Investigation of Wind and Temperature Dependence for Dimensioning of Laminated Inserts

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This study proposes a review of different design approaches allowing to dimension a laminated insert connection. It focuses on façade assemblies, where the critical load is due to wind, and where the stiffness relies on a highly viscoelastic material SentryGlas®. The high variability of material stiffness is considered using three different approaches. The first approach determines critical wind speeds and temperature values using a site-specific probabilistic analysis. The second approach uses the First Order Reliability Method to perform a probabilistic design. The final approach suggests a method for evaluating the wind and temperature dependence. This method is based on the Monte-Carlo simulation method and shows potential for optimizing the dimensioning of such façade connections.

Keywords: Structural glass, Laminated inserts, Structural reliability, Monte-Carlo simulation

1. Introduction
Recent developments in the structural glass industry have led to the use of increasingly large structural glass elements. Specifically, improved laminating techniques have allowed designers to use more complex glass build-ups: larger dimensions and more layers of glass. A comparison of the reverse pyramid of the Louvre (1993) and the Apple Store 5th Avenue (2012) demonstrates clearly the advances in structural glass design. The behaviour of large structural glass elements depends widely on the stiffness of the interlayer material. Earlier interlayers, such as Poly-vinyl Butyral (PVB) were soft and highly dependent on temperature. However, developments within the adhesive industry have allowed designers to use stiffer and more stable interlayer materials, including ionomers such as SentryGlas®.

These innovations have influenced the design of structural glass connections. Now, it is possible to rely on the interlayer stiffness to transfer forces between structural glass elements. This technique, known as laminated inserts, consists of metal pieces embedded in the structural glass elements. It has several advantages compared to some more classic connections:

- Aesthetically: flush surface and reduced visual impact.
- Practically: Reduced work on site
- Mechanically: less stress concentration, even distribution of stresses in glass and clearer load paths.

![Fig. 1 Assembly of the reverse Pyramid of the Louvre Museum (left), Laminated insert connection (right).]
Working with laminated inserts, specifically SentryGlas®, remains a challenge. Its complex mechanical behaviour makes it difficult for designers to fully utilise the structural properties of the interlayers. In this article we suggest a method allowing to better account for the structural properties of SentryGlas® and refine the design of a typical laminated insert.

2. Context

2.1. SentryGlas® material

Adhesives have been used in the construction industry for many years; epoxy resins have been used in concrete structures since the seventies. In glass applications, structural silicone glazing has been used since the eighties. However, SentryGlas® mechanical behaviour differs from these two adhesives. The difference of behaviour between these different adhesives is due to the molecular structure of the adhesive.

Adhesive materials are made of polymers. The interactions between these long molecules influence the overall mechanical behaviour of different adhesives: Epoxy resins are made of polymers which have high crosslink ratios, Structural silicones have a lower crosslink ratio. Finally, SentryGlas® has the lowest crosslink ratio. These differences in polymers are reflected in the mechanical behaviour. Epoxy resins are the stiffest adhesives; they have an elastic behaviour. Structural silicones have a hyper-elastic behaviour. Finally, the low crosslink ratio of SentryGlas® explains their visco-elastic behaviour.

![Molecular structure of different types of adhesives](Fig. 2 Molecular structure of different types of adhesives (M.Overend, 2012).)

Visco-elastic materials are complex to use in structural applications as they are subject to creep. Specifically, the SentryGlas® glass transition temperature is 42°C, a temperature which is reached on many glass façades. In practice it is necessary to perform an analysis accounting for different load durations and different temperatures. This complex behaviour of SentryGlas® means that conservative assumptions on mechanical properties are made during the design. For instance, the European technical agreement on SentryGlas® imposes a Young modulus $E = 100 \text{ MPa}$ for wind load, well below the mechanical properties given by the manufacturer ($E = 612 \text{ MPa}$, for $T = 20 \degree\text{C}$, for a load duration of 3 seconds). This is because design codes do not give precise information on how to consider the influence of temperature on the resistance of SentryGlas®.

<table>
<thead>
<tr>
<th>Load type</th>
<th>$E$ [MPa]</th>
<th>$G$ [MPa]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Wind load</td>
<td>100</td>
<td>33</td>
</tr>
<tr>
<td>Snow load</td>
<td>10</td>
<td>3.3</td>
</tr>
</tbody>
</table>

Table 1: SentryGlas® mechanical properties according to Document Technique d’Application 6/15-2253_V1.

