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Efficiency Comparison of Various Parking Charge Schemes Considering Daily Travel Cost in a Linear City

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In this paper, we introduce a new duration dependent parking fee regime based on the travel cost for an entire day, rather than a single commute trip. Commuters are assumed to reside at one end of a linear city and work in a business center at the other end. A two-stage differential method is used to derive user equilibrium travel patterns for both morning and evening rush hour commutes. Both individual travel cost and system travel cost are derived as functions of travel demand. We then compare the efficiency of the duration dependent parking fee regime with that of three previously proposed pricing regimes in the context of elastic travel demand. Results show that a pricing regime with both time-varying road tolls and location dependent parking fees is most efficient, followed by a regime with time-varying road tolls alone. Depending on the parking fee rate, a uniform duration-based parking fee regime may or may not be more efficient than a no-pricing regime. Under the duration dependent parking fee regime, an optimal parking fee rate can be obtained by minimizing system cost or maximizing social surplus, which gives rise to a system-wide performance no worse than that in the no-pricing regime.

Keyword: parking, daily commute, Nash equilibrium, user equilibrium, road toll, parking fee

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1. Introduction

In many countries car ownership levels show an increasing trend, both at the national level and in big cities. As a result, demand for parking space in city centers increasingly exceeds supply. High costs, lack of available land, and livability or environmental concerns make it difficult for municipalities to build their way out of the problem with new parking lots. Therefore, they must look to efficiently manage current parking supply. Various aspects of parking management have been investigated in the past. Verhoef et al. (1995) and Bifulco (1993) studied parking policies of governments. Hester et al. (2002) analyzed downtown parking behavior. Lam et al. (1998) examined how economic issues affect parking demand. Arnott et al. (1991), Glazer and Niskanen (1992), Miller and Everett (1982), Shoup (1982), and Arnott and Rowse (1999) investigated the parking price optimization problem. Shoup (2006) developed a model of evaluating how drivers choose whether to cruise or to pay, and predicted several practically meaningful results under different parking charge schemes.

Recently, Zhang et al. (2008) presented an integrated analysis of downtown parking that considered commuting behavior, bottleneck dynamics and static parking fee schemes. Zhang et al. (2008) examined three different parking fee and road toll schemes where parking fee is location-dependent and invariant upon time. The schemes were labeled, respectively, regime f—no pricing (both morning and evening commutes are free of road tolls and parking fees); regime r—optimal road tolls are charged to both morning and evening commutes; and regime o—optimal time-varying road tolls and location dependent parking fees. These pricing scheme models provide a good basis for investigating the efficiency of various parking management practices.

One limitation of the models in Zhang et al. (2008) is that the parking fee is static, and does not account for the temporal aspects of parking. To amend this limitation, in this paper we introduce a duration-dependent parking fee scheme that we call regime u. Our model integrates the travel costs of both morning and evening commutes and sets a duration-dependent parking fee and no charge for road use during either morning or evening commutes. We evaluate the efficiency of the model, in terms of social cost or social surplus by comparing it with the schemes proposed by Zhang et al.

The remainder of this paper is organized as follows. Section 2 describes the assumptions made in the models of this paper. Section 3 derives the travel patterns in the regime of a duration dependent parking fee. In Section 4, we compare the efficiency of the duration dependent parking fee regime with that of three previously proposed pricing regimes in the context of elastic travel demand. Analytical results of the models are given in Section 5. Section 6 concludes the paper with extended remarks as well as practical suggestions for parking management policy.

2. Model assumptions

As in Zhang et al. (2008), the basic modeling approach used in our study is dynamic traffic assignment. Following the past work of Vickrey (1969), Hurdle (1981), Fargier (1983), and de Palma and Lindsey (2002), we examine the situation of commuters travelling to a central business district (CBD), passing through a bottleneck of limited capacity, and encountering congestion delay at the bottleneck. Travelers therefore adjust their departure times to minimize their queuing delay and late/early arrival penalty, and in the equilibrium no one can (strictly) reduce his/her trip cost by changing his /her departure time unilaterally.

As in Zhang et al. (2008), we investigate the daily commute problem in depth by taking into account morning trips, evening return trips, and daytime parking fees at locations near the workplace. First we establish the evening commute pattern given a fixed set of commuter parking locations. We then integrate the travel costs of the evening commute with those of the morning commute into a daily travel cost. Next we analyze the traffic pattern assuming a duration dependent parking fee. Finally, we examine the demand elasticity considering daily commuting costs, and determine the optimal demand level under the pricing regime.

This study is based on a typical network with a single origin-destination pair connected by a traffic corridor of two routes. The origin represents a residential area and the destination a CBD. Only residents living in the origin zone who travel to the CBD for work in daytime and then return after work are considered. Each of the two parallel paths connecting the origin and the destination has a bottleneck with limited capacity, and the travel times on the line haul parts of these two paths are simply ignored. This network is the same as the one used in Zhang et al. (2005, 2008), and is shown in Figure 1.

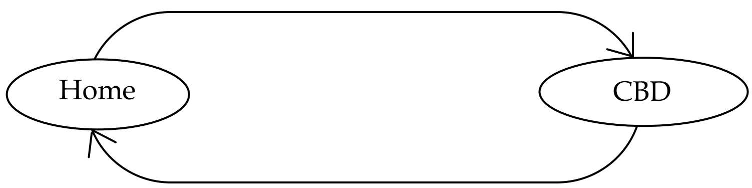


Figure 1. A bi-direction bottleneck network

In Figure 1, the capacity of the bottleneck is $\underrightarrow{s}$ in the home-to-work direction, and $\underleftarrow{s}$ in the work-to-home direction. Note that, to differentiate the morning commute from the evening commute, we use an arrow "$\rightarrow$" to underline some notations associated with the morning commute, and an arrow "$\leftarrow$" for the evening reverse commute. Let N denote the number of commuters who travel from the origin to the destination for work. A parking lot is set around the CBD. The parking spots are treated as continuous variables and indexed by $\underrightarrow{n}$ in order of increasing distance from the CBD. Let $w\underrightarrow{n}$ represent the walking time to the workplace from location $\underset{\rightarrow}{n}$, where w is the time it takes to pass one parking spot on foot. We do not consider the in-vehicle travel time within the parking lot.

Furthermore, let $\underset{\rightarrow}{t}^{*}$ denote the official work starting time, and $\underset{\leftarrow}{t}^{*}$ the official off work time. Symbol $\underset{\rightarrow}{\beta}$ represents the cost of one unit of early arrival time in the morning, and the penalty for late arrival for work is infinite. In the evening, the penalty for early departure from work is infinite, and the cost of one unit late departure time is $\underset{\leftarrow}{\gamma}$. The unit cost of in-vehicle travel time is α (for both morning and evening trips), and the unit cost of walking time is λ. To ensure a deterministic equilibrium it is assumed that $\alpha > \underset{\rightarrow}{\beta}$ and $\alpha > \underset{\leftarrow}{\gamma}$. Since in general people prefer to drive rather than to walk, hence $\lambda > \alpha$.

3. Travel pattern with a duration dependent parking fee

Parking fees are generally duration dependent (either in a continuous way, or in fixed steps of, for example, one hour). A continuous duration parking fee usually equals the parking duration time multiplied by a charging rate μ (Arnott and Rowse, 1999). This section explains how commuters react to a duration-based uniform-rate parking fee and the efficiency of such charging schemes. We model this scheme by segmenting the day into two periods: morning and evening. We assume parking fees are charged based on the parking duration given a uniform parking fee rate μ. Parking costs are assigned to the home-to-work and the work-to-home trips respectively using an intermediate time point t^{Δ}. Time t^{Δ} is positioned between the end of the morning commute and the beginning of the evening commute. A typical t^{Δ} could be the midday time.

