

Optimising time-varying network flows by low-revenue tolling under dynamic user equilibrium

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This paper presents a dynamic congestion charging framework that includes a dynamic user equilibrium (DUE) network model to optimize a tolling scheme, whose objective(s) may be determined by the transport planner. The framework presented may be utilised to evaluate potential charging regimes; this paper concentrates on the feasibility of extending the concept of minimal-revenue tolls from the static to time-varying environment. We discuss the limitations in the transference of this optimisation objective and propose a pragmatic approach designed to allow the planner to balance the effect of extracting additional toll revenue against achievable network benefit. We investigate the ability of this framework to incorporate various tolling scenarios and to illustrate the production of low-revenue tolls. The numerical tests use a fixed toll across the entire time period for a range of demand scenarios and a fairly stable tolling solution is observed.

Keywords: congestion charging; low-revenue tolling; dynamic traffic assignment; dynamic user equilibrium; time-varying network flows

1. Introduction

1.1 Background

The benefits of congestion charging have been much discussed and debated in the past few decades (e.g. Emmerink et al., 1995; Yang and Huang, 2005; Ungemah and Collier, 2007; Zhang and van Wee, 2012), and many congestion charging projects have been implemented and are now operational on a permanent basis around the world; including the projects in London, Singapore, Milan, California of the USA and Stockholm. These projects charge traffic only in a certain area within one or a few time periods during the day. Some only toll a fixed amount for the use of a charging zone during the charging period, such as in London and Milan. Some set a different constant toll for each time interval in the charging period (known as step-tolling); such as tolling schemes in Singapore and Stockholm.

Intuitively it would seem logical that if road tolls are to be implemented that they should in some way be optimal; that is they should be as effective as possible with regard to specified criteria. It may be a political objective to maximise revenue whilst not seeking particularly to discourage road users or to lessen congestion, which would lead to relatively cheap tolls. If instead congestion reduction were the primary objective then tolls would be set very high to discourage

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car usage, an extreme case of which would be to completely restrict traffic and impose high fines for violation. More commonly, some practical balance is desired between the competing objectives of raising revenue, restricting demand and traffic redistribution both temporally and spatially. Within the charging zone, the traffic may be distributed to maximise network benefits by applying different tolls to each link and theoretically minimising the collective travel time or cost for the users such that the System Optimal (SO) flow solution is achieved.

The theoretical economic basis for road tolling requires that drivers should pay for their externalities (e.g. Pigou, 1920; Walters, 1954; Vickrey, 1955), which results in marginal social cost price tolls (MSCP) where each driver is charged an equal amount to the marginal costs they impose on the network and such tolls produce the SO flow pattern. Theoretical marginal cost tolls in which a toll is placed on every link have been perceived (a) to be problematic to implement and (b) to impose tolls which are too high to be implemented politically, although they would imply the best solution regarding congestion reduction. That the practical problems in imposing a link-based charging system have led to the pursuit of 'sub-optimal' cordon based schemes (Meng et al., 1999) is understandable, but the current advances in technology regarding electronic payment and vehicle tracking make such schemes a viable alternative for the future (although efforts have been made to 'optimise' cordon based schemes with respect to certain criteria (May et al, 2000)). Tolling that is high clearly creates political problems regarding public acceptance and equity, although there is evidence to show that there is a greater public appreciation for the merits of traffic congestion charging (particularly from city centre residents) if the revenue could be suitably hypothecated (Glaister et al., 1998).

1.2 Problem description

This paper is concerned with optimising time-varying network flows by low-revenue tolling given the traveller behaviour of dynamic user equilibrium (DUE). Dynamic Traffic Assignment (DTA) models extend the static concept of UE to DUE where the system is said to be in DUE where no user can unilaterally reduce their origin-to-destination (OD) travel time or cost (e.g. Wu et al., 1998; Lo and Szeto, 2002; Carey and Ge, 2012). Recently, as two alternatives to DUE, tolerance-based dynamic user optimal (DUO) and DUO with variable tolerance are proposed respectively in Szeto and Lo (2006) and Ge and Zhou (2012) and compared to DUE in Ge et al. (2013); these two states allow the differences in travel costs between the used paths of an OD pair to be positive and the differences reflect travellers' tolerances, which enables us to find a stable state on a road network when a DUE solution does not exist. This paper does not seek these states but focus on obtaining DUE or conventional DUO states. DSO (Dynamic System Optimal) is a state on a road network where the total travel cost of all travellers is minimised over the time period under consideration (see e.g. Friesz, 2007; Chow, 2007).

At present, congestion charging in a time-varying context has been investigated by means of the DTA technique (e.g. Yang and Huang, 2005; Chung et al., 2012). In the same manner as the general research on DTA, the research on dynamic congestion charging (DCC) has two streams. One focuses on the within-day scale and the other on the day-to-day scale. This paper will focus on within-day DCC and the congestion charging toll will be imposed on each path. In this paper tolls will be determined to minimize the total revenue with an emphasis on the observation of the effects of the change in the tolling revenue - as path-specific tolls vary - on the resulting total network travel costs. We will see that minimal revenue tolls exist for each of the numerical examples presented later.