2.2. Design approach

The Eurocode accounts for the uncertainty in the mechanical resistance of materials by providing partial factors. Similarly, the uncertainty on variable loading, such as wind, is taken into account by sets of standard values. The coincidence of climatic events is accounted for by load factor combinations. This framework is the basis of the so-called semi-probabilistic approach. This approach involves a fundamental assumption: the probabilistic independence of climate loads. However, this assumption can be examined in the case of dimensioning a visco-elastic material subject to wind load, such as a laminated insert in a façade. It is likely that the critical wind load would correspond to a storm in autumn and winter. In contrast the reduced resistance of the connection would occur at high temperatures, most likely in summer. Therefore, in a typical Eurocode approach, assuming the independence of climatic events amounts to considering that two incompatible events could occur simultaneously.
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In this analysis we suggest different approaches to account for the wind and temperature dependence during the design of a laminated insert. First, we will apply a semi-probabilistic approach consistent with the Eurocode procedure. Then we will apply a probabilistic approach based on the First Order Reliability Method. Finally, we will carry out an analysis accounting for the dependence of climate loads, using a Monte-Carlo simulation method.

2.3. Wind and temperature analysis

A first preliminary analysis is carried out to assess the potential of accounting for wind-temperature dependence during the design of a laminated insert. A study of the wind-temperature dependence was carried out for four different climate typologies. Weather data was gathered from the Energy+ network with several climates considered: a polar climate was represented by Reykjavik (Iceland), a temperate climate was represented by Brest (France), a sub-tropical climate was represented by Oklahoma City (USA) and finally a tropical climate was represented by Fort-De-France (France). The correlation between wind and temperature was assessed by plotting daily maximum wind speeds and temperatures together.

The observation of the plots confirms that, in temperate regions, high wind and temperature values do not occur at the same time. On the contrary, it may not be true for the other climate systems. Therefore, the analysis will be restricted to temperate regions. It should also be noted that Energy+ weather files only covers a year of weather data, which is not enough to study extreme climates. More extensive weather data would be needed to assess critical climate loads.

![Fig. 3 Annual weather data for four different locations representative of different climate systems. Each point corresponds to the daily maximum value of temperature and wind.](image)

3. Assessing load and resistance

To study the influence of wind and temperature dependence on the dimensioning of a laminated insert, a specific example is chosen. This example consists of a splice between two glass panels. Splicing of the glass panels is achieved by laminated inserts. To dimension this connection, it is necessary to assess: first, the wind load the insert must withstand and secondly, the stiffness of the insert which is determined by the glass temperature. Dimensioning the connection consists of checking that the following inequality is verified for a given level of reliability:

\[
F_{Ed}(W) < F_{Rd}(T)
\]  

(1)

In the next part, the procedure allowing to assess wind and temperature loading is summarized.

3.1. Weather data

In the Eurocode, the assessment of climate loads on structures is based on a Bayesian approach. This approach consists of assessing the behaviour of a given climate load based on previously sampled data. Then parameters of this sample allow the calculation of climate loads using a probability density. Finally, critical loads are determined based on this probabilistic model. This ensures a reliable design framework.

3.1.1. Wind loads

The probabilistic model used to evaluate wind loads is the Gumbel distribution (Generalized Extreme Value Distribution Type-I) given by the following expression:
The parameters of this distribution, noted $\alpha_w$ and $b_w$, can be linked to the previously sampled wind data using the following equations:

$$E(W) = a_w + 0.5772b_w ; \sigma_w = \frac{\pi}{\sqrt{6}}b_w$$

(3) (4)

Where $E(W)$ and $\sigma_w$ are respectively the mean value and the standard deviation of the wind sample.

Finally, using this model of wind extreme values, it is possible to determine a critical value corresponding to a given level of reliability. In the Eurocode, the characteristic wind value is noted $v_{b,0}$ and corresponds to a wind value that has a 2% chance of occurring every year (i.e.: that will happen on average once every 50 years). Calculation of this value based on the Gumbel law is consistent with the value given by the NF EN 1991-1-4 :2005 for the corresponding area.