3.1 The evening commuting pattern

Since the travel cost encountered before t^{Δ} for each commuter is already determined, this cost does not affect the choice of departure time in the evening commute. Therefore each commuter only minimizes the cost occurs after time t^{Δ}, in the evening commute modeling. The travel cost of an individual who leaves the bottleneck at time $\underset{\leftarrow}{t}$ and parks in parking spot n is

$$\underset{\leftarrow}{C}^{\mathrm{u}}\left(\underset{\leftarrow}{t}, n, \mu\right)=\underset{\leftarrow}{\gamma}\left[\underset{\leftarrow}{t}-\frac{\underset{\leftarrow}{D}(\underset{\leftarrow}{t})}{\underset{\leftarrow}{s}}-wn-\underset{\leftarrow}{t}^{*}\right]+\lambda wn+\alpha\frac{\underset{\leftarrow}{D}(\underset{\leftarrow}{t})}{\underset{\leftarrow}{s}}+\mu\left[\underset{\leftarrow}{t}-\frac{\underset{\leftarrow}{D}(\underset{\leftarrow}{t})}{\underset{\leftarrow}{s}}-t^{\Delta}\right].$$

where $\underleftarrow{D}(\underleftarrow{t})$ is the length of the queue upon the arrival of a commuter who leaves the bottleneck at time $\underleftarrow{t}$.

At the beginning of evening rush hour, since there is no queue, then we have $\frac{d\underleftarrow{D}(\underleftarrow{t})}{dt} \geq 0$.

Therefore, $\frac{\partial \underleftarrow{C}^{u}(\underleftarrow{t}, n, \mu)}{\partial \underleftarrow{t}} > 0$ holds. This means that the travel cost of a commuter increases with the increase of her/his leaving time $\underleftarrow{t}$ from the bottleneck. Intuitively, commuters depart as early as possible to avoid a higher late departure penalty and an increasing queuing delay, Therefore, some commuters depart from the office immediately after $\underleftarrow{t}^{*}$, the official work end time, and those who parked closer to the city center can arrive at the bottleneck earlier simply because they have shorter walking time. In this paper, it is assumed that $w < 1/\underleftarrow{s}$, which means that the arrival rate to the bottleneck is greater than its capacity, and a queue is growing at the entrance of the bottleneck. In such a situation, arrival rate to the bottleneck is $\underleftarrow{r}_{1}^{u}(\mu) = 1/w$.

The commuters who parked far from the city center are willing to wait in the office to avoid the long queue. In other words, a steady state is reached where the travel cost remains unchanged with postponed departure time, namely

$$\frac{\partial \underleftarrow{C}^{u}(\underleftarrow{t}, n, \mu)}{\partial \underleftarrow{t}} = \mu + \underleftarrow{\gamma} - \frac{\underleftarrow{\gamma} + \mu - \alpha}{\underleftarrow{s}} \frac{d\underleftarrow{D}(\underleftarrow{t})}{dt} = 0,$$

which leads to $\frac{d\underleftarrow{D}(\underleftarrow{t})}{dt} = \frac{(\mu + \underleftarrow{\gamma})\underleftarrow{s}}{\underleftarrow{\gamma} + \mu - \alpha}$. Therefore, in the latter situation, the arrival rate is

$$\underleftarrow{r}_{2}^{u}(\mu) = \frac{(\alpha - \mu - \underleftarrow{\gamma})\underleftarrow{s}}{\alpha}.$$

Given the above two arrival rates and in view of the fact that the first and last arrivals encounter no queue, the traffic pattern is as shown in Figure 2. In the figure, *ABC* is the arrival curve to the bottleneck, and *AC* is the departure curve from the bottleneck. At the turning point, $\tilde{k}(\mu) = \frac{(\mu + \underleftarrow{\gamma})N}{\alpha + (\mu + \underleftarrow{\gamma} - \alpha)w\underleftarrow{s}}$, where $\tilde{k}(\mu)$ is the last commuter to leave the office immediately after work. In the figure, the vertical axis is the cumulative curve of commuters arriving at (and/or departing from) the bottleneck (rather than the order of parking spots). For

$k \le \tilde{k}(\mu)$, commuters with closer parking spots depart earlier. But for $k > \tilde{k}(\mu)$, they do not necessarily depart strictly in the order of their parking spots. Instead, their departure times need only generate an arrival pattern to the bottleneck depicted by the curve *BC* in Figure 2. If we let $n(k)$ be the parking spot of the *k*-th departure, the necessary and sufficient condition for $n(k)$ to produce an arrival curve *ABC* in Figure 2 is

$$n(k)\begin{cases} = k & \forall\ k \in [0, \tilde{k}]; \\ \le \dfrac{N}{w\underleftarrow{s}} - \dfrac{\alpha(N-k)}{(\alpha - \underleftarrow{\gamma} - \mu)w\underleftarrow{s}} & \forall\ k \in (\tilde{k}, N]. \end{cases}$$

Obviously there may be multiple solutions satisfying the above constraints, with $n(k) = k$, $\forall\ k \in [0, N]$ being a typical one. Here $n(k) = k$ implies that commuters who park closer to the city center depart earlier in the evening.

If we set $\tilde{n}(\mu) = \tilde{k}(\mu)$, individual travel cost for a commuter with parking spot n in the evening commute is

$$\underleftarrow{C}_n^{\mathrm{u}}(\mu) = \begin{cases} \dfrac{\alpha n}{\underleftarrow{s}} + (\lambda + \mu - \alpha)wn + \mu(\underleftarrow{t}^* - t^\Delta) & \text{if } n < \tilde{n}(\mu); \\ (\lambda - \underleftarrow{\gamma})wn + \dfrac{\underleftarrow{\gamma} + \mu}{\underleftarrow{s}} N + \mu(\underleftarrow{t}^* - t^\Delta) & \text{if } n \ge \tilde{n}(\mu). \end{cases}$$

The evening traffic pattern we've described is a user equilibrium, since no one can reduce their own travel cost by unilaterally changing their departure time given the departure times of other commuters. It is also shown that commuters with different parking spots have different travel costs. A commuter $n \le \tilde{n}(\mu)$ cannot depart any earlier, and departing later will increase her/his travel cost. For a commuter $n > \tilde{n}(\mu)$, arriving at the bottleneck before the queuing peak is impossible, and arriving at the bottleneck after the end of the queue will increase his/her travel cost.

The traffic pattern shown in Figure 2 is a unique user equilibrium solution, since the aggregate arrival rates to the bottleneck before and after the peak queue are uniquely determined. However, for a commuter with parking location $n > \tilde{n}(\mu)$, the departure time is not unique. The departure times of commuters who have not entered the queue before $\underleftarrow{t}^* + wN$ are flexible as long as they can give rise to the aggregate equilibrium arrival rate to the bottleneck. Furthermore, the flexibility of departure time increases as the parking spot moves from the city center to a more distant location.