1.3 Literature review

In the case of static traffic assignment the classical road tolling problem is to toll network links such that under the principle of User Equilibrium (UE) a System Optimising (SO) flow pattern is obtained (where the overall network benefit is maximised). One toll set that creates this desired effect is the aforementioned marginal social cost price tolls (MSCP). Such tolls are however often seen as too expensive to be politically implementable and toll sets that induce the SO are non-unique such that further optimisation is possible: for example, minimal revenue (non-negative)

tolls create the desired SO flow pattern at minimal additional cost to the users (Bergendorff et al, 1997; Hearn and Ramana, 1998; Yildirim and Hearn, 2005; Dial, 1999, 2000). In the single origin case, it was shown that minimal revenue tolls could be determined such that one path (for each OD) would remain un-tolled. The policy of maintaining a single un-tolled path, whilst simultaneously extracting minimal revenue from the user and achieving optimal network efficiency is highly politically attractive (Stewart, 2007) particularly regarding the existence of “free” routes from considerations of equity (see e.g. Jones, 2003; Schade, 2003; Frey, 2003). This did not hold in the multiple origin case (Dial, 2000). In the context of real-world tolling however, existing schemes do not claim to be mathematically optimal and excepting the Singapore ERP, operational schemes are generally simple fixed tolling schemes that are easy for the user to understand. Some pragmatic combination of better network performance (approaching SO) whilst permitting “free” routes to promote equity and extracting politically acceptable (lower) tolls from users is therefore desirable. Following this premise, Stewart and Maher (2006) and Stewart (2007) developed heuristics in a static stochastic environment to determine low revenue tolls that produce good sub-optimal flow patterns where total network costs approach the SO and where the “free” routes suggested from Dial’s single origin algorithm were required. It was illustrated that whilst exact SO solutions could not be achieved whilst retaining “free” paths for all OD pairs in the multi-origin case nevertheless low-revenue tolls could be derived which produced flow patterns that very closely approached the SO. The heuristics developed assumed a static modelling environment (stochastic and deterministic cases) and hence this paper seeks to examine tolling solutions which follow a broadly similar premise (that of low-revenue tolling to approach SO) within a dynamic modelling environment. Zhang and Ge (2004) took a different perspective to look at congestion charging, which aims to minimise both remaining congestion externalities as well as the total revenue that is treated as the burden/cost to the road users in an elastic-demand context.

The dynamic network tolling problem has been defined and researched with work formulating the problem as a Mathematical Programme with Equilibrium Constraints (MPEC) (Lawphongpanich and Hearn, 2004; Friesz, 2007) and explicitly as a Dynamic Optimal Toll Problem with Equilibrium Constraints (DOTPEC) (Chung et al., 2012). The game theoretical approach has also been utilised where a Stackelberg game, i.e. a two-stage dynamic game, (e.g. Ritzberger, 2002) may be formulated to describe Network Design problems generally (e.g. Zhang et al., 2005) and dynamic network tolling problems more specifically (see Joksimovic, 2007). Joksimovic (2007) further solved a range of optimal network tolling problems, utilising step-tolls in single link, 2-link and 4-link problems and utilises existing DTA-assignments with tolls and a grid-search procedure to determine optimal tolls.

This paper seeks to utilise existing DUE models to investigate their ability to incorporate tolling scenarios to produce low-revenue tolls which create desired flow profiles. As pointed out in Chung et al., (2012), “Most of the literature on congestion pricing focuses on deterministic problems in static transportation networks... The time-dependent nature of traffic flow ... is not considered in strategic analyses based solely on static equilibrium.” Therefore, the key contribution of this piece of work is to consider the minimisation of the total revenue from charging time-varying traffic flow. As pointed out before, the concept of minimal-revenue congestion pricing were discussed in static settings (Bergendorff et al. 1997; Hearn and Ramana 1998; Yildirim and Hearn 2005; Dial 1999, 2000; Zhang and Ge, 2004; Stewart and Maher, 2006; Stewart, 2007) and this paper will investigate this concept for the first time in the dynamic settings. The numerical analysis presented is to demonstrate how the change in the tolling revenue – due to the variation in path-specific tolls – may result in the change of the total travel costs, which reveals the possibility and reasonability of implementing minimal revenue tolling schemes.

1.4 Structure of this paper

The structure of this paper is as follows. Section 2 presents the DCC framework to be utilised in this research, which corresponds conceptually to a bilevel problem, in which the upper level is to optimise the objective (e.g. minimal revenue) of a congestion charging scheme while the lower level captures the traveller choice behaviour that is assumed in this research to follow DUE. Section 3 presents the scenario settings for numerical analysis, which is to be carried out first on a 2-link example road network that illustrates the initial concept and then on a 4-link, 3-path network to show the DCC framework more generally and the effects of varying the demand levels on the toll sets produced. Section 4 presents and analyses numerical results and finally Section 5 closes up with some concluding remarks.

2. Methodology: model and algorithm

The tolling schemes considered in this research are path-based and all tolls are assumed to be collected at the entry of a path across the charging period. It is assumed that the toll for each path is flat/constant over the charging period. The path-specific tolls are endogenous variables. Figure 1 illustrates the DCC framework we have defined for use in this research to determine path-specific optimal tolls. Each component of the framework is further discussed below.

2.1 A framework for dynamic congestion pricing design

Suppose that we consider a charging period $[t_p^s, t_p^e]$, which is a subset of the time horizon $[0, T]$. Let $C_p^{toll}(t)$ denote the toll (per vehicle) collected at the entry of path p at time t , which satisfies the following condition:

$$C_p^{toll}(t) \begin{cases} \geq 0 & \text{if } t \in [t_p^s, t_p^e] \\ = 0 & \text{otherwise} \end{cases} \quad (2.1)$$

The charging period $[t_p^s, t_p^e]$ may vary in general from path to path, but in this paper it will be taken as the whole of the time horizon $[0, T]$ for all paths so as to avoid boundary issue complications (Ge and Stewart 2010, Ge et al. 2012). As mentioned before, path-based tolls are not time dependent, i.e. $C_p^{toll}(t) = C_p^{toll}$. How to determine these path-specific tolls – endogenous variables – is given conceptually in Figure 1 and will be discussed later in further detail.