<table>
<thead>
<tr>
<th>$v_{b,0}$ [m/s]</th>
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<tbody>
<tr>
<td>NF EN 1991-1-4 :2005</td>
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<tr>
<td>Gumbel law</td>
</tr>
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</table>

3.1.2. Temperature loads

A similar analysis can be carried out to assess the resistance of the insert. Extensive tests of laminated inserts mechanical properties were carried out by M.Santarsiero (2015). A failure prediction modelled was developed and will be used here to determine the resistance of the connection (5).

$$F_{Rd} = f_v \frac{1}{\alpha_v} \alpha_y \alpha_T(T) A$$

(5)

In this expression, $f_v$ is the reference shear resistance for SentryGlas® (16.26 [MPa]), $\alpha_v$ is a stress factor taken as 1.06, $\alpha_y$ is the strain variation factor taken as 1.25, $A$ the adhesive surface of the laminated connection. Finally, $\alpha_T(T)$ is the temperature factor given by equation (6) and accounts for the visco-elastic behaviour of SentryGlas®.

$$\alpha_{T,SG}(T) = 0.32 \left( 1.94 + \tanh \left( \frac{28.17-T}{6.89} \right) + \tanh \left( \frac{25.27-T}{38.3} \right) \right)$$

(6)
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Given the particularly low transition temperature of SentryGlas®, the stiffness of the laminated insert varies significantly within the range of service temperatures: from a few hundred MPa at low temperatures, to barely a dozen MPa at high temperature. Therefore, to calculate the resistance of the insert it is necessary to evaluate the SentryGlas® temperature.

Glass temperature is influenced by many physical phenomena. In this analysis, the procedure defined in the DTU39 – Part 3, the reference French norm on glass temperature calculation, was applied. This norm applies a so called ‘Finite difference method’ to evaluate glass temperature. This method assumes that each physical phenomenon influencing glass temperature can be represented by a heat flow which is modelled as a simple product of the difference of temperature and the thermal conductivity. Then calculating the thermal equilibrium of a given glass build-up allows the relation between glass temperature, the parameters of the glass build-up and the parameters of the environment to be determined. It can be shown that the glass temperature is given by the following equation:

\[ T_{\text{glass}} = a\Psi + bT + c \]  

(7)

With:
- \( \Psi \) [W] the solar radiation received by the glass
- \( T \) the exterior temperature [K]
- a, b and c, parameters depending on the glass build-up and assumptions on thermal conductivities.

This gives a method to calculate for a given glass build-up the temperature of the SentryGlas® as a function of solar radiation and exterior temperature.

Then, a similar approach to that of wind loads can be carried out for assessing critical temperature values. Temperature weather data consist of daily maximum ambient temperature values. The influence of solar radiation is accounted for using a site-specific radiation analysis. This radiation analysis is carried out using the parametric tool, Ladybug plugin for Grasshopper in Rhino 3d software. The design example has a North-East orientation. Conservatively it is assumed that the daily maximum radiation value occurs simultaneously with the recorded ambient temperature. Using the ambient temperature data and the glass temperature model (Equation 7) a statistical distribution of the glass temperature can be determined.
Similarly, to the previous wind load analysis, critical temperature can be evaluated using the glass temperature data. A normal distribution law is chosen to model the temperature variations.

\[
f_T(t) = \frac{1}{\sigma_T \sqrt{2\pi}} e^{-\frac{1}{2} \left(\frac{t-a_T}{\sigma_T}\right)^2}; \quad E(X) = a_T; \quad \sigma_T = b_T
\]  

(8)

4. Assessing load and resistance

A first approach, consistent with the Eurocode is presented here.

4.1. Calculation of the design combination

A combined analysis of wind and glass temperature is carried out to determine the critical combination of wind, i.e. the load acting on the connection, and the glass temperature i.e. the resistance of the connection. This is done by generalizing in two dimension the analysis carried out earlier to determine the critical wind load.

First by assuming the independence of wind and temperature it is possible to determine a joint probability law given by equation (9). As seen earlier, in temperate regions, this assumption might be overly conservative:

\[
f_{W,T}(w, t) = f_W(w)f_T(t)
\]  

(9)

Then a level of reliability, consistent with the Eurocode approach, is chosen (i.e.: a 2% chance of failure per year). The intersection of this criteria with the joint probabilistic model determine an iso-probabilistic curve corresponding to all the combined wind and temperature events with a 2% chance of occurring every year. Then, the concavity of the iso-probabilistic curve allows to determine an envelope that encompasses the curve. This envelope is determined by a set of tangents: one corresponding to the maximum wind value, a second one corresponding to the maximum temperature value and a third intermediate tangent. The intersection of these tangents creates two points which define the design combinations. Each of these combinations correspond to a predominant load associated with a secondary load.

| Table 3: Critical wind and temperature combinations. |
|-----------------|-----------|
| **W [m/s]**     | **T[^oC]**|
| Design values   | 23.9      | 48.1      |

Fig. 7 Normal probability density of glass temperature.

