

Stochastic User Equilibrium and Value-of-time Analysis with Reference-dependent Route Choice

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Reference-dependent theory of riskless choice assumes that carriers of utility are gains and losses relative to a reference point and that individuals are loss averse, i.e. losses are valued more highly than gains. Reference-dependent money measures of the attributes in the utility functions are defined: the willingness to pay, the willingness to accept, the equivalent gain and the equivalent loss. Experimental evidence has been provided which supports reference-dependent theory for riskless route choice. A natural next development is the application of the theory to network analysis. This is an under-researched area.

In the paper, the multi-class reference-dependent stochastic user equilibrium (RDSUE) problem under the status-quo assumption for the reference point is formulated. Conditions that guarantee the properties of existence and uniqueness of RDSUE are considered. The property of reflexivity of RDSUE is also considered to verify if the equilibrium is maintained when the reference point is updated to the new status quo. A methodology for the reference-dependent valuation of travel time changes over a network is provided.

Data from a survey are used to estimate a reference-dependent route choice model and the attendant reference-dependent values of time. The estimation results are in agreement with econometric literature supporting loss aversion. The application to the case of a town bypass with toll illustrates the reference-dependent approach to network analysis and the implications of its use for policy making. It is shown that, if the interventions on the supply are phased, it is possible to exploit loss aversion and obtain advantages in terms of toll revenues and travel time spent.

Keywords: reference-dependent theory, riskless choice, loss aversion, stochastic user equilibrium, reflexive equilibrium, value of time

1. Introduction

There is a large body of field and experimental evidence that choices are best explained by assuming that carriers of utility are not states but gains and losses relative to a reference point. There is an asymmetry in choices in the sense that gains are valued differently from losses; this asymmetry has the sign of loss aversion, i.e. losses are valued more heavily than gains. Loss aversion effects are found in data across choices in several domains. Reviews are in Kahneman et al. (1991) and in Ho et al. (2006).

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Tversky and Kahneman have proposed new theories of choice where the above suppositions are incorporated. Reference-dependent theory considers riskless choices (Tversky and Kahneman, 1991). In this theory, the utilities of the alternatives are characterised by n attributes whose outcomes are certain. Prospect theory (Kahneman and Tversky, 1979), which has evolved into cumulative prospect theory (Tversky and Kahneman, 1992), considers risky choices. The alternatives are characterised by a prospect value which depends on attributes whose outcomes are uncertain. In the literature, prospect theory is traditionally restricted to consideration of one uncertain attribute only. Reference-dependent theory and (cumulative) prospect theory belong to the approach of behavioural economics which integrates psychological insights into formal economic models.

The paper here is concerned with the application of reference-dependent theory to the analysis of road networks. Riskless route choices with certain route attributes are considered. The interest is in the possibility to model loss aversion effects in the trade-off among attributes, primarily travel time and money cost. The paper addresses, in particular, two aspects: the modelling of the equilibrium and the valuation of travel time changes over a network. These are still under-researched areas.

A stream of transportation literature on route choice modelling has dealt with the experimental validation of reference-dependent theory. De Borger and Fosgerau (2008) and Hess et al. (2008) have formulated reference-dependent route choice models following the framework proposed by Tversky and Kahneman (1991). The utility functions depend asymmetrically on gains and losses in the attributes.

The model in De Borger and Fosgerau (2008) has a more complex structure as it incorporates diminishing sensitivity but restricts to two attributes only: travel time and money. Hess et al. (2008) use a reference-dependent formulation with constant sensitivity, as done by Suzuki et al. (2001) who had made an application to represent the choice of carriers in the airline sector. The model by Hess et al. (2008) considers four attributes: free-flow travel time, slowed-down travel time, running cost and tolls.

Both De Borger and Fosgerau (2008) and Hess et al. (2008) estimate a random utility version of their models based on multinomial logit assumptions for the stochastic terms. Both find clear evidence of asymmetrical response in gains and losses. De Borger and Fosgerau (2008) find that the coefficients conform to the loss aversion hypothesis for both travel time and money. Hess et al. (2008) find that the loss aversion hypothesis is supported for some attributes and users' segments though not for all. The econometric investigations by De Borger and Fosgerau (2008) and by Hess et al. (2008) support the adoption of reference-dependent models for route choice.

A further step consists in the use of reference-dependent route choice models in network analysis. In particular, the paper addresses stochastic user equilibrium (SUE). SUE is the name for the fixed-point solution to the problem of finding flows and travel times over a network where users' route choices are made according to a random utility model and link travel times are dependent on the link flows (Daganzo and Sheffi, 1977). The paper addresses the version of SUE where a reference-dependent route choice model is adopted. This version will be called hereafter reference-dependent SUE (RDSUE).

For the aims here, it is relevant to look at the stream of literature that has considered the application of cumulative prospect theory to equilibrium problems (Avineri, 2006; Connors and Sumalee, 2009; Sumalee et al., 2009; Xu, Lou, Yin, Zhou, 2011). Travel times are uncertain and the attitude of users towards risk is modelled. The users are assumed to know the distribution of travel time on each route. The equilibrium is deterministic: an extension of Wardrop's principle is considered with equilibrium being defined as the condition where all used routes have equal maximum prospect value. In the paper by Connors and Sumalee (2009), a stochastic version of the equilibrium is also considered: the perception of the uncertain travel times varies from user to

user with equilibrium being defined as the condition where each user chooses the route of maximum random prospect value. This latter model by Connors and Sumalee includes as a particular instance the riskless case of RDSUE.

However, when riskless route choice is considered, it is more appropriate to adopt new hypotheses for the reference point, different from those in Connors and Sumalee (2009). They assume that there is an exogenous single reference point for all users of each origin-destination (OD) pair. How the reference point is fixed is an open research question in the application of prospect theory where the problem is to set a reference point in terms of travel time which the theory treats as uncertain. This suggests that the reference point should be related to users' expectations (the issue is dealt with in: Avineri, 2006; Gao et al., 2010; Xu, Lou, Yin, Zhou, 2011; Xu, Zhou, Xu, 2011). Connors and Sumalee (2009) argue that, with a stochastic equilibrium, future research should consider a random distribution of reference points across users of the same OD.

In contrast, in the application of reference-dependent theory, the attributes of routes are assumed to be known with certainty and it is most natural to interpret reference points as the status quo. This assumption, which is adopted in the paper here, implies departures from the model in Connors and Sumalee (2009).

First, there is a multiplicity of reference points for users of the same OD pair because in the status quo users choose different routes, each having a distinct (certain, hence deterministic) travel time. This gives rise to an equilibrium problem with multiple user classes, with each class having a distinct reference point determined by the travel time in the status quo.

Second, the property of reflexivity of the equilibrium needs to be considered. The idea of reflexive equilibrium appears in economics literature and is not yet found in transport applications. So far it has been considered in the context of trade equilibrium (Munro and Sugden, 2003; Munro, 2009). If it is assumed that reference points are current endowments, trades induce changes in endowments and therefore in reference points. Thus, the question arises whether what results to be an equilibrium when viewed from initial endowments is still an equilibrium when viewed from the equilibrium itself, i.e. from the endowments it gives rise. In the case the answer is affirmative the equilibrium is called reflexive. The issue of reflexivity appears to be relevant to network equilibrium as soon as it is assumed that users always adopt as reference point the status quo, i.e. the current route choice. After an RDSUE is set, users update the reference point to the current status quo. Thus, there is a need to verify if the equilibrium still holds under the new reference points.

The other aspect addressed by the paper is the valuation of travel time changes. A corollary of reference-dependent theory is the money valuation of attributes. In the classical, non reference-dependent, theory there is one measure of the money value of an attribute. This is the usual trade-off value calculated from utility. In reference-dependent theory, as utility depends on losses and gains in the money attribute and in other attributes, it is possible to define four different measures according to the different combinations of gains and losses (as proposed by Bateman et al., 1997). Two measures provide the money values of a gain in the attribute: the willingness-to-pay (*WTP*) and the equivalent gain (*EG*). Other two measures provide the money values of a loss in the attribute: the willingness-to-accept (*WTA*) and the equivalent loss (*EL*).

De Borger and Fosgerau (2008) have considered the four measures of the money value of travel time (*WTP*, *EG*, *WTA*, *EL*). They show that, given the loss aversion hypothesis, the four measures need to satisfy certain inequalities among each other. Hess et al. (2008) conclude that the use of asymmetric route choice models opens the way for an innovative approach to the valuation of travel time changes over a network. Instead of the usual valuation according to a single value of time, time changes should be valued based on gains and losses in travel time and money using multiple measures of the value of time.

The paper here provides the following original contributions:

- formulates the RDSUE model with multiple user classes under the assumption that the reference point is the status quo, and identifies conditions under which the properties of existence, uniqueness and reflexivity hold;
- gives a formal treatment of the methodology for the valuation of travel time changes over a network according to the four reference-dependent money values of travel time (*WTP*, *EG*, *WTA* and *EL*);
- carries out an econometric test of the loss aversion hypothesis based on the calibration of a reference-dependent route choice model;
- applies the RDSUE and the reference-dependent value-of-time analysis to the classical town bypass case with toll and investigates the policy implications that can be derived on the basis of the adoption of reference-dependent models.

The paper is organised as follows. Sections 2 and 3 present the models (respectively, the first and the second contribution above). Section 4 presents the numerical results (third and fourth contribution). Finally, directions for further research are identified.

2. Network equilibrium

The road network is represented by a graph, with directed links $a = 1, \dots, N$ connecting the nodes. The OD pairs on the network are denoted by $r = 1, \dots, R$.