The toll is only part of the cost travellers consider in their trip decision-making behaviour; the travel cost also includes schedule delay costs and travel time costs. It is assumed that a preferred arrival window exists, denoted as $[W^l, W^r]$, which is assumed to be the same for all destinations. If a traveller arrives within the window, there will be no schedule delay costs involved. Otherwise, a penalty will appear for early or late arrival at a destination. Denote the schedule delay cost of traffic entering path p at time t as $C_p(t + \tau_p(t))$, which depends on the entry time t plus the travel time $\tau_p(t)$. Since the travel time $\tau_p(t)$ is the time traffic entering path p at time t would experience, $t + \tau_p(t)$ is the time the traffic exits path p . We adopt the following widely used form of schedule delay costs in the literature (see Ge et al., 2012 and references therein),

$$C_p(t + \tau_p(t)) = \begin{cases} \beta[w^l - (t + \tau_p(t))] & 0 \leq t + \tau_p(t) < w^l \\ 0 & w^l \leq t + \tau_p(t) < w^r \\ \gamma[(t + \tau_p(t)) - w^r] & w^r \leq t + \tau_p(t) \leq T \end{cases} \quad (2.2)$$

where β and γ respectively represent the shadow costs of early and late arrivals. As customary, they satisfy $\beta/\gamma < 1$.

And then the generalized travel cost a traveller entering path p at time t would experience can be written as

$$\eta_p(t) = \alpha\tau_p(t) + C_p(t + \tau_p(t)) + C_p^{toll}(t) \quad (2.3)$$

in which the first term on the right hand side of the equation represents the travel time cost, the second the schedule delay cost, the third the congestion charging toll, and α the monetary value per unit time and satisfies, as customary, $\beta < \alpha < \gamma$. The values of these parameters may be dependent on origins or destinations. In this paper, they are assumed to be the same across the network under study. The travel time $\tau_p(t)$ is computed from the dynamic network loading (DNL) component, which will be described in subsection 2.2.

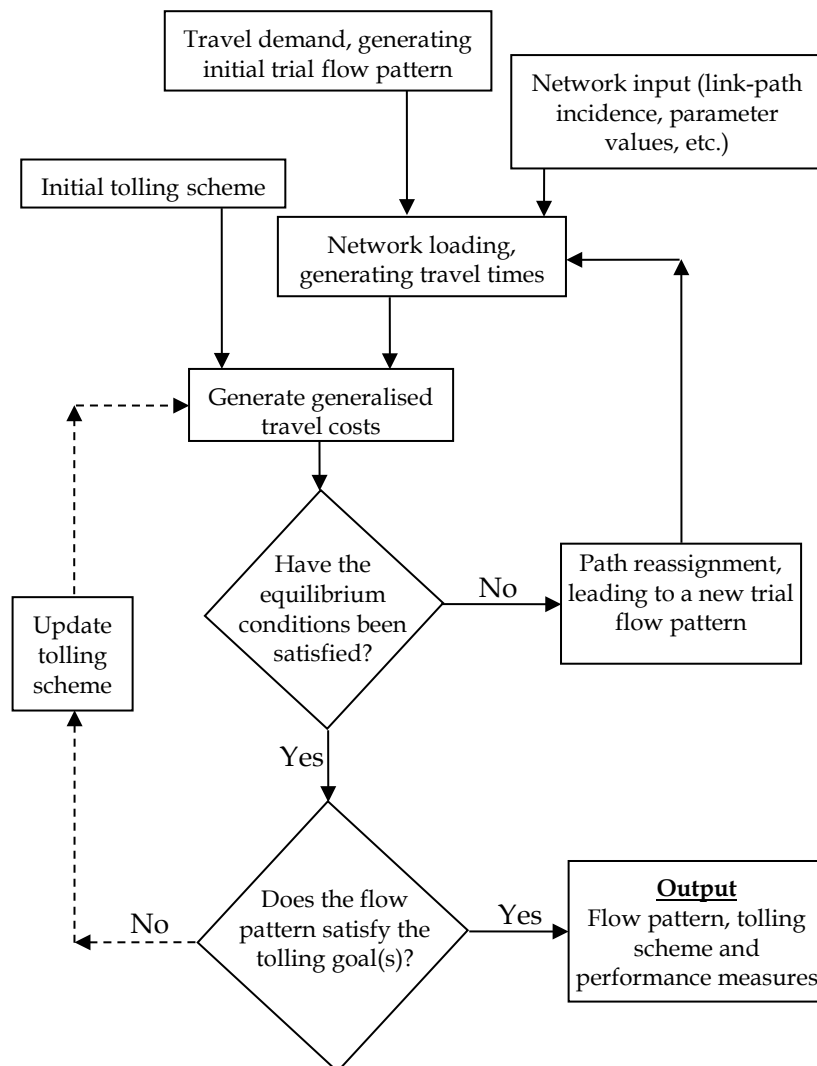


Figure 1: A framework for dynamic congestion charging

The tolling schemes to be determined are to serve to fulfil the objective of reducing the total network travel time or cost and/or total tolling revenue in general. Therefore, we are not currently seeking to produce DSO tolls, but good suboptimal tolls based on practical constraints. For the purpose of this preliminary work on network tolling under DUE, the toll sets are being taken as fixed for the purpose of illustrating that it is feasible under this constraint to produce path-based tolls that reduce the total network cost and total tolling revenue simultaneously. In practice, fixed tolls may produce undesirable boundary peaks (Ge and Stewart, 2010; Ge et al. 2012) when introduced as a strict sub-period of the total charging period, and for this reason, this paper initially considers scenarios where the tolls operate for the entire time horizon.

A further practical objective is to produce toll sets which do not impose full theoretical marginal costs upon the users, this is essentially the minimal revenue tolling objective familiar from static tolling literature, including Bergendorff et al. (1997), Hearn and Ramana (1998), Dial (1999, 2000) and Yildirim and Hearn (2005). This problem can be formulated as a bilevel mathematical program, in which the upper level has an objective defined as the tolling revenue whereas the lower level is the DUE traffic assignment model. Such a formulation can be considered as a mathematical program with equilibrium constraints (MPEC). To solve it, we may use intelligent algorithms, including genetic algorithms (e.g. Zhang and Ge, 2004) and other heuristics as discussed in Chung et al. (2012). Finding the optimal path-specific tolls is not the goal of this piece of work, what we wanted to show primarily in this paper is how the change in the tolling revenue – due to the variation in path-specific tolls – may affect the resulting total network travel costs, which illustrates the possibility and reasonability of implementing minimal revenue tolling schemes. Therefore, we chose to use a grid-search approach for this task. We always initialize our scheme with a zero-toll set, which presumes that we start from a current DUE solution based purely on the network costs. It is against the total travel cost from this initial DUE position that we will assess the effectiveness of our final tolling scheme.