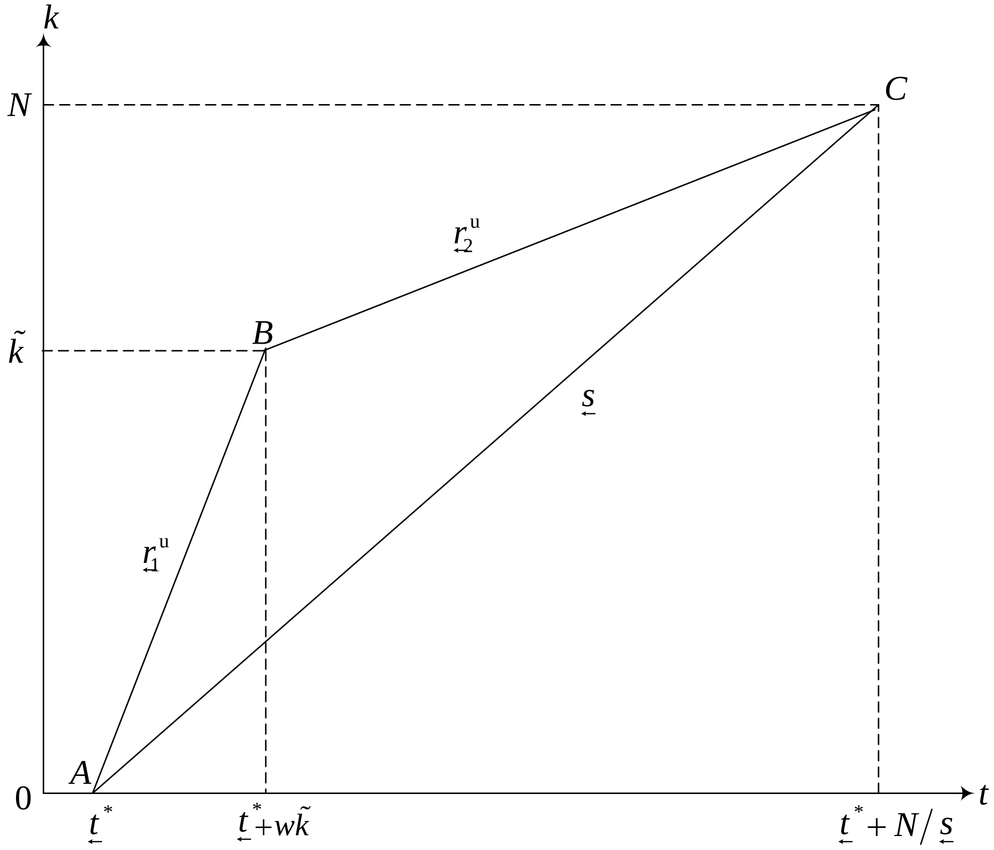


Figure 2. Traffic pattern in the evening commute, regime u

To sum up, the cumulative arrival curve to the bottleneck in the evening with duration dependent parking fee is represented in Figure 2. Although the arrival curves follow a pattern similar to regime f discussed in Zhang et al. (2008), the values of the arrival rates here are different. With the results derived in this section, the travel cost of the evening trip can be calculated for each commuter, given her/his parking location.

3.2 The morning commuting pattern

In the previous subsection, it was found that individual travel costs in the evening differ because the parking locations commuters chose in the morning affected their positions in the evening commute competition. By contrast, in the morning, we assume commuters are a priori identical since they have free choice of departure times. This raises a question: does the assumption of identical travelers in fact yield a user equilibrium traffic pattern in the morning and an equalized daily travel cost, if each commuter considers her/his travel cost for the entire day when choosing a home-to-work departure time? The answer is given as follows.

In the morning, commuters consider both home-to-work and work-to-home travel costs in choosing departure time and parking location. By attaching the evening travel cost $\underleftarrow{C}_n^u(\mu)$ to parking spot n, individual travel cost for a commuter who leaves the bottleneck at time $\underrightarrow{t}$ in the morning and parks at spot n is

$$C^{\mathrm{u}}\left(\underrightarrow{t}, n, \mu\right) = \alpha \frac{\underrightarrow{D}\left(\underrightarrow{t}\right)}{\underrightarrow{s}} + \lambda wn + \underrightarrow{\beta}\left(\underrightarrow{t}^{*} - \underrightarrow{t} - wn\right) + \mu\left(t^{\Delta} - \underrightarrow{t}\right) + \underleftarrow{C}_n^u\left(\mu\right).$$

For $n < \tilde{n}(\mu)$, we have

$$C^{\mathrm{u}}\left(\underrightarrow{t}, n, \mu\right) = \alpha \frac{\underrightarrow{D}\left(\underrightarrow{t}\right)}{\underrightarrow{s}} + 2\lambda wn + \underrightarrow{\beta}\left(\underrightarrow{t}^{*} - \underrightarrow{t} - wn\right) + \mu\left(\underleftarrow{t}^{*} - \underrightarrow{t}\right) + \frac{\alpha n}{\underleftarrow{s}} - \alpha wn,$$

and hence

$$\frac{\partial C^{\mathrm{u}}\left(\underrightarrow{t}, n, \mu\right)}{\partial n} = \left(2\lambda - \alpha - \underrightarrow{\beta} + \mu\right)\mathrm{w} + \alpha / \underleftarrow{s} > 0.$$

For $n \geq \tilde{n}(\mu)$, we have

$$C^{\mathrm{u}}\left(\underrightarrow{t}, n, \mu\right) = \alpha \frac{\underrightarrow{D}\left(\underrightarrow{t}\right)}{\underrightarrow{s}} + \underrightarrow{\beta}\left(\underrightarrow{t}^{*} - \underrightarrow{t} - wn\right) + \mu\left(\underleftarrow{t}^{*} - \underrightarrow{t}\right) + \left(2\lambda - \underleftarrow{\gamma}\right)wn + \frac{\underleftarrow{\gamma} + \mu}{\underleftarrow{s}} N,$$

which gives us

$$\frac{\partial C^{\mathrm{u}}\left(\underrightarrow{t}, n, \mu\right)}{\partial n} = \left(2\lambda - \underrightarrow{\beta} - \gamma\right)\mathrm{w} > 0.$$

Therefore, a commuter intends to park as close as possible to the city center, after leaving the bottleneck.

Since the evening travel cost $\underleftarrow{C}_n^u(\mu)$ has a different formula for $n \leq \tilde{n}(\mu)$ and $n > \tilde{n}(\mu)$, traffic equilibrium has to be derived separately for the two situations. Firstly we look at the situation of $n \leq \tilde{n}(\mu)$. Whether a queue occurs or not depends on the relative benefits of occupying a convenient parking location and arriving close to the official work start time. If occupying a convenient parking spot is more attractive than having a close arrival time, commuters will depart from home very early to compete for close parking spots and thus push forward the start of the rush hour. In such a situation, no queue occurs at the bottleneck. On the other hand, if arriving close to work start time is more attractive than occupying a close parking spot, then the rush hour concentrates tightly before the official work start time and a queue occurs.

If a queue occurs, the user equilibrium condition $\frac{dC^{\mathrm{u}}\left(\underset{\rightarrow}{t},n,\mu\right)}{d\underset{\rightarrow}{t}}=$ =0 leads to

$$\frac{d\underset{\rightarrow}{D}\left(\underset{\rightarrow}{t}\right)}{d\underset{\rightarrow}{t}}=\frac{\underset{\rightarrow}{s}}{\alpha}\left(\underset{\rightarrow}{\beta}+\mu-\frac{\alpha\underset{\rightarrow}{s}}{\underset{\leftarrow}{s}}+\left(\alpha+\underset{\rightarrow}{\beta}-\mu-2\lambda\right)w\underset{\rightarrow}{s}\right)$$

Therefore the prerequisite of a queue is $\frac{d\underset{\rightarrow}{D}\left(\underset{\rightarrow}{t}\right)}{d\underset{\rightarrow}{t}}$ >0, i.e. $\frac{\underset{\rightarrow}{s}}{\underset{\leftarrow}{s}}<\frac{\underset{\rightarrow}{\beta}+\mu}{\left(2\lambda+\mu-\alpha-\underset{\rightarrow}{\beta}\right)w\underset{\leftarrow}{s}+\alpha}$, which requires that the capacity of the home-to-work bottleneck be much smaller than the capacity of the work-to-home bottleneck intuitively. Since in real life bottlenecks have comparable capacities during morning and evening rush hours, we assume $\frac{\underset{\rightarrow}{s}}{\underset{\leftarrow}{s}}>\frac{\underset{\rightarrow}{\beta}+\mu}{\left(2\lambda+\mu-\alpha-\underset{\rightarrow}{\beta}\right)w\underset{\leftarrow}{s}+\alpha}$. In other words, we assume $\frac{d\underset{\rightarrow}{D}\left(\underset{\rightarrow}{t}\right)}{d\underset{\rightarrow}{t}}$ =0, when $n\le\tilde{n}(\mu)$.