On each OD pair r there are M^r classes of users. The set of classes on OD pair r is denoted by Φ^r . The total number of classes is $M = \sum_r M^r$. The index of class is m . The adjective disaggregate applied to flows denotes disaggregation by class.

The disaggregate demand flows are represented in the $[M \times 1]$ vector \mathbf{q} whose element $q^{m,r} > 0$ represents the demand flow of class m on OD pair r .

The disaggregate link flows are represented in the $[M \cdot N \times 1]$ vector ordered by class:

$$\mathbf{x} = [\mathbf{x}^1, \dots, \mathbf{x}^M]^T = [x_1^1, \dots, x_N^1, \dots, x_1^M, \dots, x_N^M]^T \quad (1)$$

where x_a^m is the flow of class m on link a .

The aggregate link flows are represented in the $[N \times 1]$ vector:

$$\mathbf{z} = [z_1, \dots, z_N]^T = \mathbf{A} \cdot \mathbf{x} \quad (2)$$

where the $[N \times N \cdot M]$ matrix $\mathbf{A} = [\mathbf{I}_N | \dots | \mathbf{I}_N]$ sums the disaggregate flows over each link and comprises M identity matrices \mathbf{I}_N .

On each OD pair r there are K^r simple paths. The set of simple paths on OD pair r is denoted by Ψ^r . The total number of paths on the network is $K = \sum_r K^r$. The index of path is k . It is

assumed that all the classes of a given OD pair r face each the same set of paths Ψ^r .

The disaggregate path flows are represented in the vector ordered by class and sub-ordered by OD pair:

$$\mathbf{f} = \left[\mathbf{f}^{1,1}, \dots, \mathbf{f}^{M^1,1}, \dots, \mathbf{f}^{M^R,R} \right]^T = \left[f_1^{1,1}, \dots, f_{K^1}^{1,1}, \dots, f_1^{M^1,1}, \dots, f_{K^1}^{M^1,1}, \dots, f_1^{M^R,R}, \dots, f_{K^R}^{M^R,R} \right]^T \quad (3)$$

where $f_k^{m,r}$ is the path flow of class m of OD r on path k . The vector \mathbf{f} is of size $[\Gamma \times 1]$ where $\Gamma = (M^1 \times K^1) + \dots + (M^R \times K^R)$.

The disaggregate link flows \mathbf{x} can be written as function of the disaggregate path flows \mathbf{f} using the link path incidence matrix Δ , whose elements are Kronecker delta functions $\delta_{a,k}^{m,r}$ denoting if the path k comprises the link a , for every class m and OD pair r . The disaggregate link flows are given by:

$$\mathbf{x} = \begin{bmatrix} \Delta^{1,1} & \mathbf{0} & \dots & \mathbf{0} & \dots & \mathbf{0} \\ \mathbf{0} & \bullet & \dots & \dots & \dots & \dots \\ \dots & \dots & \bullet & \mathbf{0} & \dots & \dots \\ \mathbf{0} & \dots & \mathbf{0} & \Delta^{M^1,1} & \dots & \dots \\ \dots & \dots & \dots & \dots & \bullet & \mathbf{0} \\ \mathbf{0} & \dots & \dots & \dots & \mathbf{0} & \Delta^{M^R,R} \end{bmatrix} \cdot \begin{bmatrix} \mathbf{f}^{1,1} \\ \dots \\ \mathbf{f}^{M^1,1} \\ \dots \\ \mathbf{f}^{M^R,R} \end{bmatrix} = \Delta \cdot \mathbf{f} \quad (4)$$

where the link path incidence matrix Δ is of size $[M \cdot N \times \Gamma]$, and $\Delta^{m,r}$ is the $[N \times K^r]$ component link path incidence matrix for class m of OD pair r .

Due to the assumption that all the classes of a given OD pair r face each the same set of paths Ψ^r , the component link path incidence matrices satisfy:

$$\Delta^{m,r} = \Delta^r \quad \forall m \in \Phi^r, \quad r = 1, \dots, R \quad (5)$$

where Δ^r is the $[N \times K^r]$ link path incidence matrix of OD pair r whose elements $\delta_{a,k}^r$ denote if the path k of OD pair r comprises the link a . We denote by Δ^* the reduced $[N \times K]$ link path incidence matrix $[\Delta^1 | \dots | \Delta^R]$.

The aggregate link flows \mathbf{z} are obtained as function of the disaggregate path flows \mathbf{f} using eqns (2) and (4):

$$\mathbf{z} = \mathbf{A} \cdot \mathbf{x} = \mathbf{A} \cdot \Delta \cdot \mathbf{f} \quad (6)$$

The set of feasible aggregate path flows \mathbf{f} is non-empty, compact and convex. Also, the set of feasible aggregate link flows \mathbf{z} is non-empty, compact and convex.

All classes face the same link and path travel times. The link travel times are represented by the $[N \times 1]$ vector \mathbf{t} and are functions of the aggregate link flows \mathbf{z} :

$$\mathbf{t} = [t_1, \dots, t_N]^T = \mathbf{t}(\mathbf{z}) \quad (7)$$

It is assumed that the link travel time functions are continuous in the aggregate link flows. In addition, the vector mapping of link travel times is monotonically increasing with respect to the aggregate link flows, i.e. satisfies:

$$(\mathbf{t}(\mathbf{z}_1) - \mathbf{t}(\mathbf{z}_2))^T \cdot (\mathbf{z}_1 - \mathbf{z}_2) > 0 \quad \forall \mathbf{z}_1, \mathbf{z}_2 \quad (8)$$

This does not require the link travel times to be differentiable, nor to be separable. In the case, frequent in practical applications, where link travel times are assumed separable and increasing in the corresponding flow, condition (8) is satisfied.

The path travel times are represented by the $[K \times 1]$ vector \mathbf{T} and are obtained from the link travel times by the standard link-additive model, i.e. they are the summed constituent link travel times:

$$\mathbf{T} = [T_1^1, \dots, T_{K^1}^1, \dots, T_{K^R}^R]^T = \Delta^{*T} \cdot \mathbf{t}(\mathbf{z}) \quad (9)$$

2.1 Reference-dependent route choice

The users of the classes of an OD pair perceive a utility on each path. This path utility is a random variable given by the sum of a systematic, i.e. deterministic, component and a stochastic term. The choice is deterministic for the user, it is stochastic for the modeller due to unobserved factors which influence user's behaviour, such as other attributes of the alternatives not included in the systematic component and user-specific tastes. The stochastic terms summarise these unobserved factors. The model accounts for interpersonal variation of choice because the utility perceived varies from user to user, the systematic part being the same.

A reference-dependent model is adopted for the path systematic utility according to the following hypotheses. The path systematic utility

- (1) depends on two attributes: expenditure in travel time T and expenditure in money M ;
- (2) depends on gains G and losses L in the two attributes defined relative to a reference point, and increases with gains and decreases with losses;
- (3) is linear in gains and losses and steeper for losses than for gains;
- (4) depends on the class of users because each class of a given OD pair has a distinct reference point.

It is assumed that the only difference between classes of a given OD pair is the reference point. The utility of class m of OD pair r with reference point "0" on path k is:

$$\begin{aligned} U_{k,0}^{m,r} &= V_{k,0}^{m,r} + \varepsilon_k^{m,r} \\ V_{k,0}^{m,r} &= \beta_{GT} \cdot GT_{k,0}^{m,r} + \beta_{LT} \cdot LT_{k,0}^{m,r} + \beta_{GM} \cdot GM_{k,0}^{m,r} + \beta_{LM} \cdot LM_{k,0}^{m,r} \\ GT_{k,0}^{m,r} &= \max(T_0^{m,r} - T_k^r, 0) \\ LT_{k,0}^{m,r} &= \max(T_k^r - T_0^{m,r}, 0) \\ GM_{k,0}^{m,r} &= \max(M_0^{m,r} - M_k^r, 0) \\ LM_{k,0}^{m,r} &= \max(M_k^r - M_0^{m,r}, 0) \\ \forall k \in \Psi^r, \forall m \in \Phi^r, r = 1, \dots, R \end{aligned} \quad (10)$$

where:

$U_{k,0}^{m,r}$ is the path perceived utility,

$V_{k,0}^{m,r}$ is the path systematic utility,

$\varepsilon_k^{m,r}$ is the stochastic term,

$\beta_G = [\beta_{GT}, \beta_{GM}]^T$ is the vector of gain coefficients,

$\beta_L = [\beta_{LT}, \beta_{LM}]^T$ is the vector of loss coefficients,

$GT_{k,0}^{m,r}, GM_{k,0}^{m,r}$ are the gain, respectively, in travel time and in money,

$LT_{k,0}^{m,r}, LM_{k,0}^{m,r}$ are the loss, respectively, in travel time and in money,

T_k^r, M_k^r are, respectively, the path travel time, provided by eqns (9), and the money spent on the path;

$T_0^{m,r}, M_0^{m,r}$ are the reference point for, respectively, the travel time and the money spent.

Hypothesis (ii) implies that the systematic utility is decreasing in each attribute, i.e. the gain coefficients are positive $\beta_G > 0$ and the loss coefficients are negative $\beta_L < 0$. Hypothesis (iii) implies loss aversion, i.e. $|\beta_L| > |\beta_G|$: in absolute values, the loss coefficient is larger than the gain coefficient for each attribute.

The systematic utility in eqns (10) has two terms for each attribute: a gain term and a loss term. If there is a gain in the attribute the gain term is positive and the loss term is zero. Conversely, if there is a loss the loss term is positive and the gain term is zero.