When the DUE part of the algorithm is convergent, we consider the tolling goals. We wish to produce tolls that achieve our objectives given the tolling profile constraints that we have imposed for each scenario. In the case of a set of fixed tolls not permitted to vary across the time horizon, we have only to consider the output total network cost for different combinations of path-tolls. We terminate the process when a toll increment on any of the paths serves to increase the total network cost from that already achieved.

The update step in the case of fixed tolls relates to the relative toll level on each path. First, a path hierarchy is generated, based on the relative benefit that tolling on a single path only could achieve. The path where the maximum reduction in total network cost could be achieved if tolling were permitted on a single link only is ranked first, and the remaining paths are ranked after this. The path which is ranked last should always remain un-tolled whilst following the low-revenue tolling objective. A toll increment is determined based on the accuracy of the solution required, and the tolls on the first ranked path are allowed to vary incrementally within a defined range. The second ranked path is then permitted to be tolled, and tolls are introduced incrementally upwards from zero, while tolls of the primary path are allowed to vary within a set range. Thus a toll scheme which minimises the total network cost based on two paths with non-zero tolls may be determined. It is intended to extend this update procedure to produce a similar heuristic as presented in Stewart and Maher (2006), but this is yet to be applied numerically to a more general multi-path network.

2.2 Dynamic Network Loading (DNL)

This research uses a dynamic network loading (DNL) procedure based on the cell-transmission (CT) technique and the reader may refer to Daganzo (1995a) and Lo and Szeto (2002) for the details of the implementation of the procedure. This procedure generates path travel times.

Travel demand and initial trial flow patterns

It is assumed that the total travel demand for an OD pair w at time t is $g^w(t)$. Let $f_p(t)$ denote the flow rate entering path $p \in P^w$ at time t , where P^w is a set of paths serving the OD pair w . $f_p(t)$ is respectively subject to the traffic conservation condition and the non-negativity constraint below

$$\sum_{p \in P^w} f_p(t) = g^w(t) \quad \forall t \in [0, T], \forall w \quad (2.4a)$$

$$f_p(t) \geq 0 \quad \forall t \in [0, T], \forall p \in P^w, \forall w \quad (2.4b)$$

If a flow pattern $f = \{f_p(t): \forall t \in [0, T], \forall p \in P^w, \forall w\}$ satisfies the above conditions, it is feasible.

An initial trial flow pattern should be feasible. In our numerical experiments, we assigned the demand $g^w(t)$ uniformly onto each path in P^w .

Travel time generating

For each path p , the travel time $\tau_p(t)$ can be obtained by comparing the cumulative traffic on the entry and exit of it, represented by $Q_p(t)$ (entry) and $E_p(t)$ (exit), respectively.

Specifically, $\tau_p(t) = E_p^{-1}(Q_p(t)) - t$, where E_p^{-1} is the inverse of function E_p . A detailed description can be found in Lo and Szeto (2002) and Ge and Carey (2004).

Network input (link-path incidence, parameter values.)

This requires the data for network loading, including the link-path incidence matrix, and free-flow speed, lengths, critical densities, jam densities and capacities for all links. Also, we need the data for T , the charging period, the time step size, etc.

Flow-density (q - k) relationship

Since the CT technique is an efficient way to solve a kinematic wave model, we need to specify such a model in implementing a CT-based DNL procedure. It is well known that such a kinematic wave model consists of two parts; one is a flow-density (q - k) relationship, $q = Q(k)$, and the other is a traffic conservation equation. This research used the following nonlinear q - k relationship, which is also used in Carey and Ge (2012) and Ge and Zhou (2012).

$$q = Q(k) = \begin{cases} (q^{\max} - v^f k^c) / (k / k^c)^2 + v^f k & 0 \leq k \leq k^c \\ q^{\max} [-k^2 + 2k^c (k - k^j) + (k^j)^2] / (k^c - k^j) & k^c < k \leq k^j \end{cases} \quad (2.5)$$

If it is required that the flow function q (2.5) be smooth at $k = k^c$ (i.e., the left and the right derivatives are equal) then $k^c = 2q^{\max} / v^f$ must hold.

We may also choose other forms of q - k relationships for this type of work.

2.3 Path Reassignment

In the path reassignment component of a solution procedure of the DUE problem, we determine a new trial flow pattern based on the travel costs $\eta_p(t)$ obtained from the DNL component; a detailed description of their relationship can be found in Care and Ge (2012). In this subsection, we will briefly look at the DUE principle and methods for this purpose.

Definition of dynamic user equilibrium (DUE)

The following definition of dynamic user equilibrium (DUE) is an extension of the usual static user equilibrium to a dynamic context.

A path (in)flow pattern $f \in \Omega = \{f_p(t), \forall p \in K, t \in [0, T] \mid f_p(t) \text{ satisfies equations (3.4), } \forall t \in [0, T], \forall p \in P^w, \forall w, K = \cup_w P^w\}$ is a dynamic user equilibrium (DUE) solution if and only if, at any time t , the travel costs $\eta_p(t), \forall p \in K$, are the same on all used paths and is less than on non-used paths.

Thus, $f^* \in \Omega$ is in DUE if and only if, at any time t ,

$$\eta_p(t) = \begin{cases} = \eta_w^*(t) & \text{if } f_p^*(t) > 0 \\ \geq \eta_w^*(t) & \text{if } f_p^*(t) = 0 \end{cases} \quad \forall p \in P^w, \text{ where } \eta_w^*(t) = \min_p \{\eta_p(t) \mid p \in P^w\} \quad (2.6)$$

for all OD pairs w . It should be noted that the DUE definition is with respect to the path-choice process and gives DUE across paths at any time t . Achieving DUE over the entire time horizon (i.e. both path and departure time choices) is the subject of future work.