By setting $\frac{\partial C^{\mathrm{u}}\left(\underset{\rightarrow}{t},n,\mu\right)}{\partial\underset{\rightarrow}{t}}=0$, the departure rate is given by

$$\underset{\rightarrow}{r}_1^{\mathrm{u}}(\mu)=\frac{\left(\underset{\rightarrow}{\beta}+\mu\right)\underset{\leftarrow}{s}}{\left(2\lambda+\mu-\alpha-\underset{\rightarrow}{\beta}\right)w\underset{\leftarrow}{s}+\alpha}.$$

Now we investigate the traffic equilibrium when $n\ge\tilde{n}(\mu)$. Two situations are considered, depending on the occurrence of a queue. The condition for occurrence of a queue at the bottleneck is $\underset{\rightarrow}{\beta}+\mu-\left(2\lambda-\beta-\underset{\leftarrow}{\gamma}\right)w\underset{\rightarrow}{s}$ >0. By contrast, if $\underset{\rightarrow}{\beta}+\mu-\left(2\lambda-\beta-\underset{\leftarrow}{\gamma}\right)w\underset{\rightarrow}{s}$ $\le$0, no queue occurs.

We denote this situation of queuing as regime u(a). Let $\frac{\partial C^{\mathrm{u}}\left(\underset{\rightarrow}{t},n,\mu\right)}{\partial\underset{\rightarrow}{t}}=0$, the corresponding departure rate from home is

$$\underset{\rightarrow}{r}_2^{\mathrm{ua}}(\mu)=\frac{\alpha\underset{\rightarrow}{s}}{\alpha-\underset{\rightarrow}{\beta}-\mu+\left(2\lambda-\underset{\rightarrow}{\beta}-\underset{\leftarrow}{\gamma}\right)w\underset{\rightarrow}{s}}.$$

Assuming the last commuter's arrival time at work is $\underset{\rightarrow}{t}^{*}$, we have the traffic pattern in regime u(a) as depicted in Figure 3. In the figure, ABC is the arrival curve to the bottleneck, ABF is the departure curve from the bottleneck, and ADE is the arrival curve to the workplace. Applying the two departure rates $\underrightarrow{r}_1^{u}(\mu)$ and $\underrightarrow{r}_2^{ua}(\mu)$, we can compute the individual travel cost in regime u(a) by

$$TC^{ua}(\mu)=(\beta+\mu)\left(w+\frac{1}{\underrightarrow{s}}\right)N+\frac{(\underleftarrow{\gamma}+\mu)N}{\alpha+(\underleftarrow{\gamma}+\mu-\alpha)w\underrightarrow{s}}\left[\frac{\alpha}{\underleftarrow{s}}+(2\lambda+\mu-\alpha-\beta)w-\frac{\beta+\mu}{\underrightarrow{s}}\right]+\mu\left(\underleftarrow{t}^{*}-\underrightarrow{t}^{*}\right)$$

.

And system travel cost, including queuing delay and unpunctuality cost, is

$$SC^{ua}(\mu)=\lambda wN^2+\frac{(\alpha\tilde{n}+\underleftarrow{\gamma}N-\underleftarrow{\gamma}\tilde{n})(1-w\underleftarrow{s})N}{2\underleftarrow{s}}+\frac{\beta}{2}\left(wN^2+\frac{N^2-\tilde{n}^2}{\underrightarrow{s}}+\frac{\tilde{n}^2}{\underrightarrow{r}_1^{ua}}\right)$$
$$+\frac{\alpha}{2}(N-\tilde{n})^2\left(\frac{1}{\underrightarrow{s}}-\frac{1}{\underrightarrow{r}_2^{ua}}\right).$$

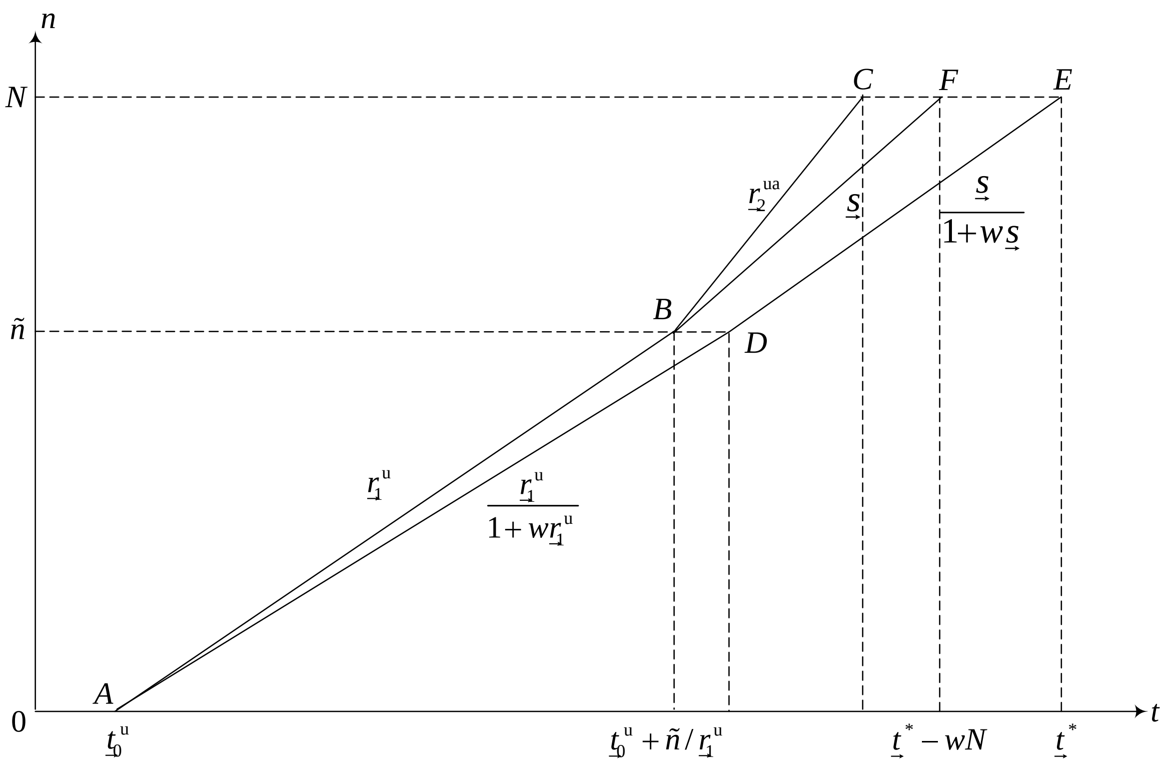


Figure 3. User equilibrium traffic pattern in the morning commute, regime u(a)

The situation without queue is denoted as regime u(b). In this regime, with $\frac{\partial C^{u}(\underrightarrow{t},n,\mu)}{\partial \underrightarrow{t}}=0$, the departure rate is

$$\underleftarrow{r}_2^{\mathrm{ub}}(\mu) = \frac{\underrightarrow{\beta} + \mu}{\left(2\lambda - \underrightarrow{\beta} - \underleftarrow{\gamma}\right) w}.$$

The traffic pattern of regime u(b) is depicted in Figure 4. In the figure, ABC is the cumulative arrival and departure curve of the bottleneck, and ADE is the cumulative arrival curve at the workplace. In regime u(b), individual travel cost is given by

$$TC^{\mathrm{ub}}(\mu) = \left(2\lambda + \mu - \underleftarrow{\gamma}\right) wN + \frac{\left(\underleftarrow{\gamma} + \mu\right) N}{\underleftarrow{s}} + \mu\left(\underleftarrow{t}^* - \underrightarrow{t}^*\right),$$

and system travel cost is

$$SC^{\mathrm{ub}}(\mu) = \lambda w N^2 + \frac{\left(\alpha\tilde{n} + \underleftarrow{\gamma} N - \underleftarrow{\gamma}\tilde{n}\right)\left(1 - w\underleftarrow{s}\right) N}{2\underleftarrow{s}} + \frac{\beta}{2}\left(wN^2 + \frac{N^2 - \tilde{n}^2}{\underleftarrow{r}_2^{\mathrm{ub}}} + \frac{\tilde{n}^2}{\underleftarrow{r}_1^{\mathrm{ub}}} \right).$$