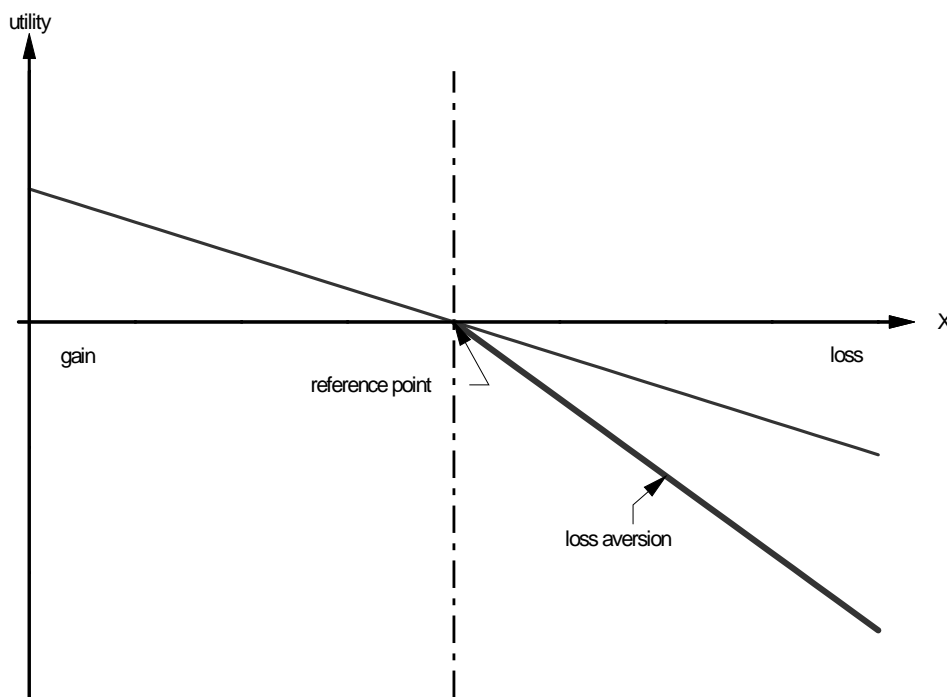


Figure 1. Single-attribute systematic utility

The single-attribute part of the systematic utility is piecewise linear in the attribute with a kink in the reference point. Thus the function is everywhere continuous in the attribute but non differentiable in the reference point. This property applied to the travel time attribute is of relevance to the equilibrium analysis below.

If the absolute values of the gain and loss coefficients were equal, the function would be symmetric about the reference point. Otherwise the function is asymmetric with slope steeper in losses than in gains if the coefficients satisfy loss aversion. This is shown in Figure 1 where the attribute (expenditure in travel time or in money) is denoted by X.

A constant additive term may be included in the systematic utility in eqn (10), e.g. to represent other time-independent path attributes, without affecting the developments below; however, without loss of generality it is left out because it has no effect either on the equilibrium or on the value-of-time analysis.

We assume that the stochastic terms $\varepsilon_k^{m,r}$ have a non-degenerate joint probability density function that is continuously differentiable, strictly positive, and independent of the path systematic utility.

Users of class m of OD pair r having "0" as reference point who choose path k are those who perceive this path to maximise their utility. The corresponding choice probability is defined as:

$$P_{k,0}^{m,r}(\mathbf{U}_0^{m,r}) = \Pr(U_{k,0}^{m,r} \geq U_{j,0}^{m,r} \quad \forall j \neq k \in \Psi^r) \quad (11)$$

$$\forall k \in \Psi^r, \forall m \in \Phi^r, r = 1, \dots, R$$

where $\mathbf{U}_0^{m,r}$ is the $[K^r \times 1]$ vector of perceived utilities for class m of OD pair r having "0" as reference point. The path choice probabilities are represented in the $[\Gamma \times 1]$ vector \mathbf{P} .

Under the assumptions made, the choice probabilities satisfy the following properties. They are single-valued. They are continuous in the path systematic utilities, and hence in the link travel times and in the link flows. In addition, the vector mapping of the choice probabilities is monotonically non-decreasing with respect to the path systematic utilities, i.e. satisfies:

$$(\mathbf{P}(\mathbf{V}_1) - \mathbf{P}(\mathbf{V}_2))^T \cdot (\mathbf{V}_1 - \mathbf{V}_2) \geq 0 \quad \forall \mathbf{V}_1, \mathbf{V}_2 \quad (12)$$

where \mathbf{V} is the $[\Gamma \times 1]$ vector of path systematic utilities.

The hypotheses made for route choice are those of the additive models where the joint distribution for the stochastic term of the path utility is independent of the systematic component. They are also referred to as translationally invariant models (Ben-Akiva and Lerman, 1985) because the choice probability is unaffected by a shift in the systematic component of the utility. The hypotheses are sufficiently general to admit a range of behavioural assumptions through the form of the joint distribution for the stochastic term, thus encompassing various models, including, but not restricting to, multinomial logit. In the case of multinomial logit the probability function takes an asymmetric "S" shape with a kink in the reference point due to the loss aversion assumption (this is illustrated graphically in Suzuki et al., 2001).

The choice model in terms of disaggregate path flows is written as:

$$\mathbf{f} = \mathbf{Q} \cdot \mathbf{P} \quad (13)$$

where $\mathbf{Q} = \text{diag } \mathbf{q}$ is the $[\Gamma \times \Gamma]$ matrix representing the expanded version of the vector \mathbf{q} of OD demand flows, such that for each constituent class m of OD pair r we have:

$$\begin{bmatrix} f_1^{m,r} \\ f_2^{m,r} \\ \dots \\ f_{K^r}^{m,r} \end{bmatrix} = \begin{bmatrix} q^{m,r} & 0 & \dots & 0 \\ 0 & q^{m,r} & & \\ \dots & & \bullet & \\ 0 & 0 & & q^{m,r} \end{bmatrix} \cdot \begin{bmatrix} P_1^{m,r} \\ P_2^{m,r} \\ \dots \\ P_{K^r}^{m,r} \end{bmatrix} \quad (14)$$

2.2 Reference-dependent stochastic user equilibrium

The network is considered in the status quo. In the status quo users of each OD pair use different paths, each with a distinct travel time and money cost. It is assumed that the status quo determines how many users are found in each path.

It is possible that interventions on the supply side modify the time-flow relationships or the money spent. Also, the intervention may consist in adding or subtracting links. The network will change from the initial status quo to a new state because users will make new choices.

We postulate the following decision-making process on the side of the users. Users behave according to a choice function which depends on the reference point as set in the previous section. The key assumption is that users adopt as reference point the status quo. Thus, on each OD pair each class of users will have a distinct reference point defined by the path chosen in the status quo. After a change in supply the network will be found in an equilibrium state where no user can improve his perceived utility by unilaterally changing route. This state is referred to as reference-dependent stochastic user equilibrium (RDSUE). After the RDSUE is set, the users update their reference point to the new status quo.

Under this decision-making process, the set of RDSUE properties of interest includes, in addition to existence and uniqueness, the property of reflexivity which is defined below.

A RDSUE is reflexive if, for each user, the path with maximum utility defined relative to the previous reference point is also the path with maximum utility defined relative to the reference point represented by the current status quo, i.e. the current choice and network state. The property implies that nobody has convenience to change choice when the reference point is updated after the RDSUE is set.

A RDSUE is a solution to the fixed-point problem:

$$\mathbf{z} = \mathbf{A} \cdot \mathbf{\Delta} \cdot \mathbf{Q} \cdot \mathbf{P}(\mathbf{V}(\mathbf{z})) \quad (15)$$

which results from chaining eqns (6), (13), (10) and (9). This is the formulation based on the aggregate link flows \mathbf{z} . A solution to (15) uniquely determines the disaggregate link flows \mathbf{x} , the disaggregate path flows \mathbf{f} , and the link and path travel times \mathbf{t} and \mathbf{T} .

Equivalently, it is possible to adopt a formulation based on the disaggregate path flows \mathbf{f} . A RDSUE is a solution to the fixed-point problem:

$$\mathbf{f} = \mathbf{Q} \cdot \mathbf{P}(\mathbf{V}(\mathbf{A} \cdot \mathbf{\Delta} \cdot \mathbf{f})) \quad (16)$$

A solution to (16) uniquely determines the aggregate link flows \mathbf{z} , the disaggregate link flows \mathbf{x} , and the link and path travel times \mathbf{t} and \mathbf{T} .

The difference of the RDSUE here with the conventional SUE lies in the systematic utilities which are piecewise linear in travel time in RDSUE, while are usually assumed linear in conventional SUE. The RDSUE would reduce to a conventional SUE if the absolute values of the gain and of the loss coefficient were equal for each attribute.

The property of existence of the RDSUE is satisfied under the assumptions made here. The proof is in the Appendix.

The property of uniqueness is guaranteed under sufficient conditions that are discussed in the Appendix. It is shown that the general assumptions made here are not sufficient to guarantee these conditions. Nevertheless, uniqueness cannot be excluded.

The property of reflexivity is satisfied under the assumptions made here if each user retains her stochastic terms when the reference point is updated to the new status quo. The hypothesis of invariance of the stochastic terms is justified having considered that no unobserved attribute of

the alternatives is changed when the reference point is updated, and that it is plausible that user-specific tastes (such as an added attraction to a given alternative) do not change. The proof in the Appendix shows that a key assumption to guarantee reflexivity is that the coefficients β_L and β_G in the path systematic utility satisfy loss aversion, i.e. it is required that $|\beta_L| > |\beta_G|$.