Generating a new trial flow pattern and convergence checking

In the existing literature, the above DUE problem is often formulated as an equivalent VI problem (e.g. Wu et al., 1998; Lo and Szeto, 2002). There are a number of solution methods proposed in the literature to generate a new trial flow pattern based on the newly generated travel costs. Carey and Ge (2012) compared some path reassignment methods by means of a CT-based DNL procedure. As suggested by the comparison in this reference, there is no preference for any one of them and here we simply choose to utilise the method in Lo and Szeto (2002). We use the convergence checking criteria developed in Carey and Ge (2012), which is a general criterion derived directly from the equilibrium conditions (2.6).

3. Scenario settings for numerical analysis

This section sets out the scenarios used for the numerical analysis. Two simple example networks were used for the numerical experiments, with each network having a single OD pair. Also, each network is assumed to be empty at time $t = 0$. In all the later analysis, the same time horizon $[0, T]$, the same forms of travel demand function and the same forms of flow-density (q - k) relationships were used, as set out below.

Time horizon: The time horizon used was $[0, T]$ where $T = 35$ time units. If one time unit is set to 6 minutes then the horizon will be 3.5 hours long, which implies that the horizon can be defined as 6am to 9:30am, and thus represent a peak am period. As customary, this research assumes that the network is initially empty. To make the scenario satisfy this assumption, the horizon should be long enough to cover a peak period plus the period during which the instantaneous demand rate rises from zero.

Travel demand: The following form of time-varying OD travel demand function was used in all cases:

$$g(t) = \begin{cases} (A + B)[\sin(a_1^{-1}T^{-1}\pi)]^{d_1} & 0 \leq t \leq t_m \\ A + B[\sin(a_2^{-1}T^{-1}\pi(t-b))]^{d_1} & t_m < t \leq T \end{cases} \quad (3.1)$$

where $A, B, a_1, a_2, b, d_1, d_2$ and t_m are the parameters. In the numerical experiments we let $t_m = 6$ and $a_1 = 0.8, a_2 = 1.2, b = 3, d_1 = 1$ and $d_2 = 5$. This demand function describes travel demands increasing from zero up to its peak $A + B$ at $t = t_m$ and then declining gradually.

At the initial iteration (i.e. determining the initial flow pattern), we assigned the OD travel demand equally among all paths for the OD pair. Also, if the inflow to a path in time interval i is greater than the receiving capacity of its first link, or cell, then the excess traffic is held over to the next time interval, in which case there is a queue for entry to the first link of a path. Any waiting times of traffic for entry to a path were included in the corresponding path travel times.

3.1 Scenario 1: 2-link, 2-path network

The first simple network used here is shown in Figure 2 and has a single origin (node O) and destination (node D). In this network, 2 links correspond to 2 paths, i.e. link l is path $p = l+1$, where $l = 0, 1$.

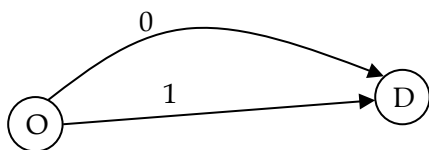


Figure 2: Example network 1

Link characteristics: To define a q - k relationship (2.5) for each link we need to specify the values for traffic parameters, including capacity (q^{\max}), free-flow speed (v^f), critical density (k^c) and jam density (k^j). Table 1 gives the values of these parameters plus the length (L) of each link l .

Table 1: Characteristics of links in Network 1

Link l	q_l^{\max} (veh/min)	v_l^f (km/min)	L_l (km)	k_l^c (veh/km)	k_l^j (veh/km)
0	26.88	0.96	1.25	56.00	160.00
1	19.60	0.70	1.05	56.00	160.00

From the data for v_l^f and L_l in Table 1 we can obtain the free flow travel time ($tt^{\text{ff}} = L/v^f$) for each link/path as follows

$$tt_0^{\text{ff}} = 1.30 \text{ mins and } tt_1^{\text{ff}} = 1.50 \text{ mins}$$

Travel demand: The travel demands over time are here obtained by assuming $A = 12.5$ and $B = 27.5$ in (3.1) which gives a peak demand $A + B = 40$. The total capacity of links 1 and 2 is clearly $26.88 + 19.60 = 46.48$; which is then larger than the peak inflow rate, thus traffic may have to wait at the entry of links 0 or 1 as the traffic assigned to either in a time interval may exceed its capacity. Consequently, for the purposes of congestion charging, the network may be assumed to experience congestion.

3.2 Scenario 2: 4-link, 3-path network

This network has one OD pair and three paths; path 1 is composed of links 0 and 2, path 2 goes along links 1 and 2 and path 3 is link 3. At Node 1, the (inflow) capacity of link 2 is no more than the (outflow) capacity of the links pointing into it (links 0 and 1), which means that the receiving capacity may be lower than the total flows waiting to enter.

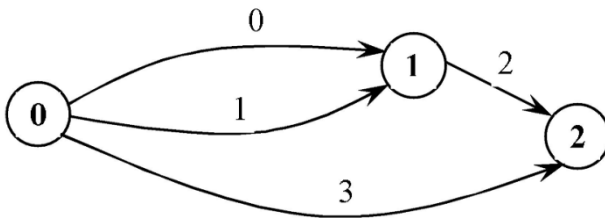


Figure 3: Example network 2

The characteristics of the links in network 2 are as follows

Table 2: Characteristics of links in Network 2

Link l	q_l^{\max} (veh/min)	v_l^f (km/min)	L_l (km)	k_l^c (veh/km)	k_l^j (veh/km)
0	26.88	0.960	1.20	56.00	160.00
1	24.50	0.875	1.05	56.00	160.00
2	100.0	1.250	0.50	160.0	280.00
3	37.50	1.000	2.00	75.00	180.00

So the free-flow travel time for each link is

$$tt_0^{\text{ff}} = 1.25 \text{ mins, } tt_1^{\text{ff}} = 1.20 \text{ mins, } tt_2^{\text{ff}} = 0.40 \text{ mins and } tt_3^{\text{ff}} = 2.00 \text{ mins;}$$

and the free-flow travel time for each path is

$$T_1^{ff} = 1.65 \text{ mins}, T_2^{ff} = 1.60 \text{ mins and } T_3^{ff} = 2.00 \text{ mins};$$

So, path 2 is the shortest among the three, path 1 the second shortest and path 3 has the longest free-flow travel time. The capacity on link-2 is set to be sufficiently large that for the purposes of this example it does not act as a bottleneck, so that the effective capacities of the three paths are defined by their initial links.