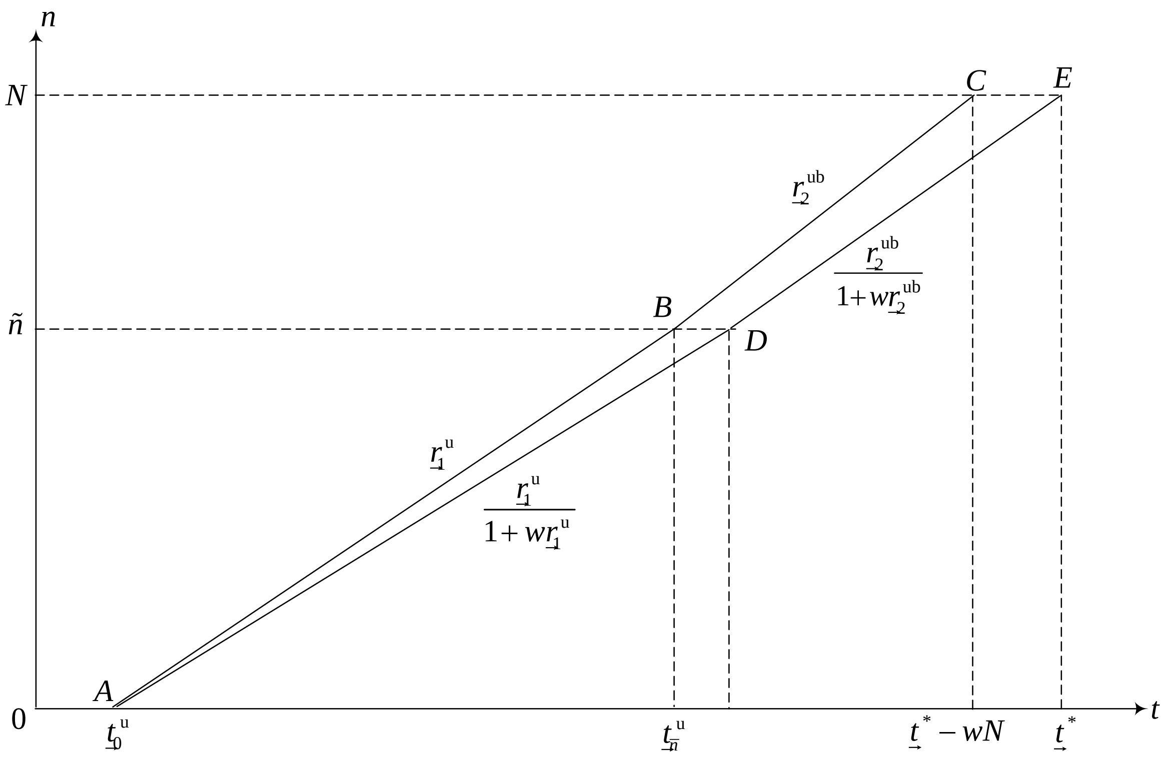


Figure 4. User equilibrium traffic pattern in the morning commute, regime u(b)

The departure curves in the morning also follow a pattern similar to Zhang et al.(2008)'s regime f, but with different departure rates. Note that in the morning rush hour, the traffic pattern follows Wardrop's user equilibrium, since everyone has an equal travel cost and no one can reduce her/his travel cost by unilaterally changing commuting decision.

In the above analysis, the parking fee rate is set freely. Actually we can compute the optimal fee rate after working out the function of daily travel cost. Let $\mu*$ denote the optimal parking fee with minimal system travel cost. Obviously $SC^{u}(\mu*) \geq SC^{u}(\mu)\big|_{\mu=0}$, which means the optimally chosen parking fee is at least as efficient as a regime with no parking fee.

4. Elastic travel demand considering daily travel cost

In existing analyses of travel demand elasticity, the number of commuters is only a function of the costs of a single trip, specifically the morning commute. But in reality, people make decisions about whether or not to make a trip and about mode of travel and departure times based on the commuting cost of the entire day. That is, before a person departs from home to work in the morning, she/he probably considers the commuting cost of the evening trip back, as well as the parking cost during the work period. In this section, we present an analysis of demand elasticity that accounts for *daily* travel cost, comprising the cost of two commutes and parking, rather than the cost of the morning commute alone.

The demand function for travel is assumed to be

$$N = D(P),\ \frac{dD}{dP} < 0,$$

where N is the number of commuters, and P is the private daily travel cost. The inverse demand function is denoted by $P = D^{-1}(N)$, which is assumed to be strictly decreasing with respect to demand N.

A commuter's surplus from travel with demand N is

$$CS(N) = \int_0^N D^{-1}(x)dx - ATC(N)N,$$

where the first term is the total gross benefit from commuting and the second term is the total user cost. The total user cost consists of total social cost and total revenue, i.e.,

$$ATC(N)N = SC(N) + R(N).$$

The social surplus from travel is the sum of the commuter's surplus and the total revenue, resulting in

$$SS(N) = \int_0^N D^{-1}(x)dx - SC(N),$$

where $SS(N)$ represents the social surplus under travel demand N.

Maximizing the social surplus leads to the following optimality condition for demand

$$N = D\left(MSC\left(N\right)\right),$$

where $MSC\left(N\right)$ stands for the marginal social cost defined below

$$MSC\left(N\right) = \frac{dSC\left(N\right)}{dN}.$$

In Figure 5, curve *DF* represents the inverse demand function, curve *ATC* is the average daily travel cost function, and curve *MSC* is the marginal social cost function. In the absence of road toll, a market equilibrium appears at point *E*. The above analysis shows that the optimal equilibrium should be at point *S* for maximizing the social surplus, i.e., the point at which the *MSC* and *DF* curves intersect. In other words, the optimal demand should be N_S, instead of N_E.

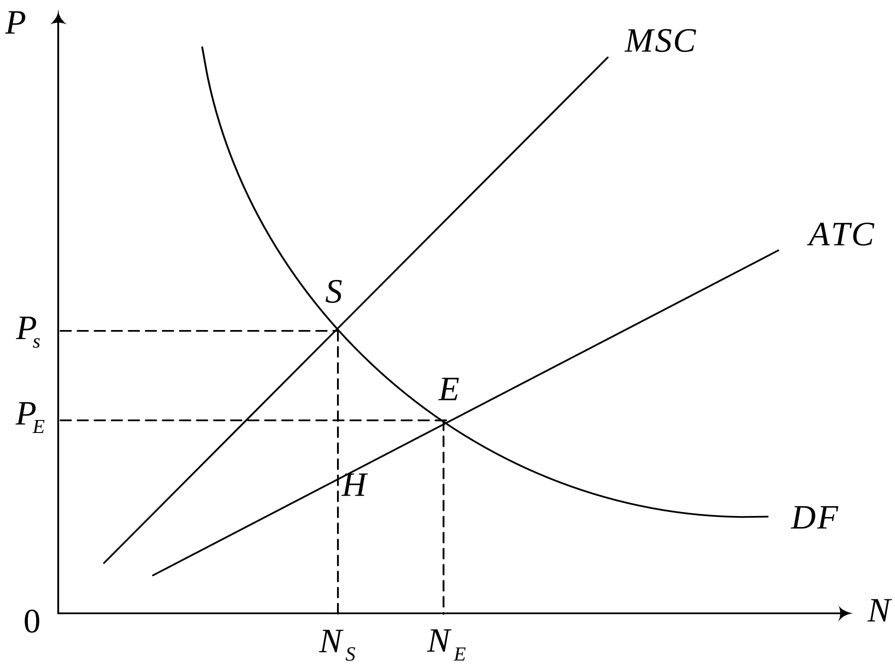


Figure 5. Demand and supply of daily commuting

We now apply our elastic demand case to evaluate four different parking pricing regimes. The first regime, presented earlier in this paper, is regime u—duration-based uniform parking fee (a temporally uniform parking fee is set but both morning and evening commutes are free of charge). The three other regimes, discussed in Zhang et al. (2008), are regime f—without pricing (both morning and evening commutes are free of road tolls and parking fees); regime r—optimal road tolls are charged to both morning and evening commutes; and regime o—optimal time-varying road tolls and location dependent parking fees. From the analytical results in Zhang et al. (2008) and section 3 of this paper, a commuter's daily travel cost is a linear function of demand. In other words, we can formulate the daily travel cost as $ATC^{\kappa}\left(N\right)=I^{\kappa}+K^{\kappa}N$ for a specific pricing regime κ, $\kappa=$f, r, u or o. Here $I^{\mathrm{f}}=I^{\mathrm{r}}=I^{\mathrm{o}}=0$, $I^{u}(\mu)=\mu\left(\underset{\leftarrow}{t}^{*}-\underset{\rightarrow}{t}^{*}\right)$, and the parameter K^{κ} takes the following values:

