling/quality indicators	2007
A-4 Madrid-R4	Brownfield	65	Shadow tolling/quality indicators	2007
A-4 Puerto Lápice-Venta de Cárdenas	Brownfield	107	Shadow tolling/quality indicators	2007
AG-11 Barbanza Motorway	Greenfield	42	Shadow tolling/quality indicators	2005
AG-56 Santiago-Brión Motorway	Greenfield	15	Shadow tolling/quality indicators	2004
AG-41 Salnés Motorway	Greenfield	17	Shadow tolling/quality indicators	2004
MA-15 Palma-Manacor Motorway	Greenfield	41	Shadow tolling/quality indicators	2004
M-203 Motorway (Madrid)	Greenfield	18	Shadow tolling/quality indicators	2005
M-407 Motorway (Madrid)	Greenfield	11.5	Shadow tolling/quality indicators	2005
CV-35 Motorway Valencia-Losa del Obispo	Greenfield	54	Shadow tolling/quality indicators	2004
Cuenca Provincial Council network	Brownfield	950	Shadow tolling/quality indicators	2006
Toledo Provincial Council network	Brownfield	989	Quality indicators	2006