Travel demand. Four levels of travel demand were defined, based on the values of the travel demand parameters as given in Table 2.

In Case 0, the demand level equals 40.00, which is greater than the capacity of each of the three paths but less than the sum of the capacities of any two paths. In case 1, the demand level is 55.00, which is greater than the sum of the two smallest capacities of the three paths but less than the sum of the two pairs including path 3. In case 2, the demand level is 68.50, which is greater than the sum of the capacities of any two paths but less than the total capacities of the three paths. Case 4 has a demand level equal to 88.00, which is just fractionally smaller than the total capacity of the three paths.

Table 3: Values of parameters A and B in demand function (3.1)

Parameter	Case			
	0	1	2	3
A	2.50	12.50	12.50	12.50
B	37.50	42.50	66.75	75.50
Peak demand A+B	40.00	55.00	68.50	88.00

4. Numerical analysis

4.1 Scenario 1: 2-link, 2-path network

In the DCC framework (Figure 1), the initial toll scheme is defined as (0, 0); i.e. zero-toll on each path (link). The tolling goal for the 2-link network is to minimise the total network cost whilst assuming the low-revenue tolling assumption that the path which would produce least benefit if tolled under DUE should remain toll-free. Consequently in the 2-path network, path 2 (=link 1) is tolled and path 1 (=link 0) remains toll free. The tolling scheme is updated by incrementally assigning tolls on link 1 until the total network cost (TNTC) is no longer reducing.

The minimising toll value may be seen in figure 4 below, where the lowest value of TNTC (=592.929) occurs at toll = (0, 0.169). The DCC framework process would then terminate at toll = (0, 0.170) and the prior value reported. At toll = (0, 0) TNTC = 602.945; thus the reduction in TNTC achieved by constant tolling is 1.66%.

Clearly, under the stated constraints and objectives and given the special structure of this network, the TNTC given above is a minimal value under minimal-revenue tolls rather than a good sub-optimal. In general however, tolls will have to vary on multiple paths and the determination of tolls to minimise TNTC will be more difficult to demonstrate as being optimal and good sub-optimality will be the stated objective.

4.2 Scenario 2: 4-link, 3-path network

In scenario 2, 4 cases for demand level were determined. These were set to allow paths to experience congestion, but not to render the whole network completely oversaturated. Thus a tolling scheme would be hoped to improve TNTC.

As may be seen from table 4; in all 4 cases, path 2 was determined to be the path on which the benefit of tolling on that path alone would be greatest, path 1 was the second most beneficial, and path 3 in all cases showed no possible reduction in TNTC by levying tolls on that path. Thus in each case, path 3 was assigned zero-toll, and tolls were varied on path 2 for increments of toll on path 1.

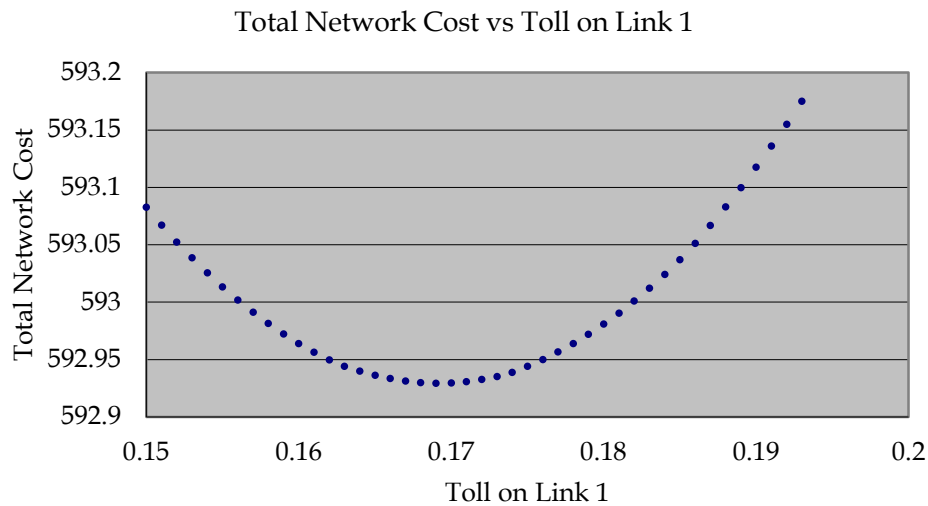


Figure 4: Example network 1 Total Network Travel Cost

Table 4: Minimum Total Network Travel Cost given single path tolled

Toll	Case			
	0	1	2	3
(0, 0, 0)	654.36	1082.59	1513.33	1871.94
(t, 0, 0)	654.21	1081.85	1510.95	1867.59
(0, t, 0)	652.63	1078.05	1505.16	1860.34
(0, 0, t)	654.36	1082.59	1513.33	1871.94

Given the structure of this network, the minimal value for the TNTC may be illustrated graphically as shown on Figures 5-8 below (The toll sets given produce the least TNTC given that the tolls are determined to 2 d.p.)