$$K^{\mathrm{f}}=\begin{cases}\dfrac{\beta}{\underset{\rightarrow}{s}}+w\beta+\dfrac{\gamma}{\alpha-\alpha w\underset{\leftarrow}{s}+\gamma w\underset{\leftarrow}{s}}\left(\dfrac{\alpha}{\underset{\leftarrow}{s}}-\dfrac{\beta}{\underset{\rightarrow}{s}}+2\lambda w-\alpha w-\beta w\right) & \text{for regime f(a);}\\ w\left(2\lambda-\underset{\leftarrow}{\gamma}\right)+\dfrac{\underset{\leftarrow}{\gamma}}{\underset{\leftarrow}{s}} & \text{for regime f(b).}\end{cases}$$

for regime f(a) and

$$K^{\mathrm{r}}=\begin{cases}\underset{\rightarrow}{\beta}\left(w+\dfrac{1}{\underset{\rightarrow}{s}}\right)+\dfrac{\gamma}{\underset{\leftarrow}{s}} & \text{if } \underset{\rightarrow}{\beta}-\left(2\lambda-\underset{\leftarrow}{\gamma}-\underset{\rightarrow}{\beta}\right)w\underset{\rightarrow}{s}\geq 0;\\ \underset{\rightarrow}{\beta}\left(w+\dfrac{1}{\underset{\rightarrow}{s}}\right)+\dfrac{\gamma}{\underset{\leftarrow}{s}}+\left(2\lambda-\underset{\leftarrow}{\gamma}-\underset{\rightarrow}{\beta}\right)w\underset{\rightarrow}{s}-\underset{\rightarrow}{\beta} & \text{if } \underset{\rightarrow}{\beta}-\left(2\lambda-\underset{\leftarrow}{\gamma}-\underset{\rightarrow}{\beta}\right)w\underset{\rightarrow}{s}<0.\end{cases}$$

$$K^{\mathrm{o}}=\left(2\lambda-\underset{\rightarrow}{\beta}-\underset{\leftarrow}{\gamma}\right)w+\frac{\underset{\rightarrow}{\beta}}{\underset{\rightarrow}{s}}+\frac{\gamma}{\underset{\leftarrow}{s}},$$

$$K^{\mathrm{u}}(\mu)=\begin{cases}\left(\beta+\mu\right)\left(w+\dfrac{1}{\underset{\rightarrow}{s}}\right)+\left(\dfrac{\underset{\leftarrow}{\gamma}+\mu}{\underset{\leftarrow}{s}}\right)\dfrac{\alpha+\left(2\lambda+\mu-\alpha-\beta\right)w\underset{\leftarrow}{s}-\beta+\mu}{\alpha+\left(\underset{\leftarrow}{\gamma}+\mu-\alpha\right)w\underset{\leftarrow}{s}} & \text{for regime u(a);}\\ \left(2\lambda+\mu-\underset{\leftarrow}{\gamma}\right)w+\dfrac{\underset{\leftarrow}{\gamma}+\mu}{\underset{\leftarrow}{s}} & \text{for regime u(b).}\end{cases}.$$

In market equilibrium, hence, the implemented demand N_E can be obtained by solving the two equations representing demand function $N=D\left(P\right)$ and cost function $P=K^{\kappa}N$, respectively.

We also find that the total social cost is proportional to the square of the demand, i.e., $SC^{\kappa}\left(N\right)=L^{\kappa}N^{2}$ for a specific pricing regime κ, $\kappa=$f, r, u or o. The parameter L^{κ} takes the following values

$$L^{\mathrm{f}}=\begin{cases}\dfrac{\beta}{\underset{\rightarrow}{s}}+w\beta+\dfrac{\gamma}{\alpha-\alpha w\underset{\leftarrow}{s}+\gamma w\underset{\leftarrow}{s}}\left(\dfrac{\alpha}{\underset{\leftarrow}{s}}-\dfrac{\beta}{\underset{\rightarrow}{s}}+2\lambda w-\alpha w-\beta w\right) & \text{for regime f(a);}\\ w\left(2\lambda-\underset{\leftarrow}{\gamma}\right)+\dfrac{\underset{\leftarrow}{\gamma}}{\underset{\leftarrow}{s}} & \text{for regime f(b).}\end{cases},$$

$$L^{\mathrm{r}} = \lambda w + \frac{\underleftarrow{\gamma}}{2}\left(\frac{1}{\underleftarrow{s}} - w\right) + \frac{\underrightarrow{\beta}}{2}\left(w + \frac{1}{\underrightarrow{s}}\right)$$

$$L^{\mathrm{o}} = \lambda w + \frac{\gamma}{2}\left(\frac{1}{\underleftarrow{s}} - w\right) + \frac{\beta}{2}\left(\frac{1}{\underrightarrow{s}} - w\right),$$

$$L^{\mathrm{u}}(\mu) = \begin{cases} \lambda w + \dfrac{\left(\alpha\tilde{m} + \underleftarrow{\gamma} - \underleftarrow{\gamma}\tilde{m}\right)\left(1 - w\underleftarrow{s}\right)}{2\underleftarrow{s}} + \dfrac{\beta}{2}\left(w + \dfrac{1-\tilde{m}^2}{\underrightarrow{s}} + \dfrac{\tilde{m}^2}{\underrightarrow{r}_1^{\mathrm{ua}}}\right) & \\ \qquad + \dfrac{\alpha}{2}\left(1-\tilde{m}\right)^2\left(\dfrac{1}{\underrightarrow{s}} - \dfrac{1}{\underrightarrow{r}_2^{\mathrm{ua}}}\right) & \text{for regime u(a);} \\ \lambda w + \dfrac{\left(\alpha\tilde{m} + \underleftarrow{\gamma} - \underleftarrow{\gamma}\tilde{m}\right)\left(1 - w\underleftarrow{s}\right)}{2\underleftarrow{s}} + \dfrac{\beta}{2}\left(w + \dfrac{1-\tilde{m}^2}{\underrightarrow{r}_2^{\mathrm{ub}}} + \dfrac{\tilde{m}^2}{\underrightarrow{r}_1^{\mathrm{ub}}}\right) & \text{for regime u(b).} \end{cases} \qquad \left(\tilde{m} = \frac{\tilde{n}(\mu)}{N}\right)$$

The marginal social cost becomes $MSC^{\kappa}\left(N\right) = 2L^{\kappa}N$; thus optimal demand N_S can be obtained by simultaneously solving demand function $N = D\left(P\right)$ and cost function $P = 2L^{\kappa}N$.

To drive the demand from N_E to N_S (i.e., from market equilibrium to optimal equilibrium), each individual must be charged in such a way that the externalities are internalized: by the difference between marginal social cost and marginal private cost. In pricing regime κ, κ =f, r, u or o, the externality caused by an additional individual commuter is

$$E^{\kappa} = MSC^{\kappa}\left(N_S^{\kappa}\right) - ATC^{\kappa}\left(N_S^{\kappa}\right) = \left(2L^{\kappa} - K^{\kappa}\right)N_S^{\kappa} - I^{\kappa},$$

where N_S^{κ} is the optimal demand level in region κ.