In practical applications the initial reference points can be determined if the solution of a SUE is available for the network. In the SUE case in each OD pair there will be as many classes as paths, each class with a distinct reference point. The SUE path flows provide the cardinality of each class of users. After the first RDSUE is set, if the application envisages the computation of a sequence of equilibria, it will be possible to take each time as reference points the RDSUE solution.

The adoption of an approach to network equilibria based on RDSUE makes a significant difference in comparison with conventional, i.e. non reference-dependent, approaches. Under a short-range perspective where the OD demand is held constant and supply is modified according to a sequence of interventions, with conventional models the final equilibrium over the network is independent of this sequence: the network flows and travel times depend on the specification of supply of the latest intervention only. We have in this case policy-path independence.

In contrast, with RDSUE, the equilibrium depends on the reference point. As it is assumed that users adopt as reference point the status quo, the equilibrium after a supply modification depends on the initial state of the network. This means that the final equilibrium over the network after a sequence of interventions will be dependent on this sequence. We have in this case policy-path dependence. An example of this policy-path dependence is illustrated in Section 4 for the case of a town bypass with toll. Policy-path dependence is a consequence of the property of reflexivity of RDSUE: reflexivity needs to be satisfied so that users retain, after each intervention on the supply, the new status quo. This implies that the resulting equilibrium depends on the initial state of the network.

It has been seen that reflexivity is guaranteed under the assumption that the stochastic terms do not change when the reference point is updated. It has been argued that the assumption is justified. Nonetheless, provided that at the current state of knowledge definitive statements on the dynamics of the stochastic terms are lacking, the modelling developments of this paper and the illustrative application should be seen having taken into account that alternative approaches to reference-dependent stochastic equilibrium are possible³.

In this respect, it is worth mentioning that, at the opposite extreme of the invariance assumption adopted here, is the assumption that the stochastic terms are redrawn for each class. This implies that the relative convenience of the alternatives may change when the reference point is updated. Therefore, a new RDSUE would need to be computed on the basis of the updated systematic utilities (this certainly yields a different fixed-point solution). The reference points would be further adjusted and a new RDSUE computed. The adjustment would be stopped when a condition of self-consistency is achieved, i.e. the reference points are those of the current flows and the number of users in each class equals the path flows resulting from the equilibrium. Under this alternative assumption, due to the endogeneity of the reference points, we would have again policy-path independence as in conventional equilibrium models.

As an aside, we note that the consideration of a deterministic user equilibrium (DUE) would be inconsistent with the analysis framework here. Here we postulate the knowledge of the status quo of the network in terms of users assigned to distinct paths. In contrast, in a DUE setting the correspondence between used paths, each with a distinct travel time and money cost (but common utility), and path flows is not determined.

³ The insightful comments of one anonymous referee on this point are gratefully acknowledged

From a computational viewpoint, a challenge of RDSUE is the number of classes which can be large as this equals the number of paths. To control the number of classes, the easiest is to select paths, with attendant reference points, from the SUE or RDSUE solution available for the initial state, and then calculate the number of users in each class by applying the route choice model with the choice set restricted to the selected paths. For selection, criteria such as the size of path flow (only paths with flow above a threshold depending on approximation errors) and others based on the realism of route choice behaviour that have been proposed in the literature (reviews are in Bekhor and Toledo, 2005; Cascetta, 2009) can be used. Ideally, there should be consistency between the paths considered in the RDSUE solution algorithm and the paths considered for class identification, in particular when the application envisages the computation of a sequence of RDSUE. Consistency is granted if path-based solution algorithms are used, as they adopt a selective approach for path enumeration which allows consideration of a user-defined set of paths. Path-based solution algorithms are the only viable in RDSUE because path utilities are not additive with respect to constituent links.

3. Value-of-time analysis

3.1 Definitions

Based on the route choice model of eqns (10) it is possible to derive the following four measures of the money value of the travel time attribute (following Bateman et al., 1997, and De Borger and Fosgerau, 2008). Two are compensating measures:

- willingness to pay (*WTP*): money one would pay in return for a unit gain in time;
- willingness to accept (*WTA*): money one would accept in return for a unit loss in time;

two are equivalent measures:

- equivalent gain (*EG*): money gain one would accept in place of a unit gain in time;
- equivalent loss (*EL*): money loss one would accept in place of a unit loss in time.

Thus we have two measures of the money value of a unit time gain: a compensating measure (*WTP*) and an equivalent measure (*EG*). Similarly, we have two measures of the money value of a unit time loss (*WTA* and *EL*).

The four measures are obtained from choices for certain pairs of route alternatives. The *WTP* can be derived from a choice between the reference alternative and an alternative with a time gain and a money loss. The measure expresses a trade-off value therefore it is obtained by imposing indifference between the two alternatives, i.e. by equalising the systematic utilities:

$$\beta_{GT} \cdot GT + \beta_{LM} \cdot LM = 0 \quad (17)$$

From this we obtain:

$$WTP = -\frac{\beta_{GT}}{\beta_{LM}} \quad (18)$$

The other measures are obtained as follows:

- the *WTA* from a choice between the reference alternative and an alternative with a time loss and a money gain;
- the *EG* from a choice between an alternative with a time gain and an alternative with a money gain;

- the EL from a choice between an alternative with a time loss and an alternative with a money loss.

Mathematically:

$$\begin{array}{l}
 WTA \left\{ \begin{array}{l} \beta_{LT} \cdot LT + \beta_{GM} \cdot GM = 0 \\ WTA = -\frac{\beta_{LT}}{\beta_{GM}} \end{array} \right. \\
 EG \left\{ \begin{array}{l} \beta_{GT} \cdot GT = \beta_{GM} \cdot GM \\ EG = \frac{\beta_{GT}}{\beta_{GM}} \end{array} \right. \\
 EL \left\{ \begin{array}{l} \beta_{LT} \cdot LT = \beta_{LM} \cdot LM \\ EL = \frac{\beta_{LT}}{\beta_{LM}} \end{array} \right.
 \end{array} \quad (19)$$

We note that, given the constant sensitivity assumption for the utilities, the four measures of the value of time are independent of the size of the time difference. If the coefficients satisfy loss aversion for both time and money, i.e. $|\beta_L| > |\beta_G|$, the following inequalities hold:

$$WTP < \min\{EG, EL\} \leq \max\{EG, EL\} < WTA \quad (20)$$

Figure 2 shows how the four measures can be derived graphically from the indifference curves. This construction is not found in the literature. The quantity T on the horizontal axis represents the time expenditure and the quantity M on the vertical axis the money expenditure. The indifference curves, i.e. the curves obtained by setting systematic utility equal to a constant, are piecewise linear. The vertical and horizontal lines passing through the reference values of the two attributes are reference lines which define four quadrants corresponding to the different combinations of gains and losses. Each combination is denoted in the figure with the pair: gain/loss in time, gain/loss in money (GL, LL, LG and GG). The indifference curves have kinks at each point where they cross the reference lines. The indifference curve passing through the reference point has one kink, all other indifference curves have two kinks each. The four measures are obtained graphically as soon as it is assumed that the two indifference curves above and below the reference point refer to a unit value of the gain and of the loss in time.

The case in Figure 2 is one where the coefficients satisfy loss aversion for both attributes. It is seen from the figure that under loss aversion the “better than” set for each indifference curve (i.e. the set of points having at least that utility) is convex and the relative magnitude of the four measures satisfy inequalities (20).