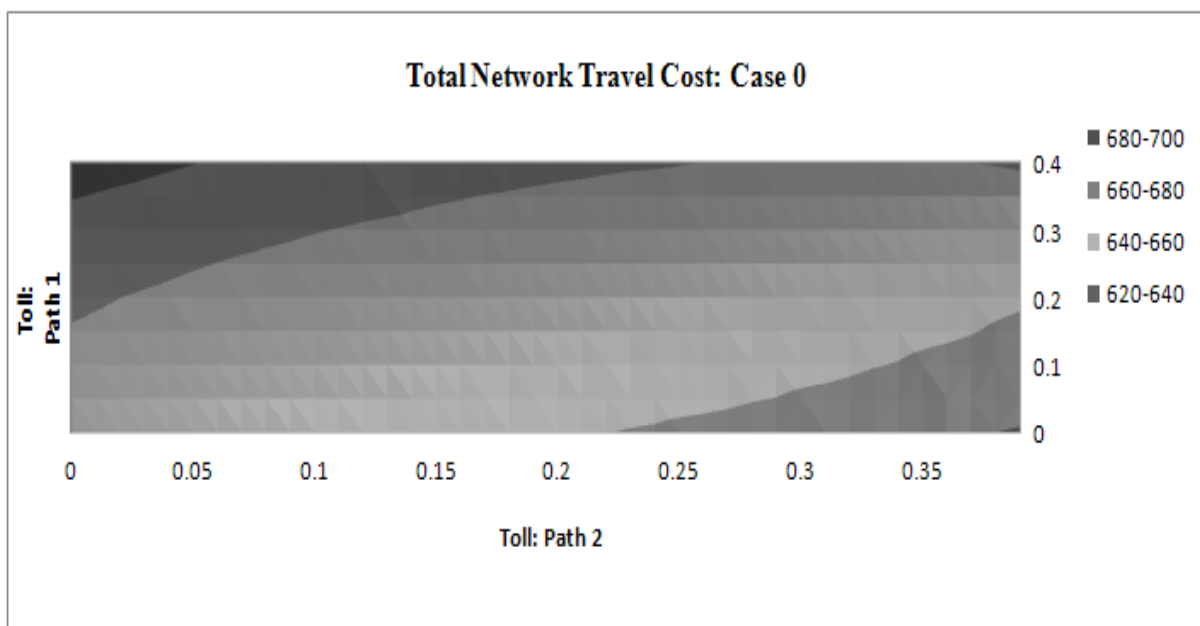


Figure 5: $T = (0.2, 0.23, 0)$ Min TNTC = 643.93 Reduction = 1.59%

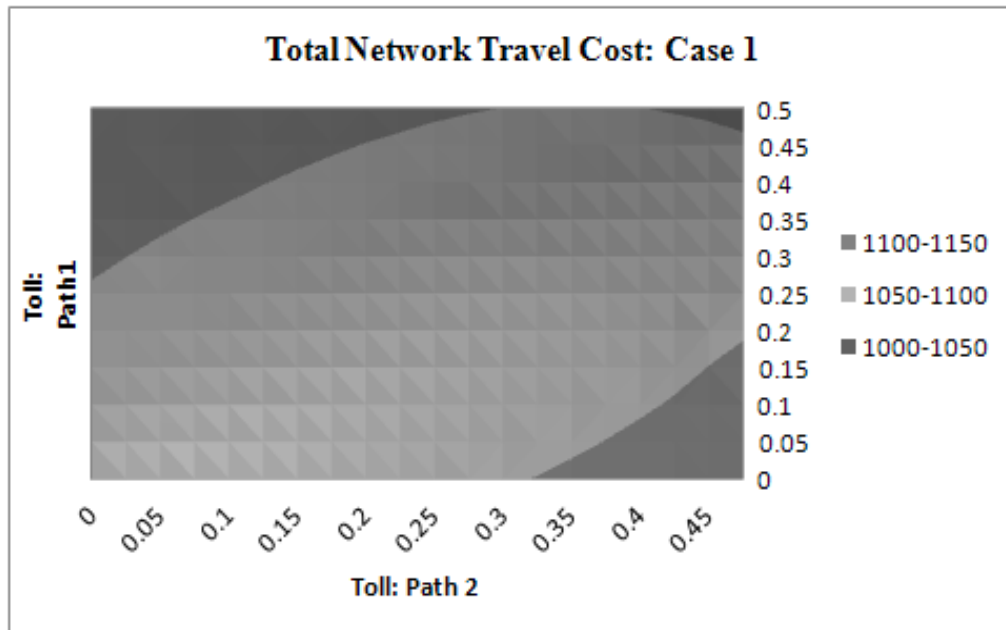


Figure 6: $T = (0.2, 0.25, 0)$ Min TNTC = 1064.22 Reduction = 1.70%

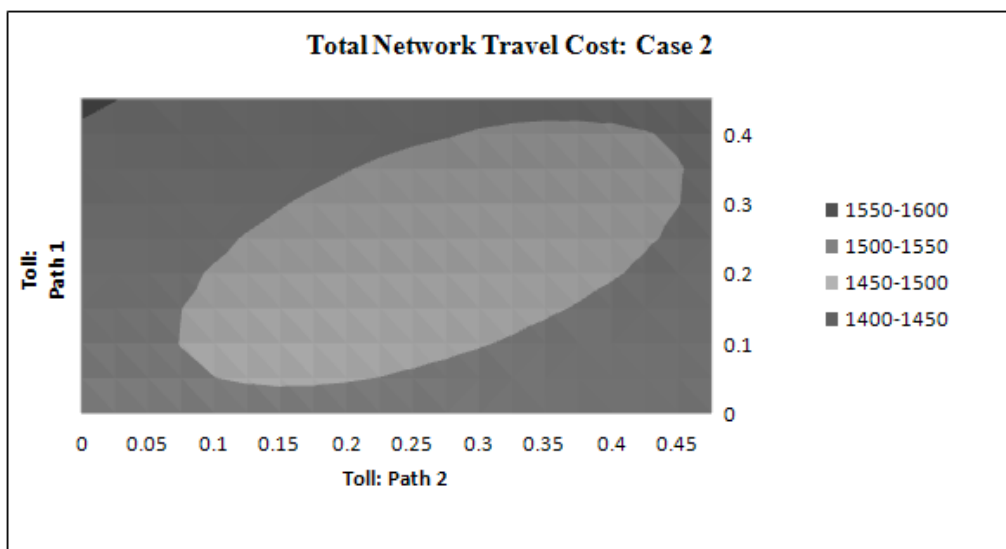


Figure 7: $T = (0.25, 0.275, 0)$ Min TNTC = 1484.44 Reduction = 1.91%

It may be seen that the low-revenue toll set determined in each case is in a relatively similar location irrespective of the demand value, and as the overall demand rises is tending to increase slightly in value in both the tolled paths. This illustrates the relative stability of this tolling scheme which is beneficial in practical terms when the exact value of demand will vary day to day.