In Figure 5, SH represents the externality. Note that the purpose and implications of the externality-based charge here is different from the previously derived road tolls and parking fees. The externality-based charge balances demand and supply to the optimal level, whereas the road tolls and parking fees derived earlier serve to eliminate queues and minimize schedule delays. Furthermore, the externality is anonymous to everyone, so it can be implemented by charging each commuter an additional constant parking fee (separate from the location and duration dependent parking fees), or a road toll. Additional policy measures can be taken to cover this externality, such as license fees, gasoline tax, and so on. Note that in regime o, the externality E^{o} =0, which means that the time-varying road toll and location dependent parking fee automatically lead to an optimal demand level.

It is not difficult to prove $L^{o} < L^{r}$, $L^{r} < L^{u}(\mu)$, and $L^{r} < L^{f}$;therefore, in the case of fixed demand the best pricing policy is regime *o*, followed by *r*. The efficiency comparison between regime f and regime u differs from case to case. In the case of elastic demand (with the same demand function), suppose the optimal demands are N_S^{o}, $N_S^{r}, N_S^{f}, N_S^{u}(\mu)$ in regimes o, r, f, and u respectively, giving us $SS^{o}\left(N_S^{o}\right) > SS^{r}\left(N_S^{r}\right)$, $SS^{r}\left(N_S^{r}\right) > SS^{u}\left(N_S^{u}(\mu), \mu\right)$, and $SS^{r}\left(N_S^{r}\right) > SS^{f}\left(N_S^{f}\right)$. This means that if demand is adjusted to the corresponding optimal level for each pricing regime, the efficiency of these regimes ranks in the same order as in the case of fixed demand. This is proved below. With the same demand function, it is obvious that $N_S^{o} > N_S^{r}$, $N_S^{r} > N_S^{u}(\mu)$, and $N_S^{r} > N_S^{f}$. For regime *o*, the optimal demand is solved from equation $D^{-1}\left(N_S^{o}\right) = 2L^{o} N_S^{o}$. Since $D^{-1}\left(\cdot\right)$ is strictly decreasing, then

$$\int_{N_S^{r}}^{N_S^{o}} D^{-1}\left(x\right)dx > 2L^{o} {N_s^{o}}^2 - 2L^{o} N_s^{o} N_s^{r}$$

and

$$2L^{o} {N_s^{o}}^2 - 2L^{o} N_s^{o} N_s^{r} = L^{o}(N_s^{o} - N_s^{r})^2 + L^{o}({N_s^{o}}^2 - {N_s^{r}}^2),$$

hence

$$\int_{N_S^{r}}^{N_S^{o}} D^{-1}\left(x\right)dx > L^{o}\left({N_S^{o}}^2 - {N_S^{r}}^2\right).$$

Therefore, we obtain

$$SS^{o}\left(N_S^{o}\right) - SS^{r}\left(N_S^{r}\right) = \int_{N_S^{r}}^{N_S^{o}} D^{-1}\left(x\right)dx - L^{o}\left({N_S^{o}}^2 - {N_S^{r}}^2\right) + \left(L^{r} - L^{o}\right){N_S^{r}}^2 > 0 .$$

Similarly, we have $SS^{r}\left(N_S^{r}\right) > SS^{u}\left(N_S^{u}(\mu), \mu\right)$ and $SS^{r}\left(N_S^{r}\right) > SS^{f}\left(N_S^{f}\right)$.

Therefore, in terms of social surplus maximization, the pricing regime with both time-varying road tolls and location dependent parking fees is the best. The second best is the pricing regime with time-varying road tolls alone. The duration-dependent uniform parking fee regime is less efficient than these two regimes, and may or may not be more efficient than the no-pricing regime, depending on the rate of the parking fee. For the duration- dependent parking fee regime, actually an optimal rate of parking fee can be chosen by maximizing $SS^{u}\left(N_S^{u}(\mu), \mu\right)$. Let $\mu*$ denote the optimal parking fee with maximal social surplus. Obviously $SS^{u}\left(N_S^{u}(\mu*), \mu*\right) \geq SS^{u}\left(N_S^{u}(\mu), \mu\right)\Big|_{\mu=0}$, which means the optimal duration-dependent parking fee regime is at least as efficient as the no-pricing regime.

5. Numerical Examples

In the network shown in Figure 1, the service rate of each bottleneck is assumed to be $\underrightarrow{s} = \underleftarrow{s} = 1.0\times10^2$ veh/min, and a total of $N = 1.0\times10^4$ commuters are assumed to live in the residential area. We assume each person drives his/her own car to work in the morning and returns home in the evening. The official work start time is $\underrightarrow{t}^*$ =09:00 and the end time is $\underleftarrow{t}^*$ =17:00.The unit cost of in-vehicle waiting time (marginal Value of Time) is α =0.6$/min. The time to pass one parking spot on foot is w =0.001s and its unit cost is λ =2.0 $/min. In the morning, the shadow cost per unit time for arriving early is $\underrightarrow{\beta}$ =0.3$/min and late arriving is not allowed. In the evening, the penalty cost per unit time for leaving late is $\underleftarrow{\gamma}$ =0.3$/min, and early arriving is not allowed. In regime u, the rate of uniform parking fee μ varies from 0 to 0.06$/min.

Applying these parameters to our model, we can derive the morning and evening traffic patterns for various pricing regimes. The corresponding individual travel costs, social costs and revenue are summarized in Table 1. Figure 6 shows the optimal time-varying road tolls in pricing regime r. Figure 7 gives the optimal time-varying road tolls and optimal location dependent parking fees in regime o.

Table 1. Main numerical results in various regimes in the case of fixed demand

Regime	O	r	f	u (with the following parking fee $/min)						
				0.01	0.02	0.03	0.04	0.043	0.05	0.06
Individual travel cost ($)	94.0	67.0	67.0	72.9	78.8	84.7	90.6	92.5	96.9	103.2
Social cost ($\$10^5$)	4.700	5.000	6.700	6.692	6.688	6.687	6.688	6.700	6.727	6.763
Revenue ($\$10^5$)	4.700	1.700	0.000	0.598	1.192	1.783	2.372	2.549	2.963	3.552

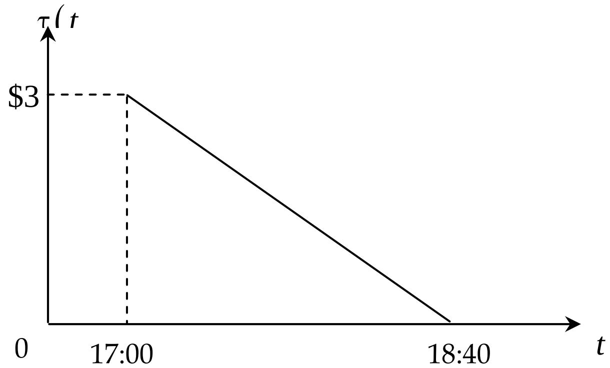
As can be seen, social costs in regime o are the lowest, followed by regime r. The efficiency comparison between regime f and regime u depends on the value of μ. When μ increases from 0 to 0.03$/min, social cost decreases from $\$6.700\times10^5$ to $\$6.687\times10^5$. However, when μ increases further, social costs increase and become equal to that in regime f when μ =0.043$/min. Note that we used different formulas to calculate the social cost, since when $\mu \le 0.04$$/min, the case is in regime u(b), and when $\mu > 0.04$$/min, the case is in regime u(a). The optimal parking fee rate with the lowest social cost in the duration dependant parking fee regime is 0.03$/min.