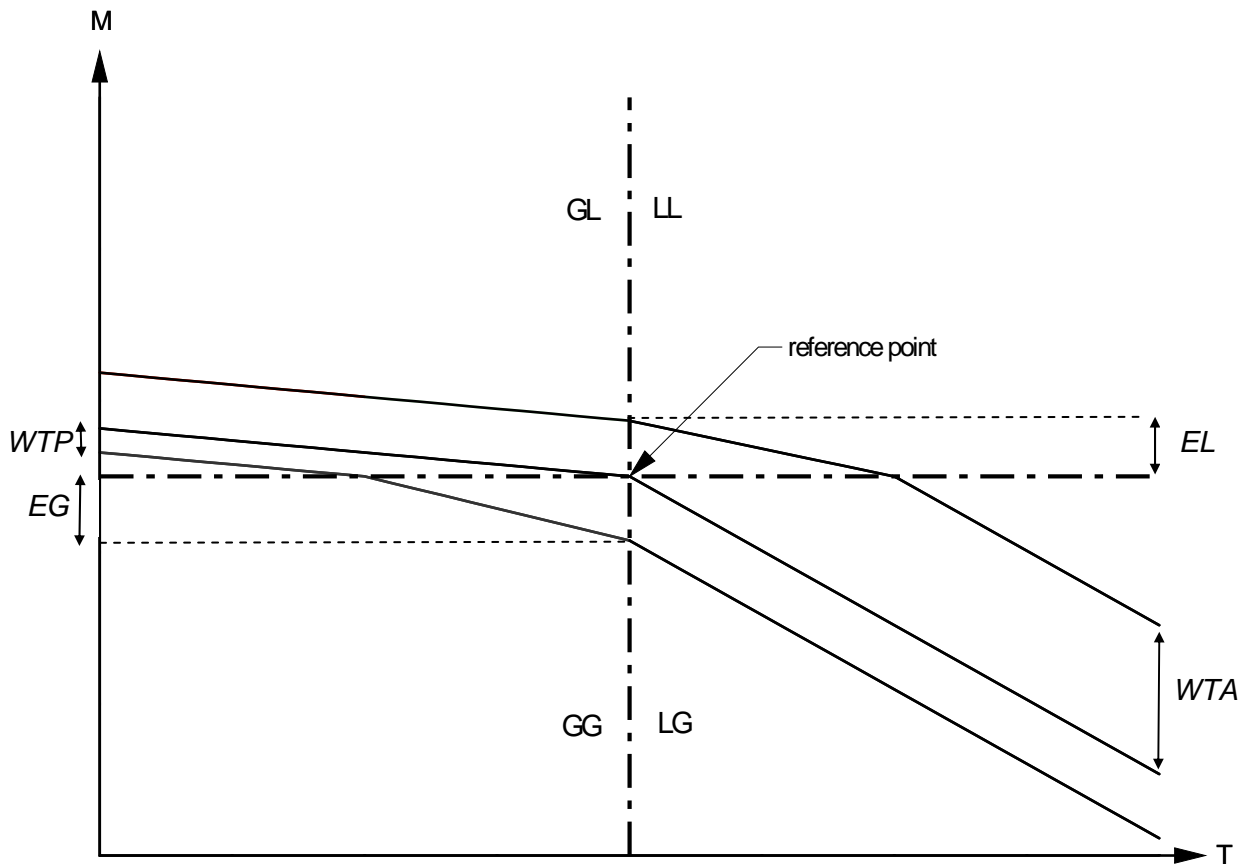


Figure 2. Indifference curves and measures of the value of time under loss aversion

3.2 Valuation of network time changes

It is customary in network models to provide estimates of the value of travel time savings. When conventional, non-reference dependent, models are used the practice is to calculate from equilibrium the time savings, relative to a reference scenario, and then use the value of time obtained as trade-off value from a demand model, e.g. route choice, to calculate the total value. When the reference-dependent models proposed in this paper are used it is possible to proceed as follows.

As it is assumed that the reference point is the status quo, the class of users m of OD pair r that uses path k at RDSUE is assigned a travel time change from $T_0^{m,r}$ to T_k^r .

The compensating measures of the total travel time change for the class m of OD pair r that uses path k at RDSUE are:

$$\Delta CT_k^{m,r} = WTP \cdot \max(T_0^{m,r} - T_k^r, 0) \cdot q^{m,r} \cdot P_k^{m,r} + WTA \cdot \max(T_k^r - T_0^{m,r}, 0) \cdot q^{m,r} \cdot P_k^{m,r} \quad (21)$$

$$\forall k \in \Psi^r, \forall m \in \Phi^r, r = 1, \dots, R$$

Eqn (21) gives rise to a WTP measure in the case of a time gain, to a WTA measure in the case of a time loss.

Similarly, the equivalent measures of the total travel time change are:

$$\Delta ET_k^{m,r} = EG \cdot \max(T_0^{m,r} - T_k^r, 0) \cdot q^{m,r} \cdot P_k^{m,r} + EL \cdot \max(T_k^r - T_0^{m,r}, 0) \cdot q^{m,r} \cdot P_k^{m,r} \quad (22)$$

$$\forall k \in \Psi^r, \forall m \in \Phi^r, r = 1, \dots, R$$

Eqn (22) gives rise to an *EG* measure in the case of a time gain, to an *EL* measure in the case of a time loss.

According to the transitions from the path in the status quo to the path in the RDSUE, it is possible, in addition, to calculate for each class the number of users in the group of non-shifters, i.e. who remain on the same path, and the number of users in the $\Psi^r - 1$ groups of shifters.

The circumstance that two measures are available for valuing time gains (or time losses), one compensating and one equivalent measure, instead of one measure only, raises the question of which one of the two is to be used in the assessment of policies. Chin and Knetsch (2006) and Knetsch (2007) address this issue. These authors argue that the choice of the appropriate measure depends on which state the individuals regard as the norm for judging their satisfaction.

Their argument is as follows. Take travel time savings. They can be valued by *WTP* or *EG*. If users regard as norm the state before the improving intervention, i.e. the inferior state, the value of the gain is the willingness to pay to move to the superior state, i.e. the *WTP*. Conversely, if they regard as norm the state after the improving intervention, i.e. the superior state, the value of the gain is the money they demand to forego the gain, i.e. the *EG*.

A similar argument can be put forward in the case of time losses. They can be valued by *WTA* or *EL*. If users regard as norm the state before the damaging intervention, i.e. the superior state, the value of the loss is the willingness to accept a compensation to move to the inferior state, i.e. the *WTA*. Conversely, if they regard as norm the state after the damaging intervention, i.e. the inferior state, the value of the loss is the money they are willing to pay to avoid the loss, i.e. the *EL*.

According to this line of reasoning, the identification of the state the users regard as norm is key for the choice of the valuation measure. Knetsch (2007) report the possibility that individuals regard as norm their legitimate expectations, or what they feel is deserving or right. Chin and Knetsch (2006) argue that road users are likely to regard as norm the free-flowing traffic conditions. This implies that, when the assessment concerns policies that mitigate congestion, the appropriate measure of the value of travel time savings would be the *EG*. Choosing the *WTP* instead would lead to an underestimation of the benefits of the policy because under loss aversion $WTP < EG$.

4. Numerical results

4.1 Route choice model and time values estimation

The data used came from a stated preference survey which took place in Rome in 2007. The survey was part of a study which aimed at the estimation of the prospective demand for a new road infrastructure, the so-called "sottopasso", planned within the recent master plan. The new infrastructure would provide a fast alternative route to current congested routes in the south-eastern area of Rome. For financial feasibility it would possibly be subject to the payment of a toll for its use.

The questionnaire was submitted to 356 individuals. A preliminary question asked all respondents the current route and the time saving the individual would have gained with the new infrastructure. Then respondents were asked to consider a reference route and to make a choice between two routes. The respondents were subdivided into three groups, each group was presented with a block. The blocks considered each a different choice situation. In the first block the reference route is the current route, the first alternative is the reference and the second is the "sottopasso" offering a time gain and subject to a toll. In the second block the reference route is the "sottopasso" offering a time gain without toll, the first alternative is the current route, the

second alternative is the “sottopasso” offering a time gain and subject to a toll. In the third block the reference route is the current choice, the first alternative is the reference, the second is the current route with a time loss (e.g. for roadworks or demand shocks) and a money reward. Each respondent was presented with three choices generated from three values for the money attribute (1 EUR, 2 EUR, 4 EUR). The idea behind the blocks was to elicit from the stated choices the willingness to pay for a time gain (first block), the willingness to pay in place of incurring a time loss (second block), the willingness to accept a reward for a time loss (third block).

The data from the survey were used to calibrate two route choice models. First, a conventional binomial logit with two attributes: the changes, relative to a reference situation, in travel time (assumed positive for a decrease) and in money spent (assumed positive for an increase). Second, an asymmetric binomial logit according to the formulation in Section 2. The two models are estimated in NLOGIT (Hensher et al., 2005). The results of the estimation are in Table 1.

Table 1. Estimation results for route choice model

	conventional logit		asymmetric logit		t statistic for difference in absolute values
	coefficient	t statistic	coefficient	t statistic	
time variation (min)	0.10796	10.868	-	-	-
time gain	-	-	0.10545	9.521	-1.5318
time loss	-	-	-0.12270	-9.827	
money variation (EUR)	-1.52248	-14.447	-	-	-
money gain	-	-	1.25287	9.481	-3.302
money loss	-	-	-1.67346	-14.075	
number of observations:	1068				
final log likelihood	-424.4524		-413.4574		
rho-squared	0.4266		0.4414		
parameters estimated	2		4		
rho-squared adjusted	0.4255		0.4393		
non-nested test of model specification: pr<0.001					

All the coefficients of both the conventional and the asymmetric model have the right sign. In particular it is seen from the asymmetric model that individuals value positively gains and negatively losses. From the t-statistic it is seen that all the coefficients of both models are statistically significant (at 5% significance level, two-tailed).

The findings for the asymmetric model support the hypothesis of loss aversion. In absolute value, the loss coefficient is higher than the gain coefficient for both travel time and money. The degree of loss aversion, defined as the ratio between the loss coefficient and the gain coefficient, is higher for money ($\beta_{LM} / \beta_{GM} = 1.34$) than for time ($\beta_{LT} / \beta_{GT} = 1.16$), i.e., individuals are more loss averse in the money dimension than in the time dimension.

We tested the statistical significance of the assumption that responses are asymmetric. We considered the null hypothesis that the difference in absolute value between the gain coefficient and the loss coefficient is zero, i.e. a symmetrical response. Based on the t-statistic in Table 1, for the time attribute we can reject the null hypothesis at 10% significance level (one-tailed, with sign consistent with loss aversion), for the money attribute we can reject the null hypothesis at 5% significance (two tailed). Therefore, data support asymmetry to a different extent according to the attribute: the asymmetry in money is more significant than in time. The results relating to the statistical significance of the coefficients need to be viewed in the light of the assumption of independence of observations made for estimation. Correlations across responses of the same

individual have not been taken into account. This has a potential to bias the significance of the estimated coefficients (Hess et al., 2008).

We then carried out comparisons between the conventional and the asymmetric model. As goodness-of-fit measure the rho-squared adjusted takes into account the different number of parameters in the two specifications. The rho-squared adjusted of the asymmetric model is higher than the rho-squared adjusted of the conventional model. We also tested model specification. The test proposed by Horowitz (Ben-Akiva and Lerman, 1985) was used to reject the conventional model. The test is used when one model is not a nested hypothesis of the other. The null hypothesis is that the conventional model is the correct specification. We obtained that under this hypothesis the probability that the difference in the rho-squared adjusted obtained from the sample $(0.4393-0.4255=0.0138)$ would be exceeded is less than 0.001. Thus the conventional model is almost certainly incorrect and the asymmetric model is the one to merit further consideration.

The values of time are in Table 2. For the conventional model we have a single value of time (*VOT*), given by the ratio of the time coefficient to the money coefficient, of 4.25 EUR/h. From the asymmetric model we have the four measures in eqns (18) and (19). Given that loss aversion is satisfied for both time and money ($|\beta_L| > |\beta_G|$), the four measures satisfy the inequalities (20). The willingness to pay (*WTP*) has the lowest value of 3.78 EUR/h, the willingness to accept (*WTA*) the highest value of 5.88 EUR/h. The two equivalent measures (*EG*, *EL*) have intermediate values between *WTP* and *WTA*.

Table 2. Values of time (EUR/h)

conventional logit	asymmetric logit			
	time gains		time losses	
<i>VOT</i>	<i>WTP</i>	<i>EG</i>	<i>WTA</i>	<i>EL</i>
4.25	3.78	5.05	5.88	4.40

4.2 Application to the town bypass case

4.2.1 The policy case

We consider a two-link network representing a town centre route and a bypass route. We assume that users have reference-dependent demand functions given by the asymmetric model estimated in the previous section. We assume a total demand of 1200 veh/h. For supply, BPR time-flow functions derived empirically for similar routes are used. The functions (in hours) are $T = 0.057[1 + (f/800)^{5.2}]$ for the town centre route, and $T = 0.045[1 + 0.68(f/1230)^{4.6}]$ for the bypass route.

In a do-nothing scenario there is only the town centre route. In this scenario the route works in congested conditions. Table 3 shows that travel time is more than 30 minutes. The do-something scenario consists in the construction of a bypass. The aim of the application is to assess tolling policies for the bypass. Two policies are compared. The first (P1) consists in opening the bypass and, at the same time, charging a toll of 1 EUR for its use. The second (P2) consists in a two-stage policy. In stage 1 the bypass is opened and nothing is charged. Only in stage 2 a toll of 1 EUR is charged on the bypass. In the next section we show that with reference-dependent models the two tolling policies bring about different results, i.e. we have policy-path dependence.

4.2.2 Equilibrium

We calculate the RDSUE for policy P1 and for the two stages of policy P2 by solving the fixed-point problem (15). Table 3 shows the results of the calculation. For each link the flow and time at equilibrium are shown.

Table 3. RDSUE results in the town bypass case

	town centre route		bypass route		toll revenues (EUR)	time spent (hours)
	flow (veh/h)	time (min)	flow (veh/h)	time (min)		
<i>do-nothing</i>	1200	31.6	-	-	0	632
<i>policy P1: bypass opened with toll=1EUR</i>	879	9.0	321	2.7	321	146
<i>policy P2: bypass opened without toll (stage 1) + toll=1EUR on bypass (stage 2)</i>						
stage 1	563	4.0	637	2.8	0	67
stage 2	867	8.6	333	2.7	333	139

It is essential to note the differences in the reference points. The assumption is that users adopt as reference the status quo, i.e. the current choice and network state. For P1 and for stage 1 of P2 the reference is the do-nothing scenario. The RDSUE is in these cases a single-class problem because in the reference we find the total of demand on the town centre route only. Conversely, for stage 2 of P2 the reference is the RDSUE of stage 1. The RDSUE is in this case a two-class problem because in the reference we find part of the demand on town centre route (class 1), part on the bypass route (class 2). Both classes participate in the equilibrium according to their respective demand functions and are assigned to the two routes. One may say that users on the town centre route would not participate in the equilibrium because the policy consists in making the other route even less attractive with a toll. This is, however, not correct because the travel time on the town centre route increases as some users from the bypass divert to it. Therefore, both classes of users take part in the equilibrium.

We note that although P1 and stage 2 of P2 represent the same intervention, i.e. opening of the bypass with a toll of 1 EUR charged for its use, the RDSUE of P1 is different from the RDSUE of P2 stage 2. The equilibrium flows and times in P1 are different from those in stage 2 of P2. The reason of the differences lies in the difference of the reference point. In contrast, with a conventional SUE the reference point does not matter and the route choice depends only on the final states in terms of time and money expenditure. Therefore, with a conventional SUE, P1 and P2 would lead to the same equilibrium.

We assess the policies based on two criteria: a financial criterion quantified by the toll revenues and a social criterion quantified by the total time spent on the network. One represents the point of view of the bypass operator, one the point of view of society. As the RDSUE is different between P1 and stage 2 of P2, the assessment of P1 will be different from that of P2. The sign of these differences are as follows.

Table 3 shows that P2 is, at its end, superior both in terms of toll revenues, which are higher than in P1, and of total time spent, which is lower than in P1. In P2 the number of users choosing the bypass at the end of stage 2 is higher than the number of users choosing it in P1. This explains the superiority of P2 in terms of revenues. In P2 less users than P1 are found in the town centre route which is the route where the time spent is highest. This explains the superiority of P2 in terms of total time spent. The implication is that it is advantageous to charge the toll on the bypass only in a later stage, after the bypass is opened.

We note that the sign of the differences in the number of users choosing the two routes between P1 and P2 is a consequence of loss aversion embodied in the route choice model. To see this consider the bypass route. In P1 the RDSUE flow is 321 veh/h resulting from the probability of choosing that route times total demand: $0.2675 \times 1200 = 321$. In P2 stage 2 the RDSUE flow is 333 veh/h resulting from the sum of the probability of choosing that route by the first class times the users of the first class and the probability of choosing that route by the second class times the users of the second class: $0.275 \times 563 + 0.279 \times 637 = 333$. We see that the probabilities of choosing the bypass for both classes are higher than the probability of choosing it in P1 (0.275 and 0.279 versus

0.2675). The probability depends on the difference of the utilities between the two alternatives. In P1 and P2 stage 2 the money component of this difference is the same (because we have no toll in one route and 1EUR toll in the other in both policies). What makes the difference between P1 and P2 is the time component. In P1 both alternatives gain on the reference and the difference in their utilities is $\Delta(P1) = \beta_{GT} \cdot (31.6-2.7) - \beta_{GT} \cdot (31.6-9) = \beta_{GT} \cdot 6.3$. In P2 the bypass gains and the town centre route loses for both classes. Taking for instance class 2 we have: $\Delta(P2) = \beta_{GT} \cdot (2.8-2.7) - \beta_{LT} \cdot (8.6-2.8) = \beta_{GT} \cdot 0.1 - \beta_{LT} \cdot 5.8$. Since $\Delta(P2) > \Delta(P1)$ this is possible only if $|\beta_{LT}| > |\beta_{GT}|$, i.e. only under loss aversion in the time attribute. Stated in words, when in P2 after stage 1 we charge a toll, some users on the bypass will change back to the town centre but loss aversion affects their willingness to change as they incur a loss. A similar argument holds for the users of class 1. This explains why we find more users on the bypass at the end of P2 than in P1.

4.2.3 Sensitivity analysis

The results obtained indicate that the differences between the two policies P1 and P2 are not large. From Table 3 we see that the percentage difference in the flow of the bypass route between P1 and P2 is only 3.7% (321 vs 333 veh/h). The percentage difference in toll revenues equals the percentage difference in flow. Therefore, from a financial standpoint the two policies differ by a mere 3.7%. With conventional, non-reference models, there would be no difference at all between P1 and P2. With reference-dependent models there is a difference which, however, is small. We conducted a sensitivity analysis with the aim of finding conditions where the difference is accentuated.

We explored the sensitivity to total demand (in the range between 600 and 1400 veh/h) and to toll (considering 2 EUR and 3 EUR toll). Changing either of these two parameters all other things being equal, we found that the difference in the bypass flow between P1 and P2 remains in the range of few percentage units.

We also explored the sensitivity to the degree of loss aversion in the time attribute, i.e. the ratio β_{LT} / β_{GT} between the loss and the gain coefficient. From the calibration based on the data used here the degree of loss aversion is 1.16. This value is low when compared with the results obtained by Hess et al. (2008). They found for the two demand segments considered a degree of loss aversion in the free flow time attribute of 1.49 and 2.44. This comparison suggests that it is meaningful to explore the sensitivity to the degree of loss aversion.

The results of this analysis for the range of β_{LT} / β_{GT} between 1.16 and 3 are shown in Figure 3 (β_{LT} is changed, β_{GT} is unchanged). The percentage difference in the bypass flow, and in toll revenues, between P1 and P2 shows a sharp sensitivity to the degree of loss aversion in the time attribute. This difference rises from 3.7% when $\beta_{LT} / \beta_{GT} = 1.16$ to 30% when $\beta_{LT} / \beta_{GT} = 3$. The percentage difference in total travel time between P1 and P2 is shown in Figure 4. Percentage values are negative because the total time in P2 is lower than in P1. The absolute value of the difference rises from 4.4% when $\beta_{LT} / \beta_{GT} = 1.16$ to 29.7% when $\beta_{LT} / \beta_{GT} = 3$. These results suggest that when the loss aversion effects at the level of demand are stronger, as it has been found in other data sets, the adoption of reference-dependent models does make a difference in terms of equilibria and policy implications compared with conventional models.

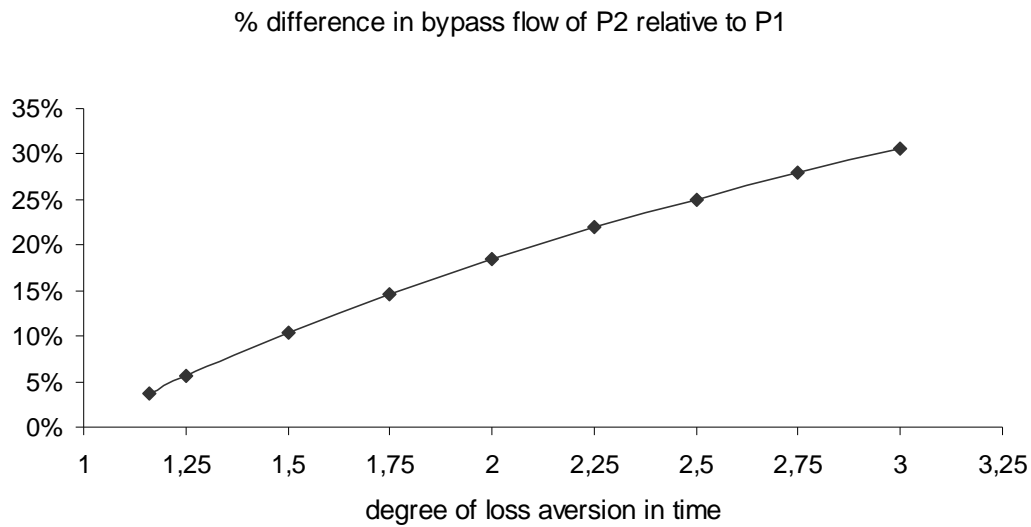


Figure 3. Sensitivity analysis on the degree of loss aversion in time: bypass flow

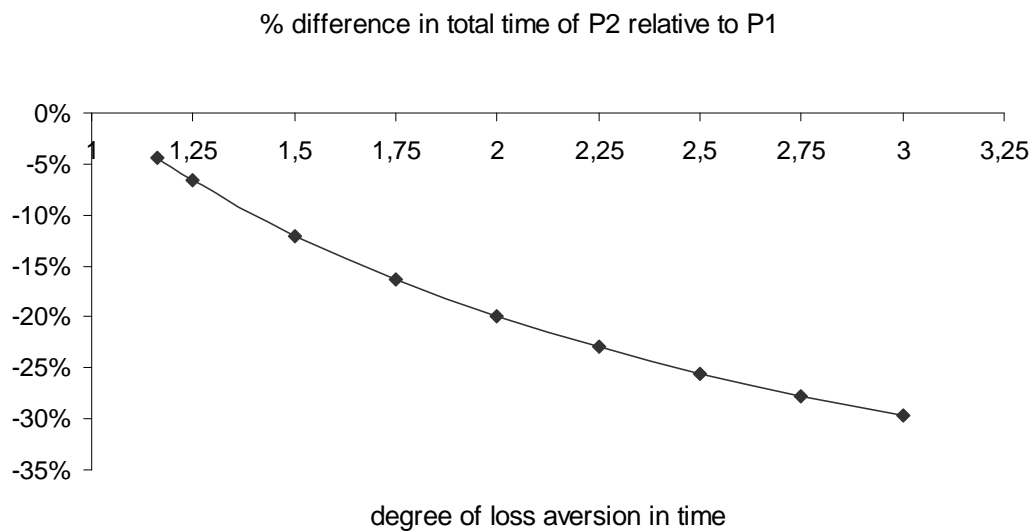


Figure 4. Sensitivity analysis on the degree of loss aversion in time: total time

4.2.4 Valuation of time changes

Reference-dependence makes it necessary to consider distinctly time gains (savings) and time losses. We have then two measures of the money value of the unit time gain (WTP , EG), and two measures of the money value of the unit time loss (WTA , EL), which can be used to provide multiple values of the time changes over the network.

The results of the time valuations for the town bypass case are shown in Table 4. The table provides a comparison between policy P1 and policy P2 based on the valuation of the travel time changes with respect to the do-nothing scenario. In both policies all users experience a time

saving on the do-nothing scenario. Both a total *WTP* value (i.e. a compensating measure) and a total *EG* value (i.e. an equivalent measure) of the time savings are provided. The table considers also stage 2 of P2 with respect to stage 1. This case is interesting because there are also time losses. Users on the bypass in stage 2 experience a time gain, users on the town centre route in stage 2 experience a time loss with respect to stage 1. The table shows the total *WTP* and *EG* values for gains, and the total *WTA* and *EL* values for losses. The total values are disaggregated by class according to eqns (21) and (22). This implies the assignment of the values of time changes distinctly to shifters and non-shifters.

Table 4. Time valuation in the town bypass case

	flow (veh/h)	time savings			time losses			
		hours	<i>WTP</i> value (EUR)	<i>EG</i> value (EUR)	hours	<i>WTA</i> value (EUR)	<i>EL</i> value (EUR)	
<i>P1 with respect to do-nothing</i>								
town centre route	879	331	1252	1672	-	-	-	
bypass route	321	155	584	781	-	-	-	
total	1200	486	1836	2453	-	-	-	
<i>P2 with respect to do-nothing</i>								
town centre route	867	337	1273	1700	-	-	-	
bypass route	333	155	584	781	-	-	-	
total	1200	492	1857	2481	-	-	-	
<i>P2 stage 2 with respect to P2 stage 1</i>								
town centre route	non shifting	408	-	-	-	32	186	139
	from bypass route	459	-	-	-	46	262	196
bypass route	from town centre route	155	3.3	12.3	16.5	-	-	-
	non shifting	178	0.25	0.9	1.3	-	-	-
total	1200	3.55	13.2	17.8	78	448	335	

5. Conclusions

The paper has considered reference-dependent models for stochastic user equilibrium and value-of-time analysis in networks of general topology. The modelling developments of the paper are motivated by the experimentally supported hypothesis that route choice is reference dependent. The analysis incorporates in probabilistic models asymmetric responses to gains and losses in travel time and money.

The analysis of the equilibrium is developed under the assumption of a decision-making process where users always adopt as reference point the status quo, i.e. the current choice and network state. This appears as the most appropriate assumption in a riskless setting where travel times are certain.

The RDSUE is formulated as a multiple user class equilibrium problem. In addition to existence and uniqueness, the property of reflexivity of RDSUE is considered. Reflexivity requires loss aversion for the coefficients in the route choice model: in a RDSUE the user is in the optimal choice when it is viewed from previous choice, and, under loss aversion, he has no convenience to change this choice when it is viewed from itself. The property holds if it is assumed that the user's stochastic terms do not shift when the reference point is updated. Reflexivity can be seen as an instance of a status quo bias, i.e. the retention of the status quo, an effect observed in a number of experiments in different contexts and explained in terms of loss aversion: the disadvantages of a move from the status quo are valued more heavily than the advantages.

A graphical construction of the four reference-dependent measures of the value of time, based on the indifference curves and new with respect to existing literature, is provided. A methodology for the valuation of travel time changes over a network based on these measures is developed.

The results of the estimation of the route choice model presented in the numerical part of the paper are in agreement with the body of econometric literature supporting reference-dependent theory. It is shown that the asymmetric logit outperforms the conventional logit, in terms of data fitting and correctness of the specification, thus supporting the supposition of asymmetric response. The relative magnitude of the coefficients supports loss aversion for both travel time and money, with individuals showing to be more loss averse in the money dimension than in the time dimension.

The application of the reference-dependent models to the town bypass case has served to identify the policy implications for tolling on the bypass that can be derived on the basis of the adoption of the reference-dependent approach to equilibrium analysis.

It is shown that the final state of the network depends on the policy path. Loss aversion justifies the advantages of the adoption of multi-stage policies, namely the introduction of a toll only in a later stage after the new infrastructure is opened. The advantages are small but become significant as the degree of loss aversion increases. The more marked the loss aversion effects embodied in the data used to calibrate the demand model, the more significant the difference made by the use of reference-dependent models in terms of equilibria and policy implications compared to conventional models.

The town bypass application has also served to illustrate the reference-dependent approach in the valuation of travel time changes over a network. It is shown that the assessment of a policy can be accompanied by a distinct valuation of time savings and time losses. These in turn can be valued according to a compensating or an equivalent measure having each a distinct interpretation. The multiplicity of the values of travel time changes is a potential source of ambiguities in policy assessment. This implication of the adoption of reference-dependent models is a reason of dismay among the economists' community.

Still in the riskless setting of certain travel times which has been tackled here, directions for further research can be identified.

One is the consideration of diminishing sensitivity in the systematic utility. This implies replacing the piecewise linear functions here with non-linear functions. The conditions that need to be satisfied to guarantee reflexivity of the equilibrium need to be investigated. Also there is a need to adapt the valuation of time changes. The approach based on the unit time values of the piecewise linear case is not applicable. The valuation of the time changes of a class of users will be dependent on the size of these time changes. The valuation will be obtained by imposing indifference conditions, as in eqns (17) and (19), distinctly for each class and path.

Another research topic relates to reference points when the reference-dependent models are applied under a long-range perspective where the OD demand varies. If it is assumed that demand increases over time, it will be possible within the framework outlined here to consider a further class of users for each OD pair constituted by the newly generated users. However, it remains unsolved the problem of how to set a reference point as for this class the status quo assumption is not applicable.

The modelling framework presented has assumed that the stochastic terms remain the same when users update the reference point to the new status quo. It has been explained why the assumption is justified but, given the absence of elements to make any definitive statement on this issue, consideration of other assumptions is of relevance. In the case the stochastic terms changed, the relative convenience of the alternatives may be altered and the property of reflexivity would not hold. Future research should consider different assumptions on the

dynamics of the stochastic terms and investigate the implications for the modelling of reference-dependent equilibrium.

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Appendix. Properties of RDSUE

A. Existence

We aim to prove that, under the assumptions in Section 2, a solution to the RDSUE fixed-point problem exists. This is established by applying the Brouwer's fixed point theorem to either the link-based formulation (15) or the path-based formulation (16). Continuity of all functions composed to form the fixed-point condition plus a feasible region which is non-empty, compact and convex are sufficient for existence. Continuity is satisfied under the assumptions of Section 2, therefore a solution exists.

B. Uniqueness

Adopting a path-based formulation as in Connors and Sumalee (2009) we have the following sufficient condition for uniqueness. First, define $\tilde{\mathbf{T}}$ the class-based expanded version of \mathbf{T} , i.e. $\tilde{\mathbf{T}}$ is the $[\Gamma \times 1]$ vector, ordered as \mathbf{P} and \mathbf{f} , such that for any class m and OD pair r we have $T_k^{m,r} = T_k^r, \forall k \in \Psi^r$.

The fixed-point solution to (16) is unique if the $\mathbf{P}(\tilde{\mathbf{T}})$ mapping satisfies:

$$\left(\mathbf{P}(\tilde{\mathbf{T}}_1) - \mathbf{P}(\tilde{\mathbf{T}}_2)\right)^T \cdot (\tilde{\mathbf{T}}_1 - \tilde{\mathbf{T}}_2) \leq 0 \quad \forall \tilde{\mathbf{T}}_1, \tilde{\mathbf{T}}_2 \quad (\text{B.1})$$

The proof is by contradiction. Suppose there are two distinct solutions \mathbf{z}_1 and \mathbf{z}_2 to the RDSUE problem. We have by eqn (8) that the link time-flow mapping is monotonically increasing, therefore:

$$\begin{aligned} (\mathbf{t}_1 - \mathbf{t}_2)^T \cdot (\mathbf{z}_1 - \mathbf{z}_2) &= (\mathbf{t}_1 - \mathbf{t}_2)^T \cdot (\mathbf{A} \cdot \Delta \cdot \mathbf{f}_1 - \mathbf{A} \cdot \Delta \cdot \mathbf{f}_2) = (\mathbf{t}_1 - \mathbf{t}_2)^T \cdot (\mathbf{A} \cdot \Delta) \cdot (\mathbf{f}_1 - \mathbf{f}_2) = \\ &= \left((\mathbf{A} \cdot \Delta)^T \cdot (\mathbf{t}_1 - \mathbf{t}_2)\right)^T \cdot (\mathbf{f}_1 - \mathbf{f}_2) > 0 \end{aligned}$$

from which we have:

$$\left(\tilde{\mathbf{T}}_1 - \tilde{\mathbf{T}}_2\right)^T \cdot (\mathbf{f}_1 - \mathbf{f}_2) > 0 \quad (\text{B.2})$$

The variational inequality (B.1) implies:

$$(\mathbf{f}_1 - \mathbf{f}_2)^T \cdot (\tilde{\mathbf{T}}_1 - \tilde{\mathbf{T}}_2) \leq 0$$

which gives a contradiction with (B.2). Therefore, if (B.1) is satisfied two distinct solutions cannot exist and the solution must be unique.

The question arises if the general assumptions made in Section 2 are sufficient to guarantee the condition (B.1). This is desirable but is, however, not the case.

To see this consider one class of an OD pair (the class and OD pair indexes are omitted for ease of notation). In the assumptions made we have that the variational inequality (12) is satisfied for the class as the probabilistic model is invariant. The inequality (12) is with respect to the path systematic utilities while the inequality (B.1) is with respect to the path travel times. In the case of a conventional model we would have the linear transformation $\mathbf{V} = -\alpha \cdot \tilde{\mathbf{T}} + \bar{\mathbf{V}}$, $\alpha > 0$, and, therefore, (12) would imply (B.1) for the class. By simple summation over classes and OD pairs we obtain that also (B.1) would be implied.

In contrast, in the reference-dependent case here \mathbf{V} is a different transformation of $\tilde{\mathbf{T}}$ and, for this reason, (12) does not imply (B.1) even if the transformation $\tilde{\mathbf{T}} \rightarrow \mathbf{V}$ is monotonic. This is because in general monotonicity is not preserved under composition: the composition of two monotonic mappings is in general not monotonic. In other words, while usually in most cases for any two points $\tilde{\mathbf{T}}_1$ and $\tilde{\mathbf{T}}_2$ the expression in the left-hand side of (B.1) for one class takes the negative sign, it is possible to find pairs $\tilde{\mathbf{T}}_1$ and $\tilde{\mathbf{T}}_2$ such that this expression takes the positive sign. This is ascertained by simple numerical inspection. Thus (B.1) is not implied by (12). Nevertheless, it cannot be excluded that under the assumptions of Section 2 the fixed point is unique.

C. Reflexivity

We aim to prove that, under the assumptions in Section 2, the RDSUE is reflexive. We adopt the hypothesis that each user retains her stochastic terms when the reference point is updated to the new status quo.

We establish the property for a generic class m of OD pair r (the class and OD pair indexes are omitted for ease of notation). The users of this class have reference point denoted by "0" and will be found on K path alternatives $k=1, \dots, K$ after the RDSUE is set. We prove reflexivity considering a generic path alternative i . The aim is to prove that if i is the path alternative with maximum utility in RDSUE, it is the alternative of maximum utility also if the reference point is changed from "0" to the reference point represented by alternative i in the RDSUE network state.

Consider the two sets:

$$\Omega_{i,0} = \{(\varepsilon_1, \dots, \varepsilon_K) : U_{i,0} > U_{j,0} \quad \forall j \neq i\}$$

$$\Omega_{i,i} = \{(\varepsilon_1, \dots, \varepsilon_K) : U_{i,i} > U_{j,i} \quad \forall j \neq i\}$$

We prove reflexivity by proving that $\Omega_{i,i} \supseteq \Omega_{i,0}$. Stated in words, the group of the users who choose path i is a set of the stochastic terms $(\varepsilon_1, \dots, \varepsilon_K)$. The proof consists in showing that all the users who in the RDSUE choose path i when the reference point is "0", choose path i when the reference is changed to path i in the RDSUE itself.

The sets $\Omega_{i,0}$ and $\Omega_{i,i}$ satisfy:

$$\Omega_{i,0} = \{(\varepsilon_1, \dots, \varepsilon_K) : \varepsilon_j < V_{i,0} - V_{j,0} + \varepsilon_i \quad \forall j \neq i\}$$

$$\Omega_{i,i} = \{(\varepsilon_1, \dots, \varepsilon_K) : \varepsilon_j < V_{i,i} - V_{j,i} + \varepsilon_i \quad \forall j \neq i\}$$

We prove that $\Omega_{i,i} \supseteq \Omega_{i,0}$ by proving that the following inequality holds $\forall j \neq i$:

$$V_{i,i} - V_{j,i} \geq V_{i,0} - V_{j,0} \quad (C.1)$$

We have:

$$V_{i,0} - V_{j,0} = AT + AM + BT + BM$$

$$V_{i,i} - V_{j,i} = CT + CM$$

where:

$$AT = \beta_{GT} \cdot GT_{i,0} + \beta_{LT} \cdot LT_{i,0}$$

$$AM = \beta_{GM} \cdot GM_{i,0} + \beta_{LM} \cdot LM_{i,0}$$

$$BT = -\beta_{GT} \cdot GT_{j,0} - \beta_{LT} \cdot LT_{j,0}$$

$$BM = -\beta_{GM} \cdot GM_{j,0} - \beta_{LM} \cdot LM_{j,0}$$

$$CT = -\beta_{GT} \cdot GT_{j,i} - \beta_{LT} \cdot LT_{j,i}$$

$$CM = -\beta_{GM} \cdot GM_{j,i} - \beta_{LM} \cdot LM_{j,i}$$

(C.1) is proved by showing that the following holds:

$$CT + CM \geq AT + AM + BT + BM$$

for which it is sufficient to consider time terms and prove that

$$CT \geq AT + BT$$

as a similar argument holds for money terms.

The different occurrences are shown in the following table where G and L denote gains and losses with respect to the reference time T_0 of, in order, T_i and T_j .

CT	$AT+BT$	result	
$T_j > T_i$			
	GG	$\beta_{GT} \cdot (T_j - T_i)$	$CT > AT+BT$
$-\beta_{LT} \cdot (T_j - T_i) > 0$	GL	$-\beta_{LT} \cdot T_j + \beta_{LT} \cdot T_0 + \beta_{GT} \cdot (T_0 - T_i)$	$CT > AT+BT$
	LG	-	-
	LL	$-\beta_{LT} \cdot (T_j - T_i)$	$CT = AT+BT$
$T_j < T_i$			
	GG	$\beta_{GT} \cdot (T_j - T_i)$	$CT = AT+BT$
$-\beta_{GT} \cdot (T_i - T_j) < 0$	GL	-	-
	LG	$-\beta_{GT} \cdot (-T_j) - \beta_{GT} \cdot T_0 + \beta_{LT} \cdot (T_i - T_0)$	$CT > AT+BT$
	LL	$-\beta_{LT} \cdot (T_j - T_i)$	$CT > AT+BT$

The result in the right-hand column of the table is consequence of the loss aversion assumption for the coefficients, i.e. $|\beta_L| > |\beta_G|$. This establishes the property.