The relative reduction achieved in TNTC with respect to the zero-toll case increases with increased demand levels, which is as expected, as a greater network improvement should be possible the higher the levels of congestion. Whilst the possible reductions in TNTC appear small in these numerical examples, these are not indicative of the scale of any real reductions and simply illustrate the comparative cases.

The solutions given here are approximate; it is straightforward to achieve more accurate solutions by reducing the tolling increments when the approximate position of the solution has been ascertained.

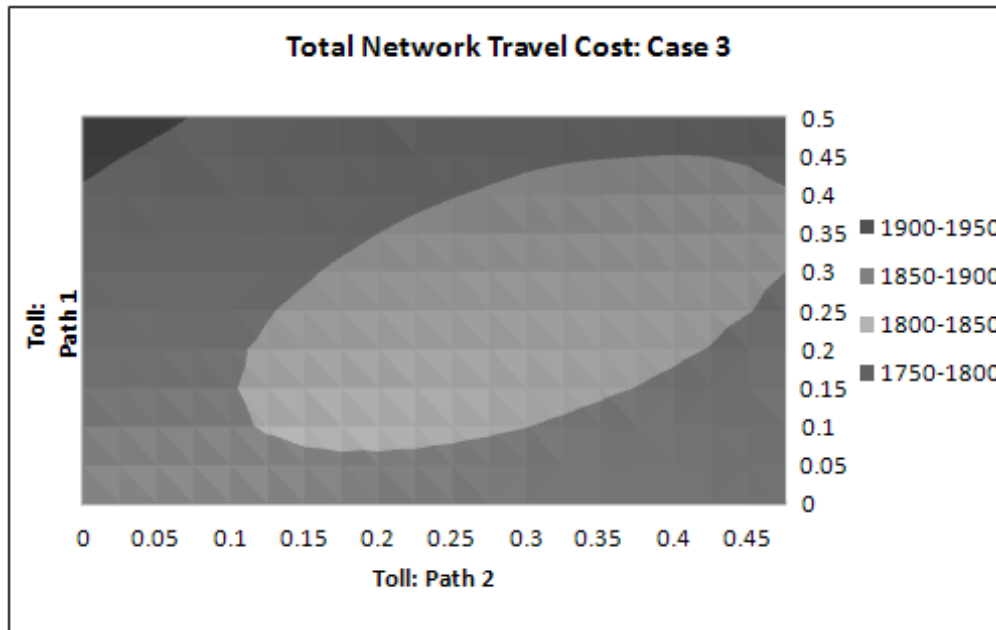


Figure 8: $T = (0.25, 0.3, 0)$ Min TNTC = 1833.91 Reduction = 2.03%

5. Concluding remarks

This paper is intended to illustrate the feasibility of determining low-revenue toll sets to reduce (tending to minimise) the total network cost of a network under DUE and illustrates the relative stability of the derived toll sets relative to scenarios varying overall traffic demand.

A Dynamic Congestion Charging (DCC) Framework has been presented which allows a general tolling scheme to be added onto a standard Dynamic User Equilibrium (DUE) algorithm. Existing algorithms have been utilised for both the dynamic network loading (DNL) and path-flow reassignment parts of DUE. The DNL part has (in this paper) assumed a predefined flow profile whilst the path flow reassignment follows DUE such that for any time t an individual driver may not benefit by changing routes. The results for the numerical examples presented represent preliminary exploratory investigations into the various components of a DCC process. We have assumed a constant path-specific toll level which applies to the whole time period being modelled but which allows all links to take their own constant-toll. The toll level allows us to illustrate clearly the existence of an optimal solution under the given constraints. Tolls determined have shown stable patterns with respect to increasing demand level in that the toll structure is similar with increasing congestion, but that larger tolls need to be levied to combat higher congestion levels. However the benefit of imposing tolls at higher congestion levels is greater, producing greater reductions in total network cost achievable when the demand level increases. We have sought to determine minimal or low-revenue tolls, by assigning one path to always have a zero-toll. Due to the small scale of the networks considered, this has allowed us to determine the position of the optimal tolls easily. If the number of paths in the network were to increase, it is proposed to move through possible paths in turn (based on a hierarchy of tolling benefit) and incrementally increase the tolls path by path to seek a reduction in overall network cost.

Whilst a fully dynamic set of first-best optimal time varying tolls would be seen as an ultimate objective, this research has focussed on determining sub-optimal toll sets subject to certain practical constraints. One such constraint is that the toll profile should be pre-determined (as is often the case in reality due to implementation constraints). Thus future work will utilise work on the toll profiles defined in Ge and Stewart (20010) and Ge et al. (2012), including flat, stepwise-

linear, quadratic and bell-shaped tolling profiles. Cases both where the tolls span the entire time horizon and where they are permitted to be a subset of that time horizon will be considered, as will the effect of varying the arrival time penalty by reducing the size of the arrival time window from the entire period to a specified sub-period.

Finally, in the 4-link 3-path network presented here, the capacity of each path was determined by the first link encountered; allowing bottlenecks to exist at a later position in the network will require full exploration. This would correspond to reducing the capacity of link 2 in the 4-link 3-path network. The model used here was defined to allow tolls to be located only at the entry point to each path; it would be desirable in general to allow tolls to be levied on links rather than on paths.

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