Now we consider the case of elastic demand. The demand function is specified as

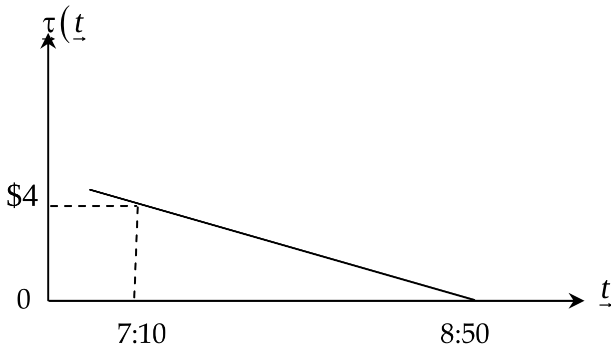
$$N(P) = 1.5\times10^4 - 80P,$$

where P stands for the daily private travel cost.

In regime o, the equilibrium demand automatically leads to the system optimum, and no externality need be charged. In regimes f, r and u, externalities have to be charged to achieve optimal demand and maximal welfare. The results of these three regimes are summarized in Table 2. As can be seen, regime o achieves the highest social surplus followed by regime r. The optimal demand level decreases as the charging rate increases. The efficiency comparison between regime f and regime u is similar to the case of fixed demand. When μ is less than 0.043$/min, regime u is more efficient than regime f, and the optimal μ level is 0.03$/min. Otherwise if μ is greater than 0.043$/min, a parking fee charge has a negative social effect. Again the optimal parking fee rate of 0.03$/min gives the maximal social surplus.



(a) The time varying road toll in the evening



(b) The time varying road toll in the morning

Figure 6. Time-varying road tolls in regime r

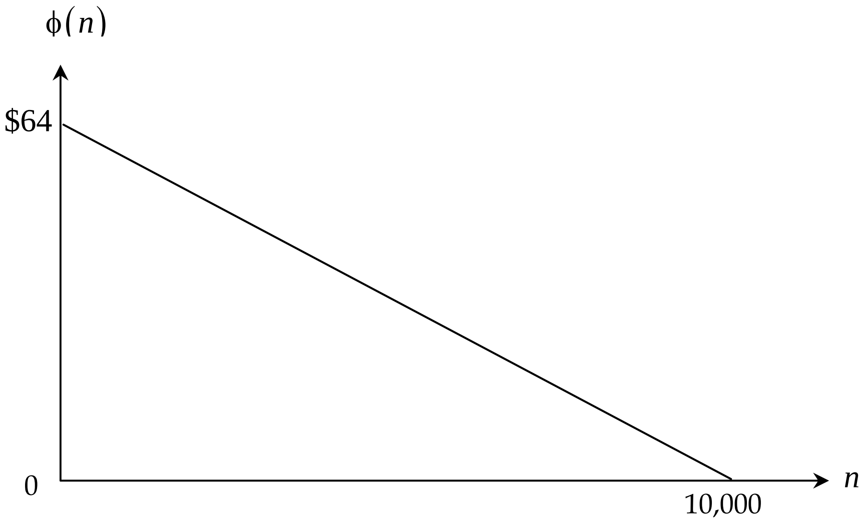


Figure 7. The location dependent parking fees in regime o

Table 2. Main numerical results in various regimes in the case of elastic demand

Regime	o	r	f	u (with the following parking fee $/min)						
				0.01	0.02	0.03	0.04	0.43	0.05	0.06
Optimal demand	8562	8333	7240	7244	7246	7247	7246	7240	7225	7219
Externality	0	27.499	48.508	42.829	37.182	31.572	25.988	24.355	20.535	15.167
Individual travel cost ($)	80.483	55.831	48.508	54.132	59.742	65.346	70.936	72.660	76.664	82.473
Social surplus (10^5)	8.027	7.812	6.788	6.791	6.793	6.794	6.793	6.788	6.773	6.768
Marginal social cost ($)	80.475	83.338	97.016	96.960	96.925	96.919	96.925	97.015	97.199	97.641
Social cost (10^5)	3.445	3.472	3.512	3.512	3.512	3.512	3.512	3.512	3.511	3.524
Revenue (10^5)	3.445	1.180	0.000	4.094	8.173	1.224	1.285	1.749	2.028	2.429

6. Concluding remarks

In this paper, we investigated a new duration dependent parking fee regime, based on a linear city with a residential area on one end and a workplace center on the other end. By setting an intermediate time point, we divide the duration dependent parking fee into morning and evening parts, which are then added to morning and evening travel costs. The joint morning and evening rush hour commutes are then investigated by a two-stage differential method. In the first stage, the evening work-to-home commuting pattern was identified, and travel costs were derived for individual commuters with different parking locations and durations. Then in the second stage, We attached the evening travel costs to different parking locations to establish the morning home-to-work commuting pattern, accounting for the total travel cost of an entire day.

We evaluated the efficiency of the duration dependent parking regime by comparing it with three other pricing regimes. Demand elasticity was modeled considering users' daily travel cost, including morning home-to-work commuting cost, parking fee when at work, and evening work-to-home commuting cost. The four pricing regimes were analyzed and compared in terms of system efficiency. Market equilibrium demand could be computed according to the demand function and the individual cost function. The optimal demand level was computed using the demand function and marginal social cost function. System efficiency was defined by social benefit, given by the gross user benefit minus the system travel cost. Analytical comparison results revealed that the pricing regime with both time-varying road tolls and location dependent parking fees was most efficient. Second best was the pricing regime with time-varying road tolls alone. Results from the comparison of the duration-based uniform parking fee regime and the no-pricing regime were indeterminate in general but case-specific.

We acknowledge that our analysis is based on the assumption of an infinite penalty for late arrival and early departure from work, which does not fit empirical observations very well. If more realistic and moderate penalties for non-punctual behavior are allowed, commuters may not follow exactly the traffic pattern described in the paper. In particular, they may not compete to enter the bottleneck as early as possible at the beginning of the evening peak. In the future, we shall investigate the existence and determination of the equilibrium traffic pattern when the assumption on the non-punctuality penalties is relaxed.

Our assumption of a linear city is also somewhat unrealistic. In a real planar city, parking spots become less available as one moves towards the city center, and thus the competition for closer parking spots may be more intensive than modeled here. Relaxing the linear city assumption may bring nonlinear rates of departure and arrival rates, as well as nonlinearly varying parking fees.

Another limitation of the paper is that we did not make an assumption on the required work time, since in reality each commuter has an ideal amount of time spent at work. Nevertheless, if commuters share the same criteria of work duration, an early arrival in the morning should correspond with an early departure in the evening. The good news in the paper is that the mapping between early arrival in the morning and early departure in the evening is a feasible and typical traffic pattern. As our models show, early-arriving commuters park closer to the city center in the morning, and commuters who have parked close to the city center depart earlier from the workplace in the evening.

Two more points are worth noting. First, we followed Arnott et al. (1991) in this paper and ignored the in-vehicle travel time within the parking area. This assumption can be relaxed by adding a term $\alpha\rho n$ to the travel cost function for both the morning and evening commutes, where ρ is the driving time required to pass one parking spot. The equilibria still exist after the relaxation, and the resulting traffic patterns remain intact.The second point is that the study assumed a uniform parking fee rate over time. In practice, parking fee rates can be set to vary over time, as well as depend on duration. In the future, we shall investigate traffic patterns and evaluate the efficiency for a regime with dynamic parking fee rates.

The modeling results in the paper are practically useful to policy-makers in traffic management. Policy-makers in traffic management may draw four practical lessons from our modelling results.

First, the charge rate of a uniform duration-dependent parking fee has to be set carefully since it may or may not improve travel efficiency. The duration-dependent parking fee with optimal charge rate can improve (or at least preserve/ not worsen) efficiency. Second, for both fixed and elastic traffic demand, the joint implementation of a time-varying road toll and location-dependent parking fee is the best pricing scheme for system optimum. Third, if a location-dependent parking fee cannot be implemented, the second best pricing scheme is a time-varying road toll. A road toll alone cannot achieve system optimization, but it is more efficient than a duration-dependent parking fee scheme. Finally, although a uniform duration-dependent parking fee alone is not as efficient as a location-dependent parking fee and time-varying road toll together, it is a practical choice if the charge rate is appropriate, due to its easy implementation.

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