Fig. 8 Determination of design combination for winds and temperatures values.
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4.2. Application of the design combination

These combinations are now applied to a design example taken from an ongoing glass box project. This glass box consists of four glass walls supported by a stiff concrete structure. The walls are made of 12 large glass panels (7.5m x 2.7m). A splice between the panels allows to create a beam action that resists lateral wind pressure (figure 9). The splice consists of a laminated insert. The load on the connection is due to wind pressure and can be easily calculated. The resistance of the connection is given by the previously mentioned failure prediction model (equation 5) and depends on the glass temperature.

![Fig. 9 Laminated insert splice.](image)

The application of the previous load combinations to the design example shows that the temperature dominant combination is more critical than the wind dominant one. The adhesive surface of the insert is calculated as 2300 mm².

<table>
<thead>
<tr>
<th>Table 4: Critical wind and temperature combinations.</th>
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</thead>
<tbody>
<tr>
<td>Combination</td>
</tr>
<tr>
<td>Design Combinations</td>
</tr>
<tr>
<td>$W + 0.6T$</td>
</tr>
<tr>
<td>$T + 0.6W$</td>
</tr>
</tbody>
</table>

5. First Order Reliability Method

The approach developed in the previous section corresponds to the semi-probabilistic method developed in the Eurocode. The uncertainty on the climate loads and on the materials resistance is absorbed in sets of critical values and design combinations. However, it is possible to carry out the design accounting for the variability of the loads applied to the connection and the variability of the resistance of the laminated insert itself. Such analysis is now carried out by implementing the First Order Reliability Method (FORM).

In this approach, the load and the resistance are modelled as random variables to which a probability density can be associated. Dimensioning the laminated insert is done by making sure the failure probability of the connection matches the failure probability rates defined by the Eurocode 0.

The failure of the connection is defined by the event ‘Resistance inferior to Action’. This probability is calculated as a double integral over the failure domain.

$$P_f = P(F_R < F_A)$$

$$P_f = \iint_{R < A} f_R(r)f_A(a)\,dr\,da$$

The Eurocode 0 sets failure probability according to the type of structure. Buildings are classified according to three ‘Consequence Classes’: if consequences in case of failure are low, the building would be associated with the CC1 class (storage buildings, greenhouses). If the consequences of a failure are high the building would be in the CC3 class (Concert halls). Finally, if the consequences in case of failure are moderate, the building would belong to the CC2 category (Office buildings). With each of these buildings, a reliability index is associated. This index is then associated to a probability of failure. A typical building, with moderate consequences of failure (CC2) would be associated with a failure probability defined by $P_f = 10^{-4}$. 
In the case of our façade connection, the load variability is due to the variation of wind pressure and the variation of resistance is due to the variation of the temperature. The previously defined probability density function associated to wind (Gümbel law) and temperature (Normal law) can therefore be used to model the load and the resistance of the connection. The failure probability is then calculated in a wind-temperature space, different from the load-action space defined previously. The definition of the failure domain is found by solving equation (12), which associates to any temperature a maximum wind speed corresponding to the failure of the insert.

\[ F_R(T) - F_A(V_{\text{lim}}) = 0 \]  

(12)

The dimensioning process consists in calculating the failure probability and iterating the adhesive surface according to a target failure probability of \( P_f = 10^{-4} \). Failure probability is calculated numerically. Result of the iteration process shows a necessary adhesive surface \( A = 2100 \, \text{mm}^2 \) which is consistent with the area previously calculated according to the semi-probabilistic approach: the FORM analysis confirms that the dimensioning according to the previous combinations matches the levels of reliability defined by the Eurocode.

Table 5: FORM results.

<table>
<thead>
<tr>
<th>Method</th>
<th>( A , [\text{mm}^2] )</th>
</tr>
</thead>
<tbody>
<tr>
<td>FORM</td>
<td>2100</td>
</tr>
</tbody>
</table>

6. Monte-Carlo simulation method

In the last part we implement a design approach using the Monte-Carlo method to calculate the failure probability of the laminated insert. Monte-Carlo simulation method is a computational approach, that relies on repeated random sampling. By simulating a large number of events, the Monte-Carlo method allows to calculate a probability of failure. It considers the dependence of wind and temperature by setting up a specific simulation process.

6.1. Simulation process

To account for the dependence of wind and temperature, the set of wind and temperature data is divided according to different ranges of temperature. This allows the definition of two limit cases. A first limit case consisting of the 25% lowest temperatures associated to the corresponding wind values. A second limit case consisting of the 25% higher temperatures and their corresponding wind values. The first limit case represents typical winter wind values. The second limit case represent typical summer wind values. It is then possible to define two limit Gümbel laws associated to each of these subsets.

The Monte-Carlo simulation process consist of the following steps:

- First a temperature value is chosen randomly, using the normal law previously defined.
- Secondly, according to the range of the chosen temperature, a wind value is randomly chosen. If the temperature value is within the lowest 25% range of temperature, the Gümbel law corresponding to the first limit case is used. If the temperature value is in the higher 25% range, the Gümbel law corresponding to the second limit case is used. If the temperature value is between the two limit cases, a Gümbel law with intermediate parameters is used. These parameters are interpolated between the parameters calculated for the two limit cases.
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The Monte-Carlo simulation is set-up in a python script. Build-in libraries (such as random.py) allow to generate random values according to a given probability distribution. Such programs are based on well-established pseudorandom number generator, such as the Mersenne Twister generator.

Comparison between the simulated data and the sampled data shows that the Monte-Carlo simulation accounts for the dependence of wind and temperature. Specifically, it allows the removal of the overestimated cases of high temperature values coinciding with high wind values (figure 12a).

Then, similarly to the FORM approach, the probability failure of the connection can be calculated by defining a failure limit above the Wind-Temperature space. The simulation of a large number of temperature and wind values allows us to estimate the probability of failure by counting the number of points above the failure limit. The probability of failure is given by the ratio of the points in the failure domain to the total number of points (12).

\[
P_f = \frac{N_{\text{fail}}}{N_{\text{tot}}} \tag{12}
\]

Given the randomness of the simulation process, the algorithm must be run several times, the probability of failure can then be estimated by calculating the average of the values calculated by each simulation. The process is found to be relatively stable for \( N_{\text{tot}} = 10^6 \) points. The simulation is carried out 10 times in order to get a correct estimate of the probability of failure variability.

The simulation process is carried out with the previous façade connection. With an adhesive surface \( A = 2100 \text{ mm}^2 \), the probability of failure is found to be lower than \( 10^{-5} \), well below the failure probability calculated without accounting for the wind and temperature dependence. Given that the target probability of failure is \( 10^{-4} \), the adhesive surface can be reduced to match the level of reliability of the Eurocode.

The simulation process is iterated, and the adhesive surface reduced until the failure probability matches the target level of reliability. The adhesive surface resulting from this process is \( A = 1200 \text{ mm}^2 \). This is 40% less compared to the previous approach, meaning that the connection visual impact could be significantly reduced thanks to this process.

<table>
<thead>
<tr>
<th>Adhesive surface</th>
<th>( P_f )</th>
<th>( \pm )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( A = 2100 \text{ mm}^2 )</td>
<td>( 0.4 \times 10^{-5} )</td>
<td>( \pm 0.08 \times 10^{-5} )</td>
</tr>
<tr>
<td>( A = 1200 \text{ mm}^2 )</td>
<td>( 0.94 \times 10^{-4} )</td>
<td>( \pm 0.03 \times 10^{-4} )</td>
</tr>
</tbody>
</table>

Fig. 11 a, b) Limit case Gumbel laws c) Interpolation of the Gumbel law parameters.

6.2. Dimensioning according to Monte-Carlo simulation method

The simulation process is iterated, and the adhesive surface reduced until the failure probability matches the target level of reliability. The adhesive surface resulting from this process is \( A = 1200 \text{ mm}^2 \). This is 40% less compared to the previous approach, meaning that the connection visual impact could be significantly reduced thanks to this process.

Table 6: Monte-Carlo simulation results.

Fig. 12 a) Application of Monte-Carlo simulation method for different adhesive surfaces b) Monte-Carlo simulation results.
7. Conclusion and further research
The reliability analysis carried out in this study only accounts for the dependence of the wind and temperature. Although, many other variable actions could be accounted for in order to complete the analysis. An analysis accounting for the dependence of solar radiation with temperature and wind could further improve the proposed design.

Also, further variable factors should be accounted for in order to complete the reliability assessing: SentryGlas® resistance, fabrication process etc. In practice, the effective area of the insert is reduced to account for delamination at the edge of the insert. This was not considered in this study.

This study shows the potential of Monte-Carlo simulation method and other probabilistic tools to refine and improve the design of a laminated insert. In our design example, we managed to reduce by 40% the dimension of the insert calculated according to the Eurocode approach. This type of analysis is available thanks to simple computational tools and allows engineers to solve relatively complex statistical problems usually accounted for by the design codes.

8. Acknowledgements
The authors would like to thank Arthur Lebée, researcher at the Laboratoire Navier, for his support on structural reliability analysis. The authors would also like to thank Jack Suddaby and Alban Berubé for their support on the comprehension of climate effects on structures.